

# Clear Thinking in an Uncertain World: Human Reasoning and its Foundations

## Lecture 7

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Accuracy and rationality are linked, they are not the same: a fool may hold a belief irrationally — as a result of a lucky guess or wishful thinking — yet it might happen to be correct. Conversely, a detective might hold a belief on the basis of a careful and exhaustive examination of all the evidence and yet the evidence may be misleading, and the belief may turn out to be wrong.

## Conceptions of Belief

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Eric Schwitzgebel. *Belief*. In The Stanford Encyclopedia of Philosophy.

Franz Huber. *Formal Theories of Belief*. In The Stanford Encyclopedia of Philosophy.

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D. Christensen. *Putting Logic in its Place*. Oxford University Press.

# Consistency Requirement

A rational agents (all-out) beliefs *should* (are rationally required to) be logically consistent.

# Preface Paradox

D. Makinson. *The Paradox of the Preface*. Analysis, 25, 205 - 207, 1965.

I. Douven and J. Uffink. *The Preface Paradox Revisited*. Erkenntnis, 59, 389 - 420, 2003.

## Preface Paradox

Suppose that in the course of his book an author makes a great many assertions:  $s_1, s_2, \dots, s_n$ .

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$$B_A(\neg(s_1 \wedge s_2 \wedge \dots \wedge s_n))$$

But  $\{s_1, \dots, s_n, \neg(s_1 \wedge \dots \wedge s_n)\}$  is logically inconsistent.

## Preface Paradox

A philosopher who asserts “all of my present philosophical positions are correct” would be regarded as rash and over-confident

A philosopher who asserts “at least some of my present philosophical beliefs will turn out to be incorrect” is simply being sensible and honest.

# Preface Paradox

1. each belief from the set  $\{s_1, \dots, s_n, s_{n+1}\}$  is rational
2. the set  $\{s_1, \dots, s_n, s_{n+1}\}$  of beliefs is rational.

1. does not necessarily imply 2.

## Preface Paradox: The Problem

“The author of the book is being rational even though inconsistent. More than this: he is being rational even though he believes each of a certain collection of statements, which *he knows* are logically incompatible....this appears to present a living and everyday example of a situation which philosophers have commonly dismissed as absurd; that it is sometimes rational to hold incompatible beliefs.”

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# Synchronic vs. Diachronic

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**Synchronic:** We assess as rational and irrational an agent's *occurrent* mental states

*It is irrational to hold inconsistent beliefs at time  $t$ .*

**Diachronic:** Rationality also involves the capacity that takes an agent from one mental state to another (either explicitly or implicitly through reasoning)

*If  $S$  believes  $p$  and believes  $q$  at time  $t$  then  $S$  should (may/will) believe  $p \wedge q$  at time  $t' > t$ .*

# Lottery Paradox

H. Kyburg. *Probability and the Logic of Rational Belief*. Wesleyan University Press, 1961.

I. Douven and T. Williamson. *Generalizing the Lottery Paradox*. British Journal of the Philosophy of Science, 57, 755 - 779, 2006.

G. Wheeler. *A Review of the Lottery Paradox*. Probability and Inference: Essays in honor of Henry E. Kyburg, Jr., College Publications, 2007.

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For each lottery ticket  $t_i$  ( $i = 1, \dots, 1000000$ ), the agent believes that  $t_i$  will lose  $B_A(\neg 't_i \text{ will win}')$

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So, the conjunction  $\bigwedge_{i=1}^{1000000} 't_i \text{ will not win}'$  should be accepted. That is, the agent should rationally accept that *no lottery ticket will win*.

But, this is a fair lottery, so at least one ticket is *guaranteed* to win!

# The Lottery Paradox

Kyburg: The following are inconsistent,

1. It is rational to accept a proposition that is very likely true,
2. It is not rational to accept a propositional that you are aware is inconsistent
3. It is rational to accept a proposition  $P$  and it is rational to accept another proposition  $P'$  then it is rational to accept  $P \wedge P'$

## Subjective Probabilities

Should a rational agent's graded beliefs satisfy the laws of probability?

J. Joyce. *Bayesianism*. in Handbook of Rationality.

Ann: “the probability it will rain tomorrow is 0.9” means “Ann's degree of belief is fairly high (0.9) that it will rain tomorrow. Of course whether it will actually rain, depends on objective events taking place in the external worlds.”

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What we need: systematic procedures for linking the probability calculus (graded beliefs) to claims about **objectively observable behavior**, such as preferences revealed by choice behavior.

## Ramsey, de Finetti and Savage (2)

Suppose we are wondering about Ann's degree of belief about whether a coin will land heads ( $H$ ) or tails ( $T$ ).

Offer Ann two bets:

- $L_1$  If the coin lands heads, you win a sports car;  
otherwise you win nothing
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otherwise you win nothing.

If Ann chooses  $L_1$ , she believes  $H$  is more probable than  $T$

If Ann chooses  $L_2$ , she believes  $T$  is more probable than  $H$

If Ann is indifferent, she believes  $H$  and  $T$  are equally probable  
(i.e.,  $p_A(H) = p_A(T) = 1/2$ )

## The Dutch Book Argument

But, why *should* a rational agent's graded beliefs satisfy the Kolmogorov axioms?

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Anyone whose beliefs violate the laws of probability is *practically irrational*.

F. P. Ramsey. *Truth and Probability*. 1931.

B. de Finetti. *La prévision: Ses lois logiques, ses sources subjectives*. 1937.

Alan Hájek. *Dutch Book Arguments*. Oxford Handbook of Rational and Social Choice, 2008.