Clear Thinking in an Uncertain World: Human Reasoning and its Foundations

Lecture 3

Eric Pacuit

Department of Philosophy
University of Maryland, College Park
pacuit.org
epacuit@umd.edu

September 11, 2013

How to think about logical reasoning

"In fact, logic is very much domaindependent in the sense that the valid schemata depend on the domain in which one reasons, *with what purpose*.

How to think about logical reasoning

“In fact, logic is very much domaindependent in the sense that the valid schemata depend on the domain in which one reasons, *with what purpose*.

“We therefore view reasoning as consisting of two stages: first one has to establish the domain about which one reasons and its formal properties (what we will call “reasoning *to* an interpretation”) and only after this initial step has been taken can one’s reasoning be guided by formal laws (what we will call “reasoning *from* an interpretation”).” (pg. 20)

The set of parameters characterizing a logic can be divided in three subsets:

1. Choice of formal language
2. Choice of a semantics for the formal language
3. Choice of a definition of valid arguments in the language

Classical Logic “Parameters”

1. *Syntax*: if φ, ψ are sentences, then so are $\neg\varphi$, $\varphi \wedge \psi$, $\varphi \vee \psi$, and $\varphi \rightarrow \psi$
2. *Semantics* (truth-functionality): the truth-value of a sentence is a function of the truth-values of its components only
3. *Semantics* (bivalence): sentences are either true or false, with nothing in-between
4. *consequence*: $\alpha_1 \ldots \alpha_n / \beta$ is valid iff β is true in all models of $\alpha_1, \ldots, \alpha_n$

Classical Logic “Parameters”

1. *Syntax*: if φ, ψ are sentences, then so are $\neg\varphi$, $\varphi \wedge \psi$, $\varphi \vee \psi$, and $\varphi \to \psi$
2. *Semantics* (truth-functionality): the truth-value of a sentence is a function of the truth-values of its components only
3. *Semantics* (bivalence): sentences are either true or false, with nothing in-between
4. *consequence*: $\alpha_1 \ldots \alpha_n / \beta$ is valid iff β is true in all models of $\alpha_1, \ldots, \alpha_n$

Domains to which classical logic is applicable must satisfy these four assumptions.

Truth-functionality without bivalence: "unknown"

"Is $2^{1257787} - 1$ prime?"

Truth-functionality without bivalence: "unknown"

"Is $2^{1257787} - 1$ prime?"

P	$\neg P$
1	0
0	1
u	u

Truth-functionality without bivalence: "unknown"

"Is $2^{1257787} - 1$ prime?"

P	$\neg P$
1	0
0	1
u	u

P	Q	$P \wedge Q$
1	1	1
1	0	0
0	1	0
0	0	0
u	0	0
u	1	u
0	u	0
1	u	u
u	u	u

Truth-functionality without bivalence: "unknown"

"Is $2^{1257787} - 1$ prime?"

P	$\neg P$
1	0
0	1
u	u

P	Q	$P \wedge Q$
1	1	1
1	0	0
0	1	0
0	0	0
u	0	0
u	1	u
0	u	0
1	u	u
u	u	u

P	Q	$P \vee Q$
1	1	1
1	0	1
0	1	1
0	0	0
u	0	u
u	1	1
0	u	u
1	u	1
u	u	u

Truth-functionality without bivalence: "unknown"

"Is $2^{1257787} - 1$ prime?"

P	$\neg P$
1	0
0	1
u	u

P	Q	$P \wedge Q$
1	1	1
1	0	0
0	1	0
0	0	0
u	0	0
u	1	u
0	u	0
1	u	u
u	u	u

P	Q	$P \vee Q$
1	1	1
1	0	1
0	1	1
0	0	0
u	0	u
u	1	1
0	u	u
1	u	1
u	u	u

P	Q	$P \rightarrow Q$
1	1	1
1	0	0
0	1	1
0	0	1
u	0	u
u	1	1
0	u	1
1	u	u
u	u	u

Non-Truth-Functional Semantics

Intuitionistic logic:

1. $\varphi \wedge \psi$ means "I have a proof of both φ and ψ"
2. $\varphi \vee \psi$ means "I have a proof of φ or a proof of ψ"
3. $\varphi \rightarrow \psi$ means "I have a construction that transforms a proof of φ into a proof of ψ"
4. $\neg\varphi$ means "Any proof of φ leads to a contradiction"

Clearly, $\varphi \vee \neg\varphi$ is not valid.

An Intensional Logic: Deontic Logic

If φ is a formula, then so is $O\varphi$.

$O\varphi$: "φ ought to be the case".

An Intensional Logic: Deontic Logic

If φ is a formula, then so is $O\varphi$.

$O\varphi$: "φ ought to be the case".

$O\varphi$ cannot be a function of the *truth* of φ (unlike '$\neg$'), since it depends on the meaning of φ: "my bike is black" and "I don't steal" are both true, but the latter *ought* to be the case.

$O\varphi$ is true provide for φ is true in all *normatively perfect alternatives*.

An Intensional Logic: Deontic Logic

If φ is a formula, then so is $O\varphi$.

$O\varphi$: "φ ought to be the case".

$O\varphi$ cannot be a function of the *truth* of φ (unlike '$\neg$'), since it depends on the meaning of φ: "my bike is black" and "I don't steal" are both true, but the latter *ought* to be the case.

$O\varphi$ is true provide for φ is true in all *normatively perfect alternatives*.

Compare: $p \to q$ to $p \to Oq$.

“Common Sense” Reasoning

(1) Bill brought his backpack to class every day of the semester.

So, [probably] (2) Bill will bring it to the next class.

"Common Sense" Reasoning

(1) Bill brought his backpack to class every day of the semester.

So, [probably] (2) Bill will bring it to the next class.

(1.1) Bill's backpack was stolen.

“Common Sense” Reasoning

(1) Bill brought his backpack to class every day of the semester.

So, [probably] (2) Bill will bring it to the next class.

(1.1) Bill’s backpack was stolen.

(3) Tweety is a bird

So, (4) Tweety flies.

(3.1) Tweety is a penguin.

Non-Monotinicity

Non-Monotinicity

If $A \vdash B$ holds then $A, C \vdash B$ also holds.

Non-Monotinicity

If $A \vdash B$ holds then $A, C \vdash B$ also holds.

Conclusions that are reasonable on the basis of specific information can become unreasonable if further information is added.

Non-Monotinicity

If $A \vdash B$ holds then $A, C \vdash B$ also holds.

Conclusions that are reasonable on the basis of specific information can become unreasonable if further information is added. Given the announced schedule for the course, and your previous experience, and that today is Thursday, it is reasonable to conclude that the course will meet in the evening.

Non-Monotinicity

If $A \vdash B$ holds then $A, C \vdash B$ also holds.

Conclusions that are reasonable on the basis of specific information can become unreasonable if further information is added. Given the announced schedule for the course, and your previous experience, and that today is Thursday, it is reasonable to conclude that the course will meet in the evening. However upon learning there is an announcement on the website that class is canceled, then it is reasonable to drop this belief.

Non-Monotinicity

If $A \vdash B$ holds then $A, C \vdash B$ also holds.

Conclusions that are reasonable on the basis of specific information can become unreasonable if further information is added. Given the announced schedule for the course, and your previous experience, and that today is Thursday, it is reasonable to conclude that the course will meet in the evening. However upon learning there is an announcement on the website that class is canceled, then it is reasonable to drop this belief. Further, if it is discovered that there was a mistake on the website, then it is reasonable to believe that there will be class.

Non-Monotinicity

If $A \vdash B$ holds then $A, C \vdash B$ also holds.

Conclusions that are reasonable on the basis of specific information can become unreasonable if further information is added. Given the announced schedule for the course, and your previous experience, and that today is Thursday, it is reasonable to conclude that the course will meet in the evening. However upon learning there is an announcement on the website that class is canceled, then it is reasonable to drop this belief. Further, if it is discovered that there was a mistake on the website, then it is reasonable to believe that there will be class.

$A \to B \vdash (A \land C) \to B$

Non-Monotinicity

If $A \vdash B$ holds then $A, C \vdash B$ also holds.

Conclusions that are reasonable on the basis of specific information can become unreasonable if further information is added. Given the announced schedule for the course, and your previous experience, and that today is Thursday, it is reasonable to conclude that the course will meet in the evening. However upon learning there is an announcement on the website that class is canceled, then it is reasonable to drop this belief. Further, if it is discovered that there was a mistake on the website, then it is reasonable to believe that there will be class.

$A \to B \vdash (A \wedge C) \to B$
'If you put sugar in the coffee, then it will taste good' can be true without 'If you put sugar and gasoline in the coffee, then it will taste good' being true.

Non-monotonic logic: What *should*/*do* I believe?

Classical consequence relation: $\varphi \vdash \psi$: ψ follows from φ using the rules of logic (there is a derivation of ψ using propositional logic and φ)

Non-monotonic logic: What *should*/*do* I believe?

Classical consequence relation: $\varphi \vdash \psi$: ψ follows from φ using the rules of logic (there is a derivation of ψ using propositional logic and φ)

Monotonicity: If $\varphi \vdash \psi$ then $\varphi, \alpha \vdash \psi$

Non-monotonic logic: What *should*/*do* I believe?

Classical consequence relation: $\varphi \vdash \psi$: ψ follows from φ using the rules of logic (there is a derivation of ψ using propositional logic and φ)

Monotonicity: If $\varphi \vdash \psi$ then $\varphi, \alpha \vdash \psi$

Doxastic reading: after coming to believe/accept φ, the agent believes/accepts ψ.

Non-monotonic logic: What *should*/*do* I believe?

Classical consequence relation: $\varphi \vdash \psi$: ψ follows from φ using the rules of logic (there is a derivation of ψ using propositional logic and φ)

Monotonicity: If $\varphi \vdash \psi$ then $\varphi, \alpha \vdash \psi$

Doxastic reading: after coming to believe/accept φ, the agent believes/accepts ψ.

Failure on monotonicity: B: Tweety is a bird; F: Tweety flies; P: Tweety is a penguin

$B \vdash F$ but $B, P \not\vdash F$.

Non-monotonic logic

$\varphi \mid\!\sim \psi$ "If φ then *typically* (*mostly*, etc.) ψ"

Nonmonotonic Reasoning

Left logical equivalence: If $\vdash \varphi \leftrightarrow \psi$ and $\varphi \mathrel{|\!\sim} \alpha$ then $\psi \mathrel{|\!\sim} \alpha$

Right weakening: If $\vdash \alpha \rightarrow \beta$ and $\varphi \mathrel{|\!\sim} \alpha$ then $\varphi \mathrel{|\!\sim} \beta$

And: If $\varphi \mathrel{|\!\sim} \alpha$ and $\varphi \mathrel{|\!\sim} \beta$ then $\varphi \mathrel{|\!\sim} (\alpha \wedge \beta)$

Or: If $\varphi \mathrel{|\!\sim} \alpha$ and $\psi \mathrel{|\!\sim} \alpha$ then $(\varphi \vee \psi) \mathrel{|\!\sim} \alpha$

Monotonicity

Monotonicity: $\varphi \mid\!\sim \alpha$ then $\varphi \wedge \psi \mid\!\sim \alpha$

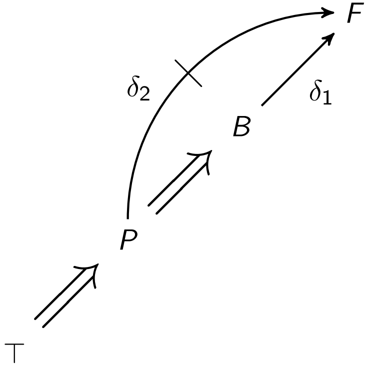
C: coffee in the cup, T: the liquid tastes good; O: oil is in the cup

$C \mid\!\sim T$ but $C \wedge O \not\mid\!\sim T$
But note that $O \not\mid\!\sim T$

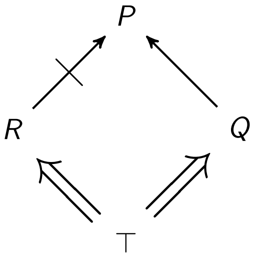
Cautious Monotonicity: If $\varphi \mid\!\sim \alpha$ and $\varphi \mid\!\sim \beta$ then $\varphi \wedge \alpha \mid\!\sim \beta$

Rational Monotonicity: If $\varphi \mid\!\sim \alpha$ and $\varphi \not\mid\!\sim \neg\beta$, then $\varphi \wedge \beta \mid\!\sim \alpha$

Tweety Triangle



Nixon Diamond



J. Horty. *Skepticism and floating conclusions*. Artificial Intelligence, 135, pp. 55 - 72, 2002.

Floating Conclusions

$$
\begin{tikzcd}
 & E & \\
H \arrow[ur, Rightarrow] & \not\Leftrightarrow & D \arrow[ul, Rightarrow] \\
R \arrow[u] & & Q \arrow[u] \\
 & \top \arrow[ul, Rightarrow] \arrow[ur, Rightarrow] &
\end{tikzcd}
$$

Floating Conclusions, II

$$
\begin{tikzcd}
 & F \lor M & \\
\neg F \land M \arrow[ur, Rightarrow] \arrow[rr, Leftrightarrow, "\mid" marking] & & F \land \neg M \arrow[ul, Rightarrow] \\
BA(\neg F \land M) \arrow[u] & & SA(F \land \neg M) \arrow[u] \\
 & \top \arrow[ul, Rightarrow] \arrow[ur, Rightarrow] &
\end{tikzcd}
$$

"But if I were told of some other individual that he is both a Quaker and a Republican, I would not be sure what to conclude. It is possible that this individual would adopt an extreme position, as either a dove or a hawk. But it seems equally reasonable to imagine that such an individual, rather than being pulled to one extreme of the other, would combine elements of both views into a more balanced, measured position falling toward the center of the political spectrum—perhaps believing that the use of military force is sometimes appropriate, but only as a response to serious provocation."

J. Horty. *Skepticism and floating conclusions*. Artificial Intelligence, 135, pp. 55 - 72, 2002.

Closed-world reasoning

Negation as failure
Suppose you are interested in whether there are any direct flights from Amsterdam to Cleveland, Ohio.

After searching online at a number of relevant sites (Expedia, Orbitz, KLM, etc.), you do not find any. You conclude that there are *no direct flights between Amsterdam and Cleveland*.

Concluding Remarks: Normatives vs. Descriptive

How can/should we incorporate *empirical data* into our *normative* theory of rationality?

Concluding Remarks: Normatives vs. Descriptive

How can/should we incorporate *empirical data* into our *normative* theory of rationality?

- *Normative*: reasoning as it should be, ideally

Concluding Remarks: Normatives vs. Descriptive

How can/should we incorporate *empirical data* into our *normative* theory of rationality?

- *Normative*: reasoning as it should be, ideally
- *Descriptive*: reasoning as it is actually practiced

Concluding Remarks: Normatives vs. Descriptive

How can/should we incorporate *empirical data* into our *normative* theory of rationality?

- *Normative*: reasoning as it should be, ideally
- *Descriptive*: reasoning as it is actually practiced
- *Prescriptive*: take into account bounded rationality (computational limitations, storage limitations)

Concluding Remarks: Normatives vs. Descriptive

How can/should we incorporate *empirical data* into our *normative* theory of rationality? (reflective equilibrium)

- *Normative*: reasoning as it should be, ideally
- *Descriptive*: reasoning as it is actually practiced
- *Prescriptive*: take into account bounded rationality (computational limitations, storage limitations)

Concluding Remarks: Positions

- Human reasoning is normatively correct. What appears to be incorrect reasoning can be explained by various maneuvers, such as different interpretation of logical terms, etc.

Concluding Remarks: Positions

- Human reasoning is normatively correct. What appears to be incorrect reasoning can be explained by various maneuvers, such as different interpretation of logical terms, etc.
- Actual human performance follows prescriptive rules, but they are not the normative rules because of the heavy demands of normatively correct reasoning

Concluding Remarks: Positions

- Human reasoning is normatively correct. What appears to be incorrect reasoning can be explained by various maneuvers, such as different interpretation of logical terms, etc.
- Actual human performance follows prescriptive rules, but they are not the normative rules because of the heavy demands of normatively correct reasoning
- Actual human reasoning falls short of prescriptive standards, so there is room for improvement by suitable education

Concluding Remarks: Positions

- Human reasoning is normatively correct. What appears to be incorrect reasoning can be explained by various maneuvers, such as different interpretation of logical terms, etc.
- Actual human performance follows prescriptive rules, but they are not the normative rules because of the heavy demands of normatively correct reasoning
- Actual human reasoning falls short of prescriptive standards, so there is room for improvement by suitable education
- Reasoning rarely happens in real life: we have developed "fast and frugal algorithms" which allow us to take quick decisions which are optimal given constraints of time and energy.

Concluding Remarks

"...a misunderstanding which has haunted the discussion of such ideas as "rule of inference"....

Concluding Remarks

"...a misunderstanding which has haunted the discussion of such ideas as "rule of inference".... In games, there are rules and there are rules.

Concluding Remarks

"...a misunderstanding which has haunted the discussion of such ideas as "rule of inference".... In games, there are rules and there are rules. There are such rules as serve to define the game, e.g., the rules of chess. I shall call them 'definitory rules'. They tell which moves are possible, or, as it is sometimes put, which moves are admissible.

Concluding Remarks

"...a misunderstanding which has haunted the discussion of such ideas as "rule of inference".... In games, there are rules and there are rules. There are such rules as serve to define the game, e.g., the rules of chess. I shall call them 'definitory rules'. They tell which moves are possible, or, as it is sometimes put, which moves are admissible. The crucial fact about definitory rules is that they say absolutely nothing about which moves are good, which ones are bad and which ones are better than others.

Concluding Remarks

"...a misunderstanding which has haunted the discussion of such ideas as "rule of inference".... In games, there are rules and there are rules. There are such rules as serve to define the game, e.g., the rules of chess. I shall call them 'definitory rules'. They tell which moves are possible, or, as it is sometimes put, which moves are admissible. The crucial fact about definitory rules is that they say absolutely nothing about which moves are good, which ones are bad and which ones are better than others. Such questions are handled by rules of another kind. I shall call them 'strategic rules'.

Concluding Remarks

"...a misunderstanding which has haunted the discussion of such ideas as "rule of inference".... In games, there are rules and there are rules. There are such rules as serve to define the game, e.g., the rules of chess. I shall call them 'definitory rules'. They tell which moves are possible, or, as it is sometimes put, which moves are admissible. The crucial fact about definitory rules is that they say absolutely nothing about which moves are good, which ones are bad and which ones are better than others. Such questions are handled by rules of another kind. I shall call them 'strategic rules'. They have to be distinguished from definitory rules. Admittedly, the notion of strategy in a given game is possible to define only after the definitory rules have been set up.

Concluding Remarks

"...a misunderstanding which has haunted the discussion of such ideas as "rule of inference"…. In games, there are rules and there are rules. There are such rules as serve to define the game, e.g., the rules of chess. I shall call them 'definitory rules'. They tell which moves are possible, or, as it is sometimes put, which moves are admissible. The crucial fact about definitory rules is that they say absolutely nothing about which moves are good, which ones are bad and which ones are better than others. Such questions are handled by rules of another kind. I shall call them 'strategic rules'. They have to be distinguished from definitory rules. Admittedly, the notion of strategy in a given game is possible to define only after the definitory rules have been set up. Only after that has been done can we hope to begin to investigate which strategies are better than others."

J. Hintikka. *Inquiry as Inquiry*. Kluwer Academic Publishers, 1999.