Clear Thinking in an Uncertain World: Human Reasoning and its Foundations

Lecture 8

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October 23, 2013

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Which is more probable?

1. Linda is a bank teller.
2. Linda is a bank teller and is active in the feminist movement.

Typically a large percentage of people asked say 2 is more probable than 1.

A. Tversky and D. Kahneman. *Extensions versus intuitive reasoning: The conjunction fallacy in probability judgment*. Psychological Review 90 (4): 293 - 315, 1983.

K. Tentori, N. Bonini and D. Osherson. *The conjunction fallacy: a misunderstanding about conjunction?*. Cognitive Science, 28, pgs. 467 - 477, 2004.

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Perhaps the “fallacy” is only that subjects conflate ‘$\wedge$’ with ‘and’.

‘and’ possesses semantic and pragmatic properties that are foreign to ‘$\wedge$’: “John had a drink and was stopped by the cops” vs. “John was stopped by the cops and had a drink”.

- It is impossible for X to be false given the truth of X-and-Y
- It is impossible for Y to be false given the truth of X-and-Y
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We have:

- X-and-$Y \models X$
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- X-and-$Y \models X \wedge Y$
- For any statements A, B, $Prob(A) \leq Prob(B)$ whenever $A \models B$.

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The only reason to deny (1) the status of a "true" conjunction fallacy is to insist that conjunction fallacies must endorse exactly *$(***)$.*

1. X-and-$Y \models X$
2. X-and-$Y \models Y$

(∗) $Prob(X\text{-and-}Y) > Prob(Y)$

Do people endorse (1), (2) as well as (∗)?

Experiment

Fifty Italian students (mean age 22.7 years, 30 males) from the University of Padua were invited to choose the most probable statement from sets of three.

SCANDINAVIA PROBLEM: The Scandinavian peninsula is the European area with the greatest percentage of people with blond hair and blue eyes. This is the case even though (as in Italy) every possible combination of hair and eye color occurs. Suppose we choose at random an individual from the Scandinavian population. Which do you think is the most probable? (Check your choice.)

- The individual has blond hair.
- The individual has blond hair and blue eyes.
- The individual has blond hair and does not have blue eyes.

Filler item: In the city of Florence there are many shops featuring crafts. Among them are shops with leather goods, jewelry, and art. Some of these shops are ample in size, and serve as schools with many apprentices. Others are smaller. Suppose we choose at random a crafts shop in Florence. Which do you think is the most probable? (Check your choice.)

- The shop sells leather goods.
- The shop does not sell leather goods and has fewer than two apprentices.
- The shop does not sell leather goods and has at least two apprentices.

Implication question for the Scandinavia problem:
Luke is in his last year of high school. One morning he met Mika, a new student from Finland. Mika has blond hair and blue eyes. Speaking together, Luca learned that Mika likes to play the piano and is in Italy because his father was transferred to the Milan branch of a large foreign bank. At home Luca tells his sister about Mika, and makes several claims about him. From among the statements shown below, please indicate which are true, which are false, and which might be either.

1. Mika was born in Helsinki.
2. Mika hates to play the piano.
3. Mika has blue eyes.
4. Mika likes living in Milan.
5. Mika has blond hair.
6. Mika says that his family moves often because of his fathers work.

Each was followed by the three choices: True, False, Might be either.

Volleyball problem: Professional volleyball players have greatly changed in the course of the last decade. In particular, they have grown younger yet taller. Women players in the first Italian division are on average taller than 1.80 m, ranging between 1.75 m for some setters to more than 1.90 m for many spikers. Suppose we choose at random a female volleyball player from the Italian first division. Which do you think is the most probable? (Check your choice.)

- The woman is less than 21 years old.
- The woman is less than 21 years old and is taller than 1.77 m.
- The woman is less than 21 years old and is not taller than 1.77 m.

Results

Forty-five of the 50 participants responded with True to both crucial items in both implication questions [(3) and (5)], thereby showing agreement with the implications in X-and-$Y \models X$ and X-and-$Y \models Y$. Of these 45 students, 29 failed in both the Scandinavia and Volleyball problems to choose the X option as most probable.

Conclusions

"Both experiments reveal numerous judgments of the form $Prob(X\text{-and-}Y) > Prob(X)$. Such judgments are fallacious if participants endorse the implication $X\text{-and-}Y \models X$, which is suggested by the experiments. The fallacy cannot be explained by the ambiguity of the word "probable"....It also appears unlikely that the fallacy results from interpreting X as $X\text{-and-not-}Y$."