Clear Thinking in an Uncertain World: Human Reasoning and its Foundations

Lecture 4

Eric Pacuit

Department of Philosophy
University of Maryland, College Park
pacuit.org
epacuit@umd.edu

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“Common Sense” Reasoning

(1) Bill brought his backpack to class every day of the semester.

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(3) Tweety is a bird

So, (4) Tweety flies.

(3.1) Tweety is a penguin.

Induction

Enumerative Induction
Given that all observed Fs are Gs, you infer that all Fs are Gs, or at least the next F is a G.

Inference to the best explanation
Holmes infers the best explanation for footprints, the absence of barking, the broken window: 'The butler wears size 10 shoes, is known to the dog, broke the window to make it look like a burglary...'

Scientific hypothetic induction
Scientists infer that Brownian motion is caused by the movement of invisible molecules.

Hume: Does positive inductive evidence support rational beliefs?

In the past, Fs have been followed by Gs (and never by non-Gs)

So, the present case of an F will be followed by a G

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The Ravens Paradox

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H: All ravens are black.

H': All nonblack things are nonravens.

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H: All ravens are black.

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But, then does a red jacket confirm H?

Goodman's New Riddle of Induction

All emeralds examined thus far are green.

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The data collected thus far seems to confirm H1 as well as H2, but H1 seems to be a "better explanation"...

N. Goodman. *Fact, Fiction and Forecast*. Bobbs-Merrill, 1965.

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There are a huge number of nonblack things as well as nonravens, the antecedent probability of finding a nonraven among nonblack things is extremely high. Consequently, finding a nonbalck nonraven only slightly increase the probability of "All ravens are black."

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e supports *h* if the probability of *h* given *e* and the background information is greater than the probability of *h* given the background information alone:

$$p(h \mid e \& b) > p(h \mid b).$$

Probability

Kolmogorov Axioms:

1. For each E, $0 \leq p(E) \leq 1$
2. $p(W) = 1$, $p(\emptyset) = 0$
3. If $E_1, \ldots, E_n, \ldots$ are pairwise disjoint ($E_i \cap E_j = \emptyset$ for $i \neq j$), then $p(\bigcup_i E_i) = \sum_i p(E_i)$

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- $p(\overline{E}) = 1 - p(E)$ ($\overline{E}$ is the complement of E)
- If $E \subseteq F$ then $p(E) \leq p(F)$
- $p(E \cup F) = p(E) + p(F) + p(E \cap F)$

Be careful about your intuitions involving probabilities...

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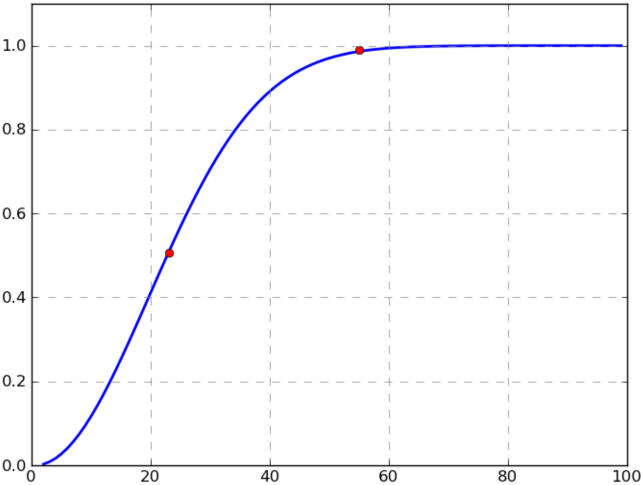
This is not a paradox but a result that people often find puzzling.


1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100

Conditional Probability

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Bayes Theorem: $p(E|F) = p(F|E)\frac{p(E)}{p(F)}$

Bayes theorem is important because it expresses the quantity $p(E|F)$ (the probability of a hypothesis E given the evidence F) — which is something people often find hard to assess — in terms of quantities that can be drawn directly from experiential knowledge.

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Three Prisoner's Paradox

Three prisoners A, B and C have been tried for murder and their verdicts will told to them tomorrow morning. They know only that one of them will be declared guilty and will be executed while the others will be set free. The identity of the condemned prisoner is revealed to the very reliable prison guard, but not to the prisoners themselves. Prisoner A asks the guard "Please give this letter to one of my friends — to the one who is to be released. We both know that at least one of them will be released".

Three Prisoner's Paradox

An hour later, A asks the guard "Can you tell me which of my friends you gave the letter to? It should give me no clue regarding my own status because, regardless of my fate, each of my friends had an equal chance of receiving my letter." The guard told him that B received his letter.

Prisoner A then concluded that the probability that he will be released is $1/2$ (since the only people without a verdict are A and C).

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Explain what is wrong with A's reasoning.

A's reasoning

Consider the following events:

G_A: "Prisoner A will be declared guilty" (we have $p(G_A) = 1/3$)

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$$p(G_A \mid I_B) = p(I_B \mid G_A)\frac{p(G_A)}{p(I_B)} = 1 \cdot \frac{1/3}{2/3} = 1/2$$

A's reasoning, corrected

But, A did not receive the information that B will be declared innocent, but rather that "the guard said that B will be declared innocent." So, A should have conditioned on the event:

I'_B: "The guard said that B will be declared innocent"

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Given that $p(I'_B \mid G_A)$ is $1/2$ (given that A is guilty, there is a 50-50 chance that the guard could have given the letter to B or C). This gives us the following correct calculation:

$$p(G_A \mid I'_B) = p(I'_B \mid G_A)\frac{p(G_A)}{p(I'_B)} = 1/2 \cdot \frac{1/3}{1/2} = 1/3$$

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$p(T|B) = p(B|T)\frac{p(T)}{p(B)} = 0.99(100/1,000,000)/[(0.99 \cdot 100 + 0.01 \cdot 999900)/1,000,000] = 1/102 \approx 0.98\%$

Monty Hall Dilemma

Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say number 1, and the host, who knows what's behind the doors, opens another door, say number 3, which has a goat. He says to you, "Do you want to pick door number 2?" Is it to your advantage to switch your choice of doors?

Monty Hall (1)

H_1: The care is behind door 1

H_2: The care is behind door 2

H_3: The care is behind door 3

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Similarly for $p(H_2 \mid E)$, so do not switch.

Monty Hall (3)

Reasoning 2: F: Monty opened door number 3

$$
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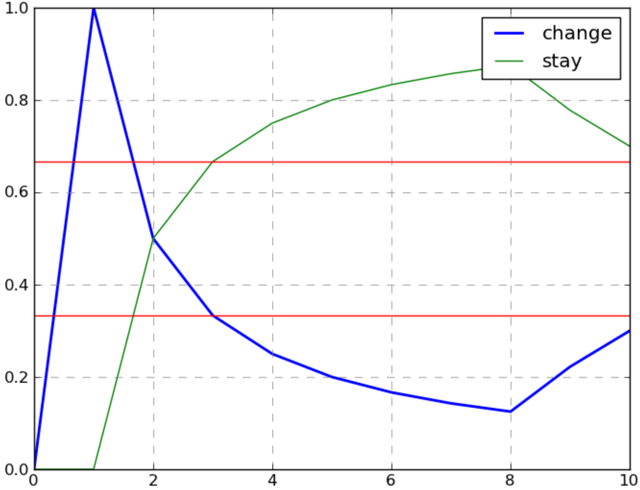
Monty Hall: Reasoning 1 vs. Reasoning 2



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