Clear Thinking in an Uncertain World: Human Reasoning and its Foundations

Lecture 5

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Base-Rate Fallacy

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Wason Selection Task

P. C. Wason. *Reasoning about a rule*. Quarterly Journal of Experimental Psychology, 20:273 - 281, 1968.

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Rule: If there is a vowel on one side, then there is an even number on the other side.

A	K	4	7

Responses

Rule: If there is a vowel on one side (P), then there is an even number on the other side (Q).

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35%	45%	5%	7%	8%

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Wason (and, until fairly recently, the great majority of researchers) assumed, without considering alternatives, that the correct performance is to turn the A and 7 cards only.

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3. K and 4
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5. All of them
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1. A
2. A and 4 (half the subjects)
3. K and 4
4. A and 7 (Very few)
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...by far the most important determinant of ease of reasoning is whether interpretation of the rule assigns it descriptive or deontic logical form, *and we explain the effect of this interpretive choice in terms of the many problems descriptive interpretation creates in the task setting, as contrasted with the ease of reasoning with deontic interpretations.* *(pg. 47)*

Deontic and Descriptive Conditionals

If P, Q

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Different Logical Forms:
$P \rightarrow Q$ vs. $P \rightarrow Ought(Q)$ vs. $Ought(P \rightarrow Q)$

Deontic Conditionals

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The proper logical forms is determined by context:

- "If someone is at the door, then it should be John" is descriptive.
- Should "In the UK, vehicles drive on the left" be interpreted deontically or descriptively?

- Some subjects think the output can be a plan for showing the rule to be true or false
- Some subjects interpolate a process of information gathering and view the task as “what information do I require to decide the rule, and how do I obtain that information.”

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an information-processing task whose output is the informaiton the subject requires for deciding the rule.
Suppose that the letters on the card can only be 'K' and 'A' and the numbers only '4' and '7'.

$$W = \{\underbrace{\mathsf{A}, \mathsf{K}, 4, 7}_{\text{partial information states}}, \underbrace{(\mathsf{A}, 4), (\mathsf{A}, 7), (\mathsf{K}, 4), (\mathsf{K}, 7), (4, \mathsf{A}), (4, \mathsf{K}), (7, \mathsf{A}), (7, \mathsf{K})}_{\text{full information states}}\}$$

$w \leq v$: "the information contained about a given card in v is an extension of, or equial to, the information about that card in w. "
$v \Vdash \varphi$ "v contains evidence for φ"
$v \models \varphi$ "v makes φ true" or "φ is true in v"

p "the card has a vowel" and q: "the card has an even number"

- A $\Vdash p$, K $\Vdash \neg p$, p is undecided on 4 and 7
- $4 \Vdash q$, $7 \Vdash \neg q$, q is undecided on A and K
- $(\mathsf{A}, 4) \Vdash p \wedge q$, $(\mathsf{A}, 7) \Vdash p \wedge \neg q$, ...

$v \Vdash p \wedge \neg q$ “there is a card (x, y) in v such that $(\mathrm{x}, \mathrm{y}) \Vdash p \wedge \neg q$.

A rule is supported by a piece of information v, denoted $v \Vdash p \rightarrow q$, if $v \nVdash p \wedge \neg q$

$v \models p \rightarrow q$ if for all $u \geq v$, $u \Vdash p \rightarrow q$.

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Do not think in terms of the information which must be *gathered*, but in terms of information which becomes *available*.

$w \models Ought(p \to q)$ iff for all v such that $R(w, v)$: $v \models p$ implies $v \models q$.

$R(\mathrm{A}, (\mathrm{A}, 4))$, $R(7, (7, \mathrm{K}))$, $\neg R(\mathrm{A}, (\mathrm{A}, 7))$, $\neg R(7, (7, \mathrm{A}))$, $R(\mathrm{K}, (\mathrm{K}, 4))$, $R(\mathrm{K}, (\mathrm{K}, 7))$, $R(4, (4, \mathrm{A}))$, $R(4, (4, \mathrm{K}))$

$w \models Ought(p \to q)$ for all states w.

The information processing task is: which cards need to be turned over to possibly violate the rule.

Modified Selection Task

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Deontic "And"

$$w \models p \sqcap q \;\text{ iff }\; \text{for all } v \text{ such that } R(w,v)\text{: } v \models p \wedge q$$

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An orthogonal issue is, which set of cards should form the domain of the model. The experimenter intends the domain to be the set of four cards....[there are] some reasons why natural language use suggests considering larger domains, of which the four cards shown are only a sample, and it presents a dialogue with a subject who has a probabilistic concept of truth that comes naturally with this interpretation of the domain. (pg. 58)

Other Logical Forms

$$p \wedge \neg e \rightarrow q$$

where $\neg e$ means "there is no exception".

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$$p \wedge \neg e \to q$$

where $\neg e$ means "there is no exception". Then, we have:

$$p' \wedge \neg q' \to e$$

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  Modus Tollens, Bayes Theorem
- *Descriptive*: reasoning as it is actually practiced
  many people do not endorse Modus Tollens or make base rate fallacies
- *Prescriptive*: take into account bounded rationality (computational limitations, storage limitations)
  closed-world reasoning, heuristics

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- Actual human reasoning falls short of prescriptive standards, so there is room for improvement by suitable education
- Reasoning rarely happens in real life: we have developed "fast and frugal algorithms" which allow us to take quick decisions which are optimal given constraints of time and energy.