Epistemic Game Theory

Lecture 5

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February 25, 2014

Plan

- Levi's argument: Deliberation crowds out prediction
- The value of information
- Newcomb's puzzle
- Fallacious reasoning in the prisoner's dilemma
- Act probabilities

"...the relevant distinction is between the *first-person* perspective of a practical deliberator and the *third-person* perspective of an observer. While the observer can predict what I will do, I can't, insofar as I deliberate upon what is to be done. Deliberating in this way is incompatible with predicting the outcome of deliberation. To put is shortly, *deliberation crowds out prediction.*" (pg. 91)

W. Rabinowicz. *Does Practical Deliberation Crowd Out Self-Prediction?*. Erkenntnis, 57, pgs. 91 = 122, 2002.

"Principles of rationality are invoked for several purposes: they are often deployed in explanation and prediction; they are also used to set standards for rational health for deliberating agents or to furnish blueprints for rational automata; and they are intended as guides for perplexed decision makers seeking to regulate their own attitudes and conduct." (pg. 19)

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"When used for self policing, the applicability of the principles [of rational choice] should be non vacuous in the sense that a nontrivial distinction may be made between feasible options which are admissible for choice and others which are not. If the principles of rational choice never eliminate any feasible option from the relevant set of feasible options, they fail to serve this functions....

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Rather than abandoning models of rationality, we should seek instead to devise techniques and therapies which enhance our capacities to do better. In this respect, models of rationality bear a closer resemblance to models of health and mental health. They are, for this reason, normative rather than explanatory and predictive." (Levi, pgs. 35, 36)

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Deliberation Crowds out Prediction

F. Schick. *Self-Knowledge, Uncertainty and Choice*. The British Journal for the Philosophy of Science, 30:3, pgs. 235 - 252, 1979.

I. Levi. *Rationality, prediction and autonomous choice*. in *The Covenant of Reason*.

I. Levi. *Feasibility*. in *Knowledge, belief and strategic interaction*, C. Bicchieri and M. L. D. Chiara (eds.), pgs. 1 - 20, 1992.

Meno's Paradox

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Levi's Argument

1. If you have access to self-knowledge and logical omniscience to apply the principles of rational choice to determine which options are admissible, then the principles of rational choice are vacuous for the purposes of deciding what to do.
2. If you do not have access to self-knowledge and logical omniscience in this sense, then the principles of rational choice are inapplicable for the purposes of deciding what do.

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If all the previous conditions are satisfied, then no inadmissible option is feasible from the deliberating agent's point of view when deciding what to do: $C(A) = A$.

"Though this result is not contradictory, it implies the vacuousness of principles of rational choice for the purpose of deciding what to do...If they are useless for this purpose, then by the argument of the previous section, they are useless for passing judgement on the rationality of choice as well." (Levi, pg. 10)

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(Earlier argument: "If X is merely giving advice, it is pointless to advise Sam to do something X is sure Sam will not do...The point I mean to belabor is that passing judgement on the rationality of Sam's choices has little merit unless it gives advice to how one should choose in predicaments similar to Sam's in relevant aspects")

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- Drop smugness: "the agent need not assume he will choose rationally...the agent should be in a state of suspense as to which of the feasible options will be chosen" (Levi)

The agent cannot simultaneously see an action as an object of choice and as an object of prediction. But he can freely switch between these two perspectives.

Suppose the agent faces a choice at t_1 with A being one of the options among which the choice is being made. A some earlier time t_0, the agent did assign a definite probability to which choice of A at t_1. However when t_1 comes, if the strong thesis is true, he can no longer assign any probability to A. How is this probability loss to be accounted for?

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"Any context where an agent engages in reasoning is a context that is distorted by the assumption of deductive omniscience, since reasoning (at least deductive reasoning) is an activity that deductively omniscient agents have no use for. Deliberation, to the extent that it is thought of as a rational process of figuring out what one should do given one's priorities and expectations is an activity that is unnecessary for the deductively omniscient. In fact any kind of information processing or computation is unintelligible as an activity of a deductively omniscient agent. " (pp. 428,429)

R. Stalnaker. *The Problem of Logical Omniscience, I*. Sythese, 89:3, 1991, pp. 425 - 440.

Discussion: The Cost of Thinking

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L. J. Savage. *Difficulties in the theory of personal probability*. Philosophy of Science, 34(4), pgs. 305 - 310, 1967.

I. Douven. *Decision theory and the rationality of further deliberation*. Economics and Philosophy, 18, pgs. 303 - 328, 2002.

Weak Thesis: In a situation of choice, the DM does not assign extreme probabilities to options among which his choice is being made.

Strong Thesis: In a situation of choice, the DM does not assign any probabilities to options among which his choice is being made.

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Strong Thesis: In a situation of choice, the DM does not assign any probabilities to options among which his choice is being made.

"...the probability assignment to A may still be available to the subject in his purely doxastic capacity but not in his capacity of an agent or practical deliberator. The agent *qua* agent must abstain from assessing the probability of his options." (Rabinowicz, pg. 3)

Common knowledge of rationality implies that each player knows that he himself is rational (=acts rationally)

Common priors on a joint actions space will have to go, since it presupposes that each player has prior probabilities for all combinations of the players' actions, inching his own. (the probabilities of an action is the sum of the probabilities of all possible action combinations that contain the action in question)

$P(B \mid A)$: the conditional probability of a consequence B given an action A is supposed to be defined as the ratio $\frac{P(B \wedge A)}{P(A)}$, which is ill-defined is the agent cannot assign probabilities to his actions.

Allow probability "gaps".

If the agent has definite conditional probabilities of rate consequences given the actions and for the actions given the consequence, then these conditional probability assignments are joints sufficient to determine his unconditional probabilities for actions:

$$P(A) = P(B) \cdot P(A \mid B) + (1 - P(B)) \cdot P(A \mid not - B)$$
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The value of information

Why is it better to make a "more informed" decision?
Suppose that you can either choose know, or perform a costless experiment and make the decision later. What should you do?

I. J. Good. *On the principle of total evidence*. British Journal for the Philosophy of Science, 17, pgs. 319 - 321, 1967.

"Never decide today what you might postpone until tomorrow in order to learn something new"

Choose between n acts $A_1, \ldots, A_n$ or perform a cost-free experiment E with possible results $\{e_k\}$, then decide.

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Then,

$$U(\text{Choose now}) = \max_j \sum_i p(K_i)U(A_j \;\&\; K_i)$$

$$= \max_j \sum_k \sum_i p(K_i)p(e_k \mid K_i)U(A_j \;\&\; K_i)$$

The value of an informed decision conditional on e:

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Compare $\max_j \sum_k \sum_i p(K_i)p(e_k \mid K_i)U(A_j \,\&\, K_i)$ and $\sum_k \max_j \sum_i p(e_k \mid K_i)p(K_i)U(A_j \,\&\, K_i)$

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$\sum_k \max_j g(k, j)$ is greater than or equal to $\max_j \sum_k g(k, j)$, so the second is greater than or equal to the first.

Newcomb's Paradox

Two boxes in front of you, A and B.

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Box A contains \$1,000 and box B contains either \$1,000,000 or nothing.

Your choice: either open both boxes, or else just open B. (You can keep whatever is inside any box you open, but you may not keep what is inside a box you do not open).

Newcomb's Paradox

A very powerful being, who has been invariably accurate in his predictions about your behavior in the past, has already acted in the following way:

1. If he has predicted that you will open just box B, he has in addition put \$1,000,000 in box B
2. If he has predicted you will open both boxes, he has put nothing in box B.

What should you do?

R. Nozick. *Newcomb's Problem and Two Principles of Choice*. 1969.

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	B = 1M	B = 0
1 Box	1M	0
2 Boxes	1M + 1000	1000

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J. Collins. *Newcomb's Problem*. International Encyclopedia of Social and Behavioral Sciences, 1999.

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What the Predictor did yesterday is *probabilistically dependent* on the choice today, but *causally independent* of today's choice.

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(the expected value of act A is a probability weighted average of the values of the ways w in which A might turn out to be true)

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Orthodox Bayesian Decision Theory: $P_A(w) := P(w \mid A)$ (Probability of w given A is chosen)

Causal Decision theory: $P_A(w) = P(A \mathrel{\Box\!\!\rightarrow} w)$ (Probability of *if A were chosen then w would be true*)

Suppose 99% confidence in predictors reliability.

B_1: one-box (open box B)
B_2: two-box choice (open both A and B)
N: receive nothing
K: receive \$1,000
M: receive \$1,000,000
L: receive \$1,001,000

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$V(B_2) = V(L)P(L \mid B_2) + V(K)P(K \mid B_2)$

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$1001000 \cdot 0.01 + 1000 \cdot 0.99$

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M: receive \$1,000,000
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$$V(B_1) = V(M)P(M \mid B_1) + V(N)P(N \mid B_1) =$$
$$1000000 \cdot 0.99 + 0 \cdot 0.01 = 990,000$$

$$V(B_2) = V(L)P(L \mid B_2) + V(K)P(K \mid B_2) =$$
$$1001000 \cdot 0.01 + 1000 \cdot 0.99 = 11,000$$

Let μ be the assigned to the conditional $B_1 \mathrel{\Box\!\!\rightarrow} M$ (and $B_2 \mathrel{\Box\!\!\rightarrow} L$) (both conditional are true iff the Predictor put \$1,000,000 in box B yesterday).

B_1: one-box (open box B)
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$1000000 \cdot \mu + 0 \cdot 1 - \mu$

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$1001000 \cdot \mu + 1000 \cdot 1 - \mu = 1000000\mu + 1000$

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The Irrational Choice

Mr. Z offers Adam two boxes, each containing \$10. Adam can choose either $S1$: to take the leftmost box and get \$10, or $S2$: to take the two boxes and get \$20. Before making his decision, Adam is informed by Z. that if he acts irrationally, Z will give him a bonus of \$100. (...to eliminate noise factors, assume that Adam believes that Z. is serious, has the relevant knowledge, is a perfect reasoner and is completely trustworthy.)

"...the bonus condition in Z's statement has truth-conditions, and once Adam has chosen it can be evaluated...It is only from the perspective of *Adam* qua *deliberating rational agent* that the bonus condition must be excluded as meaningless."

The Rational Choice

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Cassandra, a prophet of doom, used to warn people against disastrous actions, but her warnings went unheeded. She was doomed to be disbelieved by the same god who had given her the gift of foresight. And she knew it.

Imagine that, upon being asked for advice by some person, she warns the person against a certain action; but she also predicts that the person will not heed the warning. She makes thereby two predictions: that a certain action will have bad results, and that the person will take this action.

Eve's choice changes Cassandra's reliability as an expert. I cannot use an expert's advice as guide to my choosing, and at the same time use my choosing as evidence for the expert's reliability. That is, Cassandra's second prediction has no place in Eve's deliberations.

In its full generality the thesis means that, whatever information one uses in one's deliberations, one cannot use any non-trivial information about the likeliness of what one will choose.

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- What about choosing by "gut-feeling"? The non-nonsense Eve decides in certain cases to go by her feelings: *that* is her choice. She implements it when she acts according to what she feels.

- What about basing a choice on past decisions?

- What about basing a choice on past decisions? Known or believed past performance can enter into the deliberation ("I know from experience that I tend to judge right in these situations"). To be sure, very often the line between deliberation and unthinking intuition is hopelessly blurred. Someone who estimates the probability of his own pending decision, can be construed as one who has chosen to delegate the deciding authority to a partner that acts by feel, inclination, the pull of certain forces, and the like. Yet, choosing-on-impulse can be shifted to the implementing stage and considered as "external" to the deliberation, in as much as the agent can reason *about* it.

Signaling through choice

The act of choosing may itself carry some rewards, say, a feeling of being in control. But this presupposes that there is also a less "active" (do-nothing) option, and the more "active" A, is preferred because it involves doing. But then one chooses A, for the "doing" that goes with it, not for the sake of choosing A.

You can choose in bizarre ways, in order to be original.

On can also choose A, in order to impress someone else.

It is understood that if one chooses A then one actually makes A true. But we should clearly distinguish between making A true and *choosing* to make A true.

(AC) The reason for choosing A can refer to each of the available options, but they cannot refer in an essential way to the *choosing* from these options, except through considerations of signaling.

(AC*) One should not use conditional probabilities (or likeliness estimates) of choices, which are obtained by conditionalizing on some event (or parameter) upon which the choice, in the agent's judgement, has no bearing.

The choice has no bearing means that it is considered irrelevant to the event in question. Such events can be subject to probabilistic estimates outside the choice context.

Irrational Man

(straightforward reason)   \$20 is better than \$10

(c)   If Adam chooses $S2$ for the straightforward reason, then his choice is rational. Hence, he forfeits the bonus, which he could have received by choosing $S1$.

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If Mr. Z. is not assumed to be a perfect reasoner, Adam may rationally try to outsmart Z. (c) can be rephrased as a legitimate case of signaling: Adam signals (deceptively) to Z. that choosing $S1$ he is behaving irrationally. Deceptive signaling is, of course, useless if you deal with a omniscient reasoner.

Prisoner's Dilemma

(R1) Since I and player 2 are rational, either I choose C and player 2 chooses C, or I choose D and player 2 chooses D; the outcome is therefore (C, C) or (D, D) and the first preferable. I should choose C.

(R2) Since I and player 2 are twins, we reason alike; if I choose C he (very likely) chooses C for the same reasons; similarly for D. Therefore (C, C) or (D, D) is very likely. Given this, I should choose C.

(R3) Given facts F, it is likely that whatever option I realize, the other player realizes the symmetric option. Hence [with 'likely' appropriately specified] the expected utility of C is higher. I should choose C.

Newcomb's Paradox

(N1) Take one box for the reason: Given the evidence, if I take one box (make $B1$ true), I am likely to find there a very large sum; but if I take two I am likely to find the first empty, and the payoff from the second is comparatively paltry. The reasoning can be case in terms of expected utilities, where $P(E \mid B1)$ and $P(\mathit{not} - E \mid B2)$ are sufficiently high.

(N2) Take two boxes for the reason: Given the evidence, my doing does not influence in any way what the box already contains. Whatever is there, I do better by choosing $B2$.

"The problem is often posed as one of free will. Actually it is a problem of logic: the logic of agency. I can be told that I have been hypnotized to choose this, that, or whatever I choose. For all I know it might be true. But to entertain the possibility *qua* agent, is for me incoherent."

Weak Thesis: In a situation of choice, the DM does not assign extreme probabilities to options among which his choice is being made.

Strong Thesis: In a situation of choice, the DM does not assign any probabilities to options among which his choice is being made.

Spohn, Levi: on pain of contradiction or incoherence, the agent cannot have betting rates for the actions that stand at his disposal.

Given the connection between probabilities and betting rates, this means that the agent cannot coherently assign probabilities to such actions.

"Still, to understand their arguments, we should concede to supporters of the strong thesis this connection between probabilities and betting rates as their point of departure."

To pronounce a bet as fair, relative to a given agent, is thus to ascribe to the agent a certain betting disposition or a commitment to a certain betting behavior.

If the agent is prepared to buy the bet for a price C, then he should be prepared to buy it for a lower price. An if he is prepared to sell it for C, then she should be prepared to sell it for a higher price. Thus we may think of C as the highest price the agent is prepared to pay for the bet and as the lowest price he is prepared to sell it for.

If the price is increased or decreased, then the price for a fair bet is increased or decreased by the same proportion.

This follows if we *assume* that the agent is maximizing expected monetary payoffs.

$b^A_{C,S}$: A bet on proposition A that costs C to buy and pays S if won.

A bet is fair if, and only if, the agent is prepared to take each side of the bet (buy it, if offered, and sell it, if asked).

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Identification of credences with betting rates: $P(A) = C/S$

$$EU(\mathrm{Buy}\ b^{A}_{C,S}) = P(A) \cdot (S - C) + P(\overline{A})(-C)$$

$$EU(\text{Buy } b^A_{C,S}) = P(A)\cdot(S - C) + P(\overline{A})(-C) =$$
$$C/S \cdot (S - C) + (1 - C/S)\cdot -C$$

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$$EU(\text{Sell } b^A_{C,S}) = P(A) \cdot (-S + C) + P(\overline{A})(C)$$

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Suppose that A and B are alternative actions available to the agent.

$EU(A)$ and $EU(B)$ are their expected utilities for the agent *disregarding any bets that he might place on the actions themselves*.

The "gain" G for an agent who accepts and wins a bet $b^A_{C,S}$ is the *net gain* $S - C$.

If he takes a bet on A with a net gain G, his expected utility of A will instead be $EU(A) + G$. The reason is obvious: If that bet is taken, then, if A is performed, the agent will receive G in addition to $EU(A)$.

"The agent's readiness to accept a bet on an act does not depend on the betting odds but only on his gain. If the gain is high enough to put this act on the top of his preference order of acts, he will accept it, and if not, not. The stake of the agent is of no relevance whatsoever." (Spohn, 1977, p. 115)

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Take the bet "I will do action A" provided $EU(A) + G > EU(B)$ and if not, do not take the bet. *This has nothing to do with the ratio* C/S.

Suppose $EU(A) < EU(B)$, but $EU(A) + G > EU(B)$ iff $G > 4$.

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Let $C = 10$ and $S = 15$. Then, $G = 15 - 10 = 5$, which rationalizes taking the bet on A.

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Let $C = 10$ and $S = 15$. Then, $G = 15 - 10 = 5$, which rationalizes taking the bet on A.

However, if C and S drop down to 4 and 6, respectively, the quotient C/S remains unchanged but G falls below 4.

The attractiveness of the bet has nothing to do with the quotient C/S....insofar as the agent's probabilities are manifested in his betting rates, the agent has no probabilities for his own actions.
Suppose the agent's betting rate for A equals x with $0 \leq x \leq 1$. Now, if x is his betting rate, he would be expected to decline an unfair bet $b^A_{C,S}$ in which $C/S > x$. ...but, the agent may be assumed to buy the bet if only his net gain $G \ \ (= S - C)$ will be sufficiently high to put A on top of his preference ordering. Whether C/S is higher than x or not does not matter.

Criticisms:

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The agent is certain that if he takes the bet on doing the action, then he will do that action.

Betting on an action is not the same thing as deciding to do an action.

Argument II

Suppose that $P(A)$ is well-defined and $EU(A) < EU(B)$, but $EU(A) + G > EU(B)$

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The agent considers it *probable* that if he is offered a bet on A, then he will take it (but not necessarily *certain*).

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The agent considers it *probable* that if he is offered a bet on A, then he will take it (but not necessarily *certain*).

If no bet on A is offered, then the agent does not think it is probable that he will perform A, so $P(A)$ is relatively low.

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If a bet on A is offered with net gain G, then $P(A)$ increases.

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the agent thinks it probable that he will take the bet (because of the above assumption and the fact that $EU(A) + G > EU(B)$).

Thus, the probability of an action depends on whether the bet is offered or not.

If probabilities are to correspond to betting rates, the probabilities of actions under deliberation are not well-defined.

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forgetfulness

We must choose between the following 4 complex options:

1. take the bet on A & do A
2. take the bet on A & do B
3. abstain from the bet on A & do A
4. abstain from the bet on A & do B

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1. take the bet on A & do A
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Claim 1: If an agent is certain that he won't perform an option, then this option is not *feasible*

Claim 2: If the agent assigns probabilities to options, then, on pain of incoherence, his probabilities for inadmissible (= irrational) options, as revealed by his betting dispositions, must be zero.

Consider two alternatives A and B with $EU(A) > EU(B)$.

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Suppose $P(A)$, $P(B)$ are well-defined with
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Suppose $P(A)$, $P(B)$ are well-defined with
$P(A) = \text{the betting rate for } A = x$

Suppose that the agent is offered a fair bet b on A, with a positive stake S and a price C. Since b is fair, $C/S = x$. Since $1 \geq x \geq 0$ and $S > 0$, it follows that $S \geq C \geq 0$.

Thus, $G = S - C \geq 0$.

Expected utilities of the complex actions:

- $EU(b \,\&\, A) = EU(A) + G$
- $EU(\neg b \,\&\, A) = EU(A)$
- $EU(b \,\&\, B) = EU(B) - C$
- $EU(\neg b \,\&\, B) = EU(B)$

At least one of $b \,\&\, A$ and $\neg b \,\&\, B$ is admissible.

$$EU(b \,\&\, A) = EU(A) + G > EU(B) = EU(\neg b \,\&\, B)$$

This holds even if the agent's net gain is 0 (i.e., $G = S - C = 0$).

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This holds even if the agent's net gain is 0 (i.e., $G = S - C = 0$).

But then it follows that the agent should be willing to accept the bet on A even if $S = C$. Thus, the (fair) betting rate x for A must equal 1 ($P(A) = 1$), Which implies, on pain of incoherence, that $P(B) = 1 - P(A) = 0$. The inadmissible option has probability zero.

Premise-1: An agent who assigns probabilities to her present actions is required, on pain of irrationality, to assign probability of zero to any inadmissible act.

Premise-2: Once a deliberating agent assigns a subjective probability of zero to an action she no longer regards it as available for choice.

Conclusion: An agent who assigns unconditional probabilities to her own acts cannot regard any inadmissible act as available for choice.

J. Joyce. *Levi on Predicting One's Own Actions*. Philosophical Studies, 110, pgs. 69 - 102, 2002.

- Premise-1 is false
- Premise-2 is false
- We have independent reasons to think the Conclusion is false

Do we have to conclude that probabilities for one's current options must lack any connection at all to one's potential betting behavior?

Rabinowicz: Suppose that the agent is offered an opportunity to make a *betting commitment* with respect to A at stake S and price C. The agent makes a commitment (to buy or sell) not knowing whether he will be required to sell or to buy the bet.

A betting commitment is *fair* if the agent is willing to accept the commitment even if he is *radically uncertain* about what will be required of him.

In the presence of radical uncertainty, the acceptance of the commitment does not provide the agent with any extra motivation to perform the option or to abstain.

Represent the agent's uncertainty with regard to what will be required of him if he accepts the betting commitment as the set of probability distributions over $\{\text{Buy } b^A_{C,S}, \text{Sell } b^A_{C,S}\}$.

MMEU: In choice under uncertainty, the agent should maximize the minimum expected utility

P. Gärdnefors and N.-E. Sahlin. *Unreliable Probabilities, Risk-Taking and Decision-Making*. Synthese, 53, pgs. 36 - 86, 1982.

For each p, the expected utility of the betting commitment equals:

$$p[eu(\text{Buy } b^A_{C,S})] + (1-p)[eu(\text{Sell } b^A_{C,S})]$$

Letting $P(A)$ be the probability for A,
$eu(\text{Buy } b^A_{C,S}) = P(A)(S-C) + P(-A)(-C) =$
$S \cdot P(A) - C \cdot P(A) - C + C \cdot P(A) = S \cdot P(A) - C$
$eu(\text{Sell } b^A_{C,S}) = P(A)(C-S) + P(-A)(C) =$
$C \cdot P(A) - S \cdot P(A) + C - C \cdot P(A) = C - S \cdot P(A)$

Therefore, the expected utility of the betting commitment is:

$$p(S \cdot P(A) - C) + (1 - p)(C - S \cdot P(A))$$

The value of this weighted sum lies between $S \cdot P(A) - C$ and $C - S \cdot P(A)$. The minimal expected utility of the betting commitment is not lower than 0 iff none of these limiting values is lower than 0:

- $S \cdot P(A) - C \geq 0$
- $C - S \cdot P(A) \geq 0$.

1. and 2. hold iff $S \cdot P(A) = C$. I.e., (assuming $S \neq 0$), iff $P(A) = C/S$. The betting commitment with regard to A with a non-zero stake S and a price C is fair iff the quotient between the price and the stake equals the agent's probability for A.

"(...) probabilities of acts play no role in decision making. (...) The decision maker chooses the act he likes most be its probability as it may. But if this is so, there is no sense in imputing probabilities for acts to the decision maker." (Spohn (1977), pg. 115)

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- Even if it is true that we have no use for the probabilities of the options among which we choose, Spohn still needs to show that such probabilities would be positively harmful. Probabilities of actions can be of use at an earlier stage. If at the onset of deliberation, the probabilities are no longer present, they must have been contracted. (while still being accessible to the credence module, the probability of action is somehow "screened off" from the module that makes a decision.)
- Levi "I never deliberate about an option I am *certain* that I am not going to choose". If I have a low probability for doing some action A, then I may spend less time and effort in deliberation...

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- Deliberation as a *feedback* process: change in inclinations causes a change in probabilities assigned to various options, which in turn may change my inclinations towards particular options....

Deliberation is an ongoing process in which we might be inclined to different choices at different stages.

What are the players deliberating/reasoning *about*?

What are the players deliberating/reasoning *about*?

Their preferences?

What are the players deliberating/reasoning *about*?

Their preferences? The model?

What are the players deliberating/reasoning *about*?

Their preferences? The model? The other players?

What are the players deliberating/reasoning *about*?

Their preferences? The model? The other players? What to *do*?

▸ Conclusion

I. Douven. *Decision theory and the rationality of further deliberation*. Economics and Philosophy, 18, pgs. 303 - 328, 2002.

What about Common Belief of Rationality?

Replace common belief with "mutual belief": everyone believes that everyone else is rational, everyone believes that everyone believes that everyone else is rational, and so on.