LOGIC AND COMPUTATION IN PHILOSOPHY

In the Light of Logic

SOLOMON FEFERMAN

6

Gödel's Life and Work

Kurt Gödel's striking fundamental results in the decade 1929 through 1939 transformed mathematical logic and established him as the most important logician of the twentieth century. His work influenced practically all subsequent developments in the subject as well as all further thought about the foundations of mathematics.

The results that made Gödel famous are the completeness of first-order logic, the incompleteness of axiomatic systems containing number theory, and finally, the consistency of the Axiom of Choice and the Continuum Hypothesis with the other axioms of set theory. During the same decade he made other less dramatic but still significant contributions to logic, including the work on the decision problem, intuitionism, and notions of computability.

In 1940, Gödel emigrated from Austria to the United States, where he became established at the Institute for Advanced Study in Princeton. In the years following, he continued to grapple with difficult problems in set theory and at the same time began to think and write in depth about the philosophy of mathematics. Later in the 1940s he arrived at his unusual but less well-known contribution to relativistic cosmology, in which he produced solutions of Einstein's equations permitting "time travel" into the past. While Gödel's philosophical interests dominated his attention from 1950 to his death in 1978, enormous advances were made in the subject of mathematical logic during the same period. The Institute for Advanced Study became a focal point for much of this activity, largely because of his presence.

"Gödel's life and work" was first published as pp. 1–36 in Kurt Gödel, *Collected Works: Vol. I: Publications 1929–1936*, edited by Solomon Feferman et al., copyright ©1986 by Oxford University Press, New York, and is reprinted here by kind permission of the publisher. I have omitted here the sections pp. 16–28 from the original (Feferman 1986) which were devoted to a more detailed survey of Gödel's work than is found under "Life and career" in the following. However, the sections on his philosophy of mathematics and the significance and impact of his work have been retained.

A full biography of Gödel is to be found in Dawson (1996).

Gödel published comparatively little, but almost always to maximum effect; his papers are models of precision and incisive presentation. In his *Nachlass* [literary estate, presently housed at the Firestone Library, Princeton University] there are masses of detailed notes that he had made on a remarkable variety of topics in logic, mathematics, physics, philosophy, theology, and history. These have begun to reveal further the extraordinary scope and depth of his thought.

This account of Gödel's life and work is divided into three sections. The first is devoted to his life and career, and includes a description of his principle achievements. The second section concentrates on his philosophy of mathematics, and the third provides a summary assessment of the significance and impact of Gödel's work.[1,a]

Life and Career

> Kurtele, if I compare your lecture with the others, there is no comparison.
>
> —Adele Gödel[b]

In the end we search out the beginnings. Established, beyond comparison, as the most important logician of our times by his remarkable results of the 1930s, Kurt Gödel was also most unusual in the ways of his life and mind. Deeply private and reserved, he had a superb all-embracing rationality, which could descend to a maddening attention to detail in matters of everyday life. Physically, Gödel was slight of build and almost frail-looking. Cautious about food and fearful of illness, he had a constant preoccupation with his health to the point of hypochondria, yet mistrusted the advice of doctors when it was most needed. It was a familiar sight to see Gödel walking home from the Institute for Advanced Study, bundled up in a heavy black overcoat, even on warm days.

Genius will out, but how and why, and what serves to nurture it? What consonance is there with the personality, what determines the particular channels taken by the intellect and the distinctive character of what is achieved? As with any extraordinary thinker, the questions we would really like to see answered in tracing Gödel's life and career are the ones which prove to be the most elusive. What we arrive at instead is a mosaic of particularities from which some patterns clearly emerge, while the deeper ones must be left as matter for speculation, at least for the time being.

Kurt Friedrich Gödel was born 28 April 1906, the second son of Rudolf and Marianne (Handschuh) Gödel. His birthplace was Brünn, in the

---

[1]All documentation of sources is given by lettered notes at the end of the chapter. In particular, note *a* details my main sources of material and the variety of assistance I have received in preparing this chapter. Other footnotes accompany the text and are numbered. [Where no name is clearly attached to a bibliographic reference date, that is to a work of Gödel to be found listed in the References for this volume.]

Austro-Hungarian province of Moravia. This region had a mixed population which was predominately Czech but with a substantial German-speaking minority, to which Gödel's parents belonged. His father Rudolf, an energetic self-made man, had come from Vienna to work in Brünn's thriving textile industry. He was eventually to become managing director and part owner of one of the main textile firms there. The family of Kurt's mother, which came from the Rhineland region, has also been drawn to Brünn for the work in textiles. Marianne Handschuh received a better than ordinary education, partly in a French school in Brünn, in the course of which she developed life-long cultural interests.

Much of our present information concerning the family history comes from Dr. Rudolf Gödel, Kurt's brother and his elder by four years.[c] Rudolf tells us that Kurt's childhood was generally a happy one, though he was timid and could be easily upset. When he was six or seven, Kurt contracted rheumatic fever and, despite eventual full recovery, he came to believe that he had suffered permanent heart damage as well. Here are the early signs of Gödel's later preoccupation with his health. His special intellectual talents emerged early. In the family, Kurt was called *Herr Warum* (Mr. Why), because of his constant inquisitiveness.

Following the religion of his mother rather than that of his father (who was "Old" Catholic), the Gödels had Kurt baptized in the Lutheran church. In 1912, at the age of six, he was enrolled in the Evangelische Volksschule, a Lutheran school in Brünn.[2] From 1916 to 1924, Kurt carried on his school studies at the Deutsches Staats-Realgymnasium, where he showed himself to be an outstanding student, receiving the highest marks in all his subjects; he excelled particularly in mathematics, languages, and religion. (Though the latter was not given much emphasis in the family, Kurt took to it more seriously.[d]) Some of Gödel's notebooks from his young student days are preserved in the *Nachlass*, and among these the precision of the work in geometry is especially striking.

Though World War I took place during Gödel's school years, it had little direct effect on him and his family. The region of Brünn was far from the main fronts and was untouched by the devastation wrought elsewhere by the war in Europe. But the collapse of the Austro-Hungarian empire at war's end and the absorption of Moravia together with Bohemia into the new nation of Czechoslovakia was eventually to affect the German-speaking minority in adverse ways. One of the most immediate signs of the shift in national identity was the displacement of the German name "Brünn" in favor of the Czech name "Brno." For the Gödels, though, life in the years after the war continued much as before, with the family comfortably settled in a villa by that time.

Following his graduation from the Gymnasium in Brno in 1924, Gödel went to Vienna to begin his studies at the University. Vienna was to be

---

[2] A chronology with specific dates of significance in Gödel's life is in Gödel (1986), p. 37; it was prepared by John W. Dawson, Jr. [cf. also Dawson 1996].

his home for the next fifteen years, and in 1929 he was also to become an Austrian citizen. The newly created republic of Austria had entered on a difficult course following the collapse of the Austro-Hungarian empire in 1918. The political, social, and economic upheavals which followed the disappearance of monarchy and empire affected all spheres of activity, what with the economic base enormously shrunk and the *raison d'être* for the swollen bureaucracy gone. The extraordinary cultural and intellectual center that had been Vienna before the war was transformed by the changed conditions, but the Viennese spirit and ambience lived on, now sharing in the general revolutionary ferment and excitement of the 1920s. Before long, Gödel was brought into contact with the Vienna Circle, a hotbed of new thought that proved to be very significant for his work and interests [cf. Menger 1994].

At the university, Gödel was at first undecided between the study of mathematics and physics, though he apparently leaned toward the latter. It is said that Gödel's decision to concentrate on mathematics was due to his taste for precision and to the great impression that one of his professors, the number-theorist Philipp Furtwängler, made on him.[e] A description of the mathematical scene at the University of Vienna in those days is given by Olga Taussky-Todd in her reminiscences (1987) of Gödel, from which the following information is drawn. Besides Furtwängler, the professors were Hans Hahn and Wilhelm Wirtinger. Karl Menger, one of Hahn's favorite students, was an *ausserordentlicher* (associate) professor, and among the *Privatdozenten* (unsalaried lecturers) were Eduard Helly, Walter Mayer, and Leopold Vietoris. Taussky came to know Gödel as a fellow student in 1925, their first real contact coming in a seminar conducted by the philosopher Moritz Schlick on Bertrand Russell's book *Introduction to Mathematical Philosophy* (1919). Gödel hardly ever spoke, but was very quick to see problems and to point the way through to solutions. Though he was very quiet and reserved, it became evident that he was exceptionally talented. Gödel's help was much in demand and he offered such whenever needed. One could talk to him about any problem; he was always very clear about what was at issue and explained matters slowly and calmly.

Hans Hahn became Gödel's principal teacher. He was a mathematician of the new generation, had returned to Austria from a position in Bonn, and was interested in modern analysis and set-theoretic topology, as well as logic, the foundations of mathematics, and the philosophy of science. It was Hahn who introduced Gödel to the group of philosophers around Moritz Schlick, holder of the chair in the Philosophy of the Inductive Sciences (which in earlier years had been held successively by the renowned physicists Ernst Mach and Ludwig Boltzmann). Schlick's group was later baptized the "Vienna Circle" (*Wiener Kreis*) and became identified with the philosophical doctrine called logical positivism or logical empiricism.[3]

[3]For general information on Schlick's Circle and its later developments, see the articles on Moritz Schlick and logical positivism in Edwards (1967). Feigl (1969) gives

The aim of this school was to analyze knowledge in logical and empirical terms; it sought to make philosophy itself scientific and rejected metaphysical speculation. Gödel attended meetings of the Circle quite regularly in the period from 1926 to 1928, but in the following years he gradually moved away from it, though he maintained regular contact with some of its members, particularly Rudolf Carnap.

One main reason for Gödel's disengagement from the Circle was that he had developed strong philosophical views of his own which were, in large part, almost diametrically opposed to the views of the logical positivists.[f] Nevertheless, the sphere of concerns that engaged the Circle surely influenced the direction of Gödel's own interests and work. The logical empiricists had combined ideas from several sources, principally Ernst Mach's empiricist-positivist philosophy of science and Bertrand Russell's logicist program for the foundations of mathematics, both filtered through the *Tractatus Logico-philosophicus* (1922) of Ludwig Wittgenstein. The logicist ideas had been developed in great detail by Alfred North Whitehead and Russell in their famous magnum opus, *Principia Mathematica* (1910–1913), over a decade earlier. Hahn, who was at least as important as Schlick in the formation of the Vienna Circle, gave a seminar on this work in 1924 through 1925, but Gödel does not seem to have participated, since he reports first studying the *Principia* several years later.[g]

Hahn's own mathematical interest in the modern theory of functions of real numbers must also have influenced Gödel, as this involved, to a significant extent, set-theoretical considerations deriving from Georg Cantor and passing through the French school of real analysis. However, it seems that the most direct influences on Gödel in his choice of direction for creative work were Carnap's lectures on mathematical logic and the publication in 1928 of *Grundzüge der theoretischen Logik* by David Hilbert and Wilhelm Ackermann. In complete contrast to the massive tomes of Whitehead and Russell, the *Grundzüge* was a slim, unlabored, and mathematically direct volume, no doubt of greater appeal to Gödel, with his taste for succinct exposition. Posed as an open problem therein was the question whether a certain system of axioms for the first-order predicate calculus is complete. In other words, does it suffice for the derivation of every statement that is logically valid (in the sense of being correct under every possible interpretation of its basic terms and predicates)? Gödel arrived at a positive solution to the completeness problem and with that notable achievement commenced his research career. The work, which was to become his doctoral dissertation at the University of Vienna, was finished in the summer of 1929, when he was 23. The degree itself was granted in February 1930, and a revised version of the dissertation was published as Gödel (1930).[4] Al-

---

a lively picture from a more personal point of view and traces the movement of the Circle's members and their ideas. Hahn's role in the Circle is described by Menger in his introduction to the philosophical papers in Hahn (1980). [Cf. also Menger 1994.]

[4]Hahn was nominally Gödel's thesis advisor, but later in life Gödel made it known

though recognition of the fundamental significance of this work would be a gradual matter, at the time the results were already sufficiently distinctive to establish a reputation for Gödel as a rising star.

The ten years from 1929 to 1939 were a period of intense work which resulted in Gödel's major achievements in mathematical logic. In 1930 he began to pursue Hilbert's program for establishing the consistency of formal axiom systems for mathematics by finitary means. The system that had already been singled out for particular attention dealt with the general subjects of "higher" arithmetic, analysis, and set theory. Gödel started by working on the consistency problem for analysis, which he sought to reduce to that for arithmetic, but his plan led him to an obstacle related to the well-known paradoxes of truth and definability in ordinary language.[h] While Gödel saw that these paradoxes did not apply to the precisely specified languages of the formal systems he was considering, he realized that analogous nonparadoxical arguments could be carried out by substituting the notion of provability for that of truth. Pursuing this realization, he was led to the following unexpected conclusions. Any formal system S in which a certain amount of theoretical arithmetic can be developed and which satisfies some minimal consistency conditions is *incomplete*: one can construct an elementary arithmetical statement A such that neither A nor its negation is provable in S. In fact, the statement so constructed is true, since it expresses its own unprovability in S via a representation of the syntax of S in arithmetic.[5] Furthermore, one can construct a statement C which expresses the consistency of S in arithmetic, and C is not provable in S if S is consistent. It follows that, if the body of finitary combinatorial reasoning that Hilbert required for execution of his consistency program could all be formally developed in a single consistent system S, then the program could not be carried out for S or any stronger (consistent) system. The incompleteness results were published in Gödel (1931); the stunning conclusions and the novel features of his argument quickly drew wide attention and brought Gödel recognition as a leading thinker in the field.

One of the first to recognize the potential significance of Gödel's incompleteness result and to encourage their full development was John von Neumann.[i] Only three years older than Gödel, the Hungarian-born von Neumann was already well known in mathematical circles for his brilliant and extremely diverse work in set theory, proof theory, analysis, and mathematical physics. Others interested in mathematical logic were slower to absorb Gödel's new work. For example, Paul Bernays, who was Hilbert's assistant and collaborator, although quickly accepting Gödel's results, had difficulties with his proofs that were cleared up only after repeated correspondence.[j] Gödel's work even drew criticism from various quarters, which was

---

that he had completed the work before showing it to Hahn and that he made use of Hahn's (essentially editorial) suggestions only when revising the thesis for publication; see Wang (1981), pp. 653–654.

[5]The technical device used for the construction is now called "Gödel numbering."

invariably due to confusions about the necessary distinctions involved, such as that between the notions of truth and proof. In fact, the famous set-theorist Ernst Zermelo interpreted these concepts in such a way as to arrive at a flat contradiction with Gödel's results. In correspondence during 1931 Gödel took pains to explain his work to Zermelo, apparently without success.[k] In general, however, the incompleteness theorems were absorbed before long by those working in the mainstream of mathematical logic; indeed, one can fairly say that Gödel's methods and results came to infuse all aspects of that mainstream.[6]

Gödel's incompleteness work became his *Habilitationsschrift* (a kind of higher dissertation) at the University of Vienna in 1932. In his report on it, Hahn lauded Gödel's work as epochal, constituting an achievement of the first order.[l] The *Habilitation* conferred the title of *Privatdozent* and provided the *venia legendi*, which gave Gödel the right to deliver lectures at the university, but without pay except for fees he might collect from students. As it turned out, he was to lecture only intermittently in Vienna during the following years.

Meanwhile, significant changes had also been taking place in Gödel's personal life. At the age of 21 he met his wife-to-be, Adele Nimbursky (née Porkert), but the difference in their situations led to objections to their developing relationship from his parents, especially his father. Adele was a dancer, had been briefly married before, and was six years older than Kurt. Though his father died not long after, Kurt and Adele were not to be married for another ten years.

The death of Kurt's father in 1929, at the age of 54, was unexpected; fortunately he left his family in comfortable financial circumstances. While retaining the villa in Brno, Gödel's mother took an apartment in Vienna with her two sons. By then Kurt's brother Rudolf had become successfully established as a radiologist. Rudolf never married, and during their period together in Vienna the three of them frequently went out, especially to the theater. According to his brother, at home Kurt went out of his way to "hide his light under a bushel," despite his growing international fame.[m]

In the early 1930s Gödel steadily advanced his knowledge in many areas of logic and mathematics. He took a regular part in Karl Menger's colloquium in Vienna, which had begun meeting in 1929, and he also assisted in the editing of its reports, *Ergebnisse eines mathematischen Kolloquiums*. In the period from 1932 to 1936 he published thirteen short but noteworthy papers in that journal on a variety of topics, including intuitionistic logic, the decision problem for the predicate calculus, geometry, and length of proofs. Some of the results in logic were to be of lasting interest, though not of the same order as his previous work on completeness and incompleteness. During the same period he was an active reviewer for *Zentralblatt für Mathematik und ihre Grenzgebiete* and, less frequently, for *Monatshefte für Mathematik und Physik*.[7]

---

[6] For the influence of Gödel's work on logicians in the 1930s, see Kleene (1981, 1987).
[7] After 1936, Gödel never reviewed again for these or any other journals.

Menger occasionally invited foreign visitors of interest to speak in his colloquium. Among them was the Polish logician Alfred Tarski, who was shortly to become famous for his work on the notion of truth in formal languages and increasingly, later, for his leadership in the development of model theory. In early 1930 Tarski spent a few weeks in Vienna and was introduced to Gödel at that time; Gödel used the occasion to discuss the results of his 1929 dissertation. Tarski returned for a more extensive visit as a guest of Menger's colloquium during the first half of 1935.[n]

Initially, in his unsalaried position as *Privatdozent*, Gödel had to depend on the resources of his family for his livelihood. However, these means were supplemented before long by income from visiting positions in the United States of America. Gödel's first visit was to the Institute for Advanced Study in Princeton during the academic year 1933 through 1934. The Institute had been formally established in 1930, with Albert Einstein and Oswald Veblen appointed its first professors two years later by its original director, Abraham Flexner. Veblen, who was a leader in the development of higher mathematics in America and had played a principal role in building up an outstanding mathematics department at Princeton University, was largely responsible for selecting the further "matchless" mathematics faculty at the Institute: James Alexander, Marston Morse, John von Neumann, and Hermann Weyl.[o] In addition, he helped arrange postdoctoral visits for rising young mathematicians, including Gödel; no doubt Veblen had heard about Gödel from von Neumann, who regarded him as "the greatest logician since Aristotle."[8,p]

Gödel's visit in 1933 through 1934 was the first of three that he was to make to the Institute before taking up permanent residence there in 1940. He lectured on the incompleteness results in Princeton in the spring of 1934. Apparently he had already begun to work with some intensity on problems in set theory; at the same time, he felt rather lonely and depressed during this period in Princeton. Following his return to Europe, he had a nervous breakdown and entered a sanatorium for a time. In the following years there were to be recurrent bouts of mental depression and exhaustion. A scheduled return visit to Princeton had to be postponed to the fall of 1935 and then was unexpectedly cut short after two months, again on account of mental illness. More time was spent in a sanatorium in 1936, and Gödel was unable to carry on at the University of Vienna until the spring of 1937.[q] When he was finally able to resume teaching, he lectured on some of his major new results in axiomatic set theory, the development of which we now trace.

Two problems that had preoccupied workers in the field of set theory since its creation by Cantor beginning in the 1870s concerned the Well-

[8] Apropos of this, Kreisel remarks: "If Gödel's work is to be compared to that of one of the ancients, Archimedes is a better choice than Aristotle (who invented logic, but proved little about it). Archimedes did not invent mechanics, as Gödel did not invent logic. But both of them changed their subjects profoundly" (Kreisel 1980, p. 219).

Ordering Principle and the cardinality of the continuum. Zermelo had examined the first of these, both informally (1904) and then within the framework of his newly introduced system of axioms for set theory (1908, 1908a), and had shown that the Well-Ordering Principle is equivalent to the Axiom of Choice (AC). There was much intense dispute among mathematicians about the evidence for or against this new "axiom." Under its assumption, every infinite set would have a determinate cardinal number in an ordered list of transfinite cardinals. After Cantor proved that the continuum (i.e., the measurement line) is uncountable, he conjectured that its cardinal would be the least among all uncountable cardinal numbers. This conjecture became known as the Continuum Hypothesis (CH).[9]

It was to these problems in set theory that Gödel began to devote himself as his main area of concentration after obtaining the incompleteness results.[r] He considered the statements of AC and CH in the framework of axiomatic set theory (by then enlarged and made more precise through the work of Fraenkel, Skolem, von Neumann, and Bernays), to see whether they could be settled on the basis of the remaining axioms. His major result, finally achieved by the summer of 1937, was that both the Axiom of Choice and the Continuum Hypothesis (even in a natural generalized form, GCH) are consistent with the Zermelo-Fraenkel axioms (ZF) without the Axiom of Choice, and hence cannot be disproved from them if the axioms of ZF are consistent. This result at least provided a minimal guarantee of safety in the use of the seemingly problematic statements AC and GCH.

Underlying Gödel's proof was his definition within ZF of a general notion of constructibility for sets. His plan, which emerged quite early, was to show that the constructible sets form a model for all the axioms of ZF and, in addition, for the Axiom of Choice and the Generalized Continuum Hypothesis. In 1935 he was able to tell von Neumann that he had succeeded in verifying all the ZF axioms together with AC in this model, but, as noted above, it took him two more years to push his work to completion by verifying that GCH holds in the constructible sets as well. With the modest techniques then available, the details that Gödel needed to establish were formidable, and this deep and complicated work caused him much effort, especially in its final part. Perhaps that was one reason for the mental stress he suffered throughout much of the period from 1934 to 1937.[s]

The years 1937 to 1939 brought further significant changes in both Gödel's personal life and career. His mother returned to her home in Brno in 1937, though his brother remained in Vienna to continue his medical practice. That move may have eased the way for Kurt Gödel and Adele Nimbursky to be married finally, in September 1938. The marriage of Kurt and Adele proved to be a warm and enduring one, and for Kurt a source of constant support in difficult times ahead.[t]

---

[9] A full history of the emergence of the Axiom of Choice as a fundamental principle of set theory and of the controversies that surrounded it is provided by Moore (1982).

During the 1930s there were many shifts in the lives of Gödel's friends and colleagues in Vienna. Marcel Natkin and Herbert Feigl, early friends from the Vienna Circle, had already left Vienna at the turn of the decade, the first for Paris and the second for America. Rudolf Carnap left to teach in Prague in 1931; he was eventually to go to America, too. Gödel's teacher Hans Hahn died in 1934, of natural causes. Then in 1936, Moritz Schlick, the central figure of the Vienna Circle, was murdered by a deranged former student on the way to a lecture; naturally, the case created a sensation. Upset by this turn of events and the general situation in Austria, Karl Menger left the following year to take up a position at Notre Dame. Gustav Bergmann and Abraham Wald, two other contemporaries of Gödel's, also left for America in 1938.

These and related changes were taking place in the context of the difficult economic conditions that had been gripping European nations since the severe depression of 1929 and of the political situation created by the advent to power in 1933 of Adolf Hitler and the Nazis in Germany. In 1934 Austria itself fell under the rule of a semifascist regime, led by Engelbert Dollfuss until his assassination by Austrian Nazis later that same year. Dollfuss' murder was a premature attempt by the Nazis to gain power in Austria and to carry out the *Anschluss* (political and economic union) of Austria with Germany, which had been forbidden by the 1919 Treaty of Saint-Germain. There was much sentiment for *Anschluss* among certain groups of Austrians, but the main pressure came from Hitler. That mounted steadily until Hitler's threat of invasion brought down the succeeding Schuschnigg regime in the spring of 1938. Austria thenceforth became a province (*Ostmark*) of a wider Nazi Germany. The year 1938 saw the beginning of a general transformation of Austrian cultural and intellectual life, comparable to that in Germany five years previously. This led to an exodus of intellectuals, particularly those of Jewish background, for whom the move was a matter of survival, while for others emigration was a reaction to the supernationalistic and racist politics characteristic of the Nazi regime. An incidental result of all this was the final disintegration of the Vienna Circle.[u] As for Gödel, his stance was basically apolitical and noncommittal; while he was by no means unaware of what was taking place, he ignored the increasingly evident implications of the transformations around him.

At the urging of Menger, Gödel visited America once more in 1938 through 1939. He spent the fall term at the Institute for Advanced Study, where he lectured on his new results concerning the consistency of the axiom of choice and the generalized continuum hypothesis. For the spring term he joined Menger at Notre Dame, where he lectured again on his set-theoretical work and conducted an elementary course on logic with Menger. Gödel then returned to Vienna to rejoin his wife, whom he had left the previous fall only two weeks after their marriage.

Gödel planned to return to the Institute for Advanced Study in the fall of 1939, but political events intervened; his life was now directly affected by the Nazi regime in two very different ways. He was called up for a military physical examination and much to his surprise (in view of his generally poor health and his conviction that he had a weak heart) found "fit for garrison duty." Then there was the question of his situation at the University of Vienna. The unpaid position of *Privatdozent* had been abolished by the Nazis, who had replaced it by a new paid position called *Dozent neuer Ordnung*. The latter, however, required a fresh application that could be rejected on political or racial grounds. Although Gödel applied for the new position in September 1939, approval was slow in forthcoming. Questions were raised about his associations with Jewish professors (Hahn in particular), and while it was recognized that he was apolitical, his lack of open support for the Nazis counted against him. In this insecure situation and with the likely possibility that he would be drafted (war having begun in September), Gödel wrote Veblen in desperation in November 1939, seeking assistance to leave. Somehow, U.S. nonquota immigrant visas and German exit permits were arranged, and Kurt and Adele managed to leave Vienna in January 1940. As it was too dangerous at that point to cross the Atlantic by boat, they made their way instead by train through Eastern Europe, then via the Trans-Siberian Railway across Russia and Manchuria, and thence to Yokohama. From there they traveled by ship to San Francisco, and in March 1940 they finally proceeded by train to Princeton.[v]

Gödel was never to return to Europe. Ironically, his application for *Dozent neuer Ordnung* was belatedly approved in June 1940.[w] Long afterward he remained bitter about his predicament in Austria in the year 1939 through 1940, apparently blaming it more on Austrian "sloppiness" (*Schlamperei*) than on the outrageous Nazi conditions. In particular, on the occasion of his 60th birthday in 1966, he turned down an honorary membership in the Austrian Academy of Sciences. However, he couched the refusal in pseudolegalistic terms which suggested that his U.S. citizenship might be jeopardized if he were to accept membership in the academy of the country of his former citizenship.[x]

In 1940 Gödel was made an Ordinary Member of the Institute for Advanced Study, and he and his wife settled in Princeton, where they established a quiet social life. Among Gödel's closest friends were Albert Einstein and Oskar Morgenstern. The latter was another ex-Viennese, who had emigrated in 1938 and taken a position at Princeton University. Already established as a mathematical economist, Morgenstern was later to become well known to a wide public through his important and influential work with von Neumann, *The Theory of Games and Economic Behavior* (1944). (Von Neumann himself would have been less accessible to Gödel during the early 1940s, since he was frequently away from the Institute in his capacity as consultant for innumerable government war projects.[y])

Morgenstern had many stories to tell about Gödel. One concerned the occasion when, in April 1948, Gödel became a U.S. citizen, with Einstein and Morgenstern as witnesses.[z] Gödel was to take the routine citizenship examination, and he prepared for it very seriously, studying the U. S. Constitution assiduously. On the day before he was to appear, Gödel came to Morgenstern in a very excited state, saying: "I have discovered a logical-legal possibility by which the United States could be transformed into a dictatorship." Morgenstern realized that, whatever the logical merits of Gödel's arguments, the possibility was extremely hypothetical in character, and he urged Gödel to keep quiet about his discovery at the examination. The next morning, Morgenstern drove Gödel and Einstein from Princeton to Trenton, where the citizenship proceedings were to take place. Along the way Einstein kept telling one amusing anecdote after another in order to distract Gödel, apparently with great success. At the office in Trenton, the official was properly impressed by Einstein and Morgenstern, and invited them to attend the examination, normally held in private. He began by addressing Gödel: "Up to now you have held German citizenship." Gödel corrected him, explaining that he was Austrian. "Anyhow," continued the official, "it was under an evil dictatorship . . . but fortunately, that's not possible in America." "On the contrary," Gödel cried out, "I know how that can happen!" All three had great trouble restraining Gödel from elaborating his discovery, so that the proceedings could be brought to their expected conclusion.

Einstein and Gödel could frequently be seen walking home together from the institute, engaged in rather intense conversations. A number of stories concerning the two have been recounted by the mathematician Ernst Straus, who was Einstein's assistant during the years 1944 through 1948. He summarized appreciatively their unusual relationship in the following passage, take from his reminiscences (Straus 1982, p. 422).

> The one man who was, during the last years, certainly by far Einstein's best friend, and in some ways strangely resembled him most, was Kurt Gödel, the great logician. They were very different in almost every personal way—Einstein gregarious, happy, full of laughter and common sense, and Gödel extremely solemn, very serious, quite solitary, and distrustful of common sense as a means of arriving at the truth. But they shared a fundamental quality: both went directly and wholeheartedly to the questions at the very center of things.

At the institute Gödel had no formal duties and was free to pursue his research and studies as he pleased. During the first years there he continued his work in mathematical logic, along various lines. In particular, he made strenuous efforts to prove the independence of the Axiom of Choice and the Continuum Hypothesis, but only with partial success, and that just on the former problem. His efforts in this direction were never published; they remain to be deciphered (if possible) from notebooks in his *Nachlass*.[10] Another achievement early in this period (though not published

[10] The full independence results were eventually obtained by Paul Cohen (1963).

until 1958) was a new constructive interpretation of arithmetic that proved its consistency, but via methods going beyond evidently finitary means in Hilbert's sense.

From 1943 on, Gödel devoted himself almost entirely to philosophy, first to the philosophy of mathematics and then to general philosophy and metaphysics. The year 1944 marks the publication of his paper on Bertrand Russell's mathematical logic, which was extremely important both for its searching analysis of Russell's work and for its open statement of Gödel's own "platonistic" views of the reality of abstract mathematical objects.[11]

An expository paper on Cantor's continuum problem in 1947 brought out these views quite markedly in the context of set theory. One other writing of a partly philosophical character from this period did not appear until somewhat later, namely, the address in 1946 to the Princeton Bicentennial Conference on Problems of Mathematics. As for general philosophy, Gödel continued his long-pursued reading and study of Kant and Leibniz, turning also to the phenomenology of Edmund Husserl in the late 1950s.[aa] In Gödel's *Nachlass* are many notes on the writings of these philosophers.

An apparent exception to these directions of thought was Gödel's surprising work on the general theory of relativity during the period from 1947 to 1951, in which he produced new and unusual cosmological models that, in theory, permit "time travel" into the past. According to Gödel, this work did not come out of his discussions with Einstein but rather was motivated by his own interests in Kant's philosophy of space and time.[bb] Einstein himself was preoccupied, as he had been for a long time, with constructing a unified field theory, a project about which Gödel was skeptical.[cc] In this work Gödel brought to bear mathematical techniques and physical intuitions that one who was familiar only with his papers in logic would not have expected. The mathematics, however, harks back to his brief contributions to differential geometry in the 1930s, as well as to his studies of analysis with Hahn and in Menger's colloquium.

In addition to reflecting Gödel's primary interests in logic, philosophy and, to a lesser extent, mathematics and physics, the notebooks in his *Nachlass* are unexpectedly wide ranging, revealing, for example, sustained

---

[11] An amusing aside in this respect has its source in a statement by Bertrand Russell to be found in the second volume of his *Autobiography* (1968, pp. 355–356): "I used to go to [Einstein's] house once a week to discuss with him and Gödel and Pauli. These discussions were in some ways disappointing, for, although all three of them were Jews and exiles and, in intention, cosmopolitans, I found that they all had a German bias toward metaphysics [and that] Gödel turned out to be an unadulterated Platonist."

Gödel's attention was drawn to this in 1971 and he drafted a reply that is preserved in the *Nachlass*, though it was never actually sent: "As far as the passage about me [by Russell] is concerned, I have to say *first* (for the sake of truth) that I am not a Jew (even though I don't think this question is of any importance), 2) that the passage gives the wrong impression that I had many discussions with Russell, which was by no means the case (I remember only one), 3) Concerning my 'unadulterated' Platonism, it is no more 'unadulterated' than Russell's own in 1921." Fuller quotations are given in Dawson (1984a), pp. 13 and 15.

interests in history and theology. The latter even included a long-standing fascination with demonology.[dd]

Gödel was made a Permanent Member of the Institute for Advanced Study in 1946. His subsequent promotion to Professor in 1953 required him to take part in some aspects of Institute business.[12] He devoted a good deal of time to the details of these affairs and, in particular, took very seriously the increasingly frequent applications by logicians for visiting memberships.[13] When logic started to flourish in that period, the institute became a Mecca for younger logicians—many of them rising stars—and drew visits as well from older colleagues of the prewar generation, such as Paul Bernays. Gödel limited his contacts with most younger visitors, though he would give serious consideration to their work and interests and would volunteer suggestions. A few of the more advanced logicians were able to establish deeper scientific and personal relations with him and were privy to his thoughts and speculations in extensive conversations; most prominent among these were William Boone, Georg Kreisel, Gaisi Takeuti, Dana Scott, and Hao Wang. Others whose work impressed him and with whom he had some significant (though less extensive) contact were Clifford Spector and Abraham Robinson. But Gödel never had students or disciples in the usual sense of the word.

Beginning in 1951, Gödel received many honors. Particularly noteworthy are his sharing of the first Einstein Award (with Julian Schwinger) in 1951, his choice as Gibbs Lecturer for 1951 by the American Mathematical Society, and his elections to membership in the National Academy of Sciences (1955), to the American Academy of Arts and Sciences (1957), and to the Royal Society of London (1968). In 1975 he was awarded the National Medal of Science by President Ford, but because of ill health he could not attend the ceremony. A complete list of awards and honors is given in Dawson's "A Gödel chronology" (Gödel 1986, pp. 37–43).

In the last fifteen years of his life, Gödel was busy with visitors, institute business, and his own philosophical studies; during this time he returned to logic only rarely. Some papers were revised and a few notes were added to new translations. In particular, he expended a good deal of effort over a period of years on a translation and revision of his 1958 paper, which gave a constructive interpretation of arithmetic. The revised work never reached

---

[12]Questions have been raised about the relative lateness of this promotion in Ulam (1976), p. 80, and Dyson (1983), p. 48. One explanation has it that promotion was held back for Gödel's sake, so as not to burden him with the administrative responsibilities accompanying faculty status. Another has it that there were fears Gödel's exceptional attention to detail and his legalistic turn of mind would hinder the conduct of institute business if he were to assume those responsibilities. (See also Dawson (1984a), p. 15.)

[13]Concerning the latter, Gödel's colleague Hassler Whitney commented as follows: "Gödel was keenly interested in the affairs of the Institute. It was . . . hard to appoint a new member in logic since Gödel could not 'prove to himself that a number of candidates shouldn't be members, with the evidence at hand' " (quotation from *The Mathematical Intelligencer*, 1 (1978), p. 182). For a complementary view, see Kreisel (1980), p. 159.

published form, though it was found in galley proof in his *Nachlass*.[14] In the early 1970s there was a flurry of interest and excitement among logicians about notes by Gödel in which he proposed new axioms for set theory that were supposed to imply the falsity of the Continuum Hypothesis, but essential problems were found in the arguments and the notes were withdrawn. Gödel blamed his having overlooked the difficulties on the drugs he was then taking for his illness. [Cf. Gödel 1995, pp. 405–425.]

In fact, Gödel's health was poor from the late 1960s on. Among other things he had a prostate condition for which surgery had been recommended, but he would never agree to have the operation done. Along with his hypochondriacal tendencies he also had an abiding distrust of doctors' advice. (Back in the 1940s, for example, he delayed treatment of a bleeding ulcer so long that he would have died, had it not been for emergency blood transfusions.) In addition to prostate trouble, he was still convinced that his heart was weak, although there was no medical substantiation. During the last few years of his life, his wife Adele was unable to help him to the same extent as before, since she herself was partially incapacitated by a stroke and was, for a time, moved to a nursing home. Gödel's depressions returned, accompanied and aggravated by paranoia; he developed fears about being poisoned and would not eat. He died in Princeton Hospital on 14 January 1978 of "malnutrition and inanition caused by personality disturbance."[ee] Adele survived him by three years, dying on 4 February 1981. Kurt and Adele had no children, leaving Kurt's brother Rudolf as the sole surviving member of the Gödel family [subsequently deceased] . . . .

Gödel's Philosophy of Mathematics

Gödel is noted for his vigorous and unwavering espousal of a form of mathematical realism (or "platonism"). In this general direction he joins the company of such noted mathematicians and logicians as Cantor, Frege, Zermelo, Church, and (in certain respects) Bernays. These views of mathematics also accord with the implicit working conceptions of most practicing mathematicians (the "silent majority"). However, the preponderance of developed thought on the philosophy of mathematics since the late nineteenth century has been critical of realist positions and has led to a number of alternative (and opposing) standpoints, going under such names as constructivism, formalism, finitism, nominalism, predicativism, definitionism, positivism, and conventionalism. Leading figures identified with one or another of these positions are Kronecker, Brouwer, Poincaré, Borel, Hilbert, Weyl, Skolem, Heyting, Herbrand, Gentzen, and Curry, as well as Carnap. Russell veered from a distinctly realist position in his earlier work, *The Principles of Mathematics* (1903), to a more equivocal predicativist approach in *Principia Mathematica* (1910–1913) coauthored with Whitehead.

[14] It is reproduced in volume 2 of Gödel's *Collected Works* (1990) as Gödel (1972).

Critics of the realist position have raised both ontological and epistemological issues. With respect to the former, the ideas of mathematical objects as independently existing abstract entities and, in particular, of infinite classes as "completed" totalities are considered to be problematic. For the latter, questions have been raised about the admissibility of such principles as that of the excluded middle and the axiom of choice, each in its way leading to nonconstructive existence proofs. Especially in the earlier part of this century, the paradoxes of classes found by Cantor, Burali-Forti, and Russell were felt in addition to require radical reconsideration of the entire set-theoretic, philosophically platonist approach to the foundations of mathematics. This last receded in importance when it was recognized how Zermelo had rescued set theory from the obvious contradictions by means of his axiomatization and its underlying interpretation in the iterative conception of sets.

Hilbert and Brouwer were perhaps the most influential figures proposing alternative foundational schemes during the period in which Gödel was beginning his work in logic. Hilbert had elaborated a program to "secure" mathematics—including, as he hoped, Cantor's set theory—by means of finitary consistency proofs for formal axiom systems. Brouwer rejected nonconstructive existence proofs and Cantorian conceptions of "actual" infinities, seeking to rebuild mathematics according to his own intuitionistic version of constructivism. In Vienna, special attention was naturally also given to the program of the logical empiricists developed by Hahn, Schlick, Carnap, and others. Their aim was to place mathematics in a conventionalist role as the "syntax of language," thus separating it from physical science, which itself was to rest finally on empirical observation. According to his own account much later,[ff] Gödel had arrived at a general platonist viewpoint by 1925, around the time he came to Vienna. When be began to take part in the meetings of the Vienna Circle, he did so primarily as an observer, not openly disputing the approach taken, though disagreeing with it. Gödel did remark critically on the positions of Hilbert and Brouwer in the introduction of his dissertation (1929), but mainly in connection with the completeness problem. In particular, he made some trenchant remarks there concerning the idea of consistency as the criterion for existence. That view could be identified with Hilbert, though it was not a necessary part of Hilbert's program. However, this discussion was omitted from the published version (1930) of the dissertation. Gödel openly criticized the related idea, again deriving from Hilbert, of consistency sufficing for correctness when one extends a system of meaningful statements by a system of "ideal" statements and axioms. These remarks were made at the important symposium on the foundation of mathematics held at Königsberg in 1930 (see 1931a). Gödel made no further published statements on the nature of his position until the appearance of his substantial article (1944) on Russell's mathematical logic. In retrospect, however, one can recognize some brief remarks or footnotes in earlier papers as providing indications of the directions of his thought.

The main published sources for Gödel's views on the philosophy of mathematics are the papers published in 1944, 1946, 1964 (a revised and expanded version of 1947), and 1958, as well as his personal and written communications to Hao Wang reproduced in Wang (1974). Further sources that appear in Gödel (1986, 1990) but were not previously printed are the introduction to the 1929 paper, the 1972 revised and expanded version of the 1958 paper, and finally some brief notes (1972a) in Gödel (1990). All this is amplified but not modified in any significant way by unpublished manuscripts and correspondence found in Gödel's *Nachlass.** In particular, the article "Is mathematics syntax of language?" would have been Gödel's first systematic published attack on the program of the logical positivists, had it appeared in the Carnap volume as intended.[15]

The main features of Gödel's philosophy of mathematics that emerge from these sources are as follows. Mathematical objects have an independent existence and reality analogous to that of physical objects. Mathematical statements refer to such a reality, and the question of their truth is determined by objective facts which are independent of our own thoughts and constructions. We may have no direct perception of underlying mathematical objects, just as with underlying physical objects, but—again by analogy—the existence of such is necessary to deduce immediate sense perceptions. The assumption of mathematical objects and axioms is necessary to obtain a satisfactory system of mathematics, just as the assumption of physical objects and basic physical laws is necessary for a satisfactory account of the world of appearance. An example of mathematical "sense data" requiring this kind of explanation is provided by instances of arithmetical propositions whose universal generalizations demand assumptions transcending arithmetic; this is a consequence of Gödel's incompleteness theorem.[16] While mathematical objects and their properties may not be immediately accessible to us, mathematical intuition can be a source of genuine mathematical knowledge. This intuition can be cultivated through deep study of a subject, and one can thus be led to accept new basic statements as axioms. Another justification for mathematical axioms may be their fruitfulness and the abundance of their consequences; however, that is less certain than what is guaranteed by intuition.

*[See, however, chapter 8 in this volume for a possible reevaluation of this picture.]

[15]In the penultimate section of his introductory note to (1944) in Gödel (1986), Charles Parsons suggests that in one respect, at least, Gödel is more closely engaged with the ideas of the Vienna Circle than is ordinarily viewed. The relationship has to do with the thesis that mathematics is analytic. In (1944) Gödel considers two senses of the notion of analyticity of a statement, respectively (roughly speaking) that of its being true in virtue of the definitions of the concepts involved in it and that of its being true in virtue of the meaning of those concepts. In (1944) Gödel rejects the thesis that mathematics is analytic in its first sense but accepts it in its second sense (at least for the theory of types and axiomatic set theory). [Two versions of the unpublished paper "Is mathematics syntax of language?" have subsequently appeared in Gödel (1995), pp. 334–362; cf. the introductory note to those items by W. Goldfarb, *op. cit.*, pp. 324–334.]

[16]Gödel frequently refers to such propositions as arithmetical problems of Goldbach type (the conjecture that every even positive integer is a sum of two odd primes).

Gödel discussed these ideas most explicitly in connection with set theory and Cantor's continuum problem, particularly his 1947 and 1964 papers (see especially the supplement to 1964). There he argues that Cantor's notion of (infinite) cardinal numbers is definite and unique, and hence that the Continuum Hypothesis CH has a determinate truth value, even though efforts to settle it thus far have failed.[17] One can begin by examining the question of its demonstrability with reference to presently accepted axioms for set theory. These axioms (for example, the Zermelo-Fraenkel system ZF) are evidently true for the iterative structure of sets in the cumulative hierarchy. This is a perfectly self-consistent conception that is untouched by the paradoxes. In Gödel's view, the axiom of choice is just as evident for this notion as are the other axioms, and hence Cantor's cardinal arithmetic is adequately represented in the axiom system.

In his 1940 paper Gödel had shown that AC and CH are consistent with ZF, by use of his model L of constructible sets. But he conjectured in 1947 that CH is false, hence underivable from the true axioms ZF + AC. After Cohen (1963) proved the independence of CH from ZF + AC, Gödel could expand on the anticipated undecidability of CH by presently accepted axioms. This confirmed what he had long expected, namely, that new axioms would be needed to settle CH. In particular, he mentioned the possibility of using strong axioms of infinity (or large cardinal axioms) for these purposes, pointing out once more that in view of the incompleteness theorem, such axioms are productive of arithmetical consequences. But he also thought that axioms based on new ideas may be called for.[18] He argued again that such axioms need not be immediately evident, but may be arrived at only after long study and development of the subject.

There are briefer discussions or indications by Gödel in other of his publications concerning his belief in the objectivity of mathematical notions outside set theory: abstract concepts (in 1944), absolute demonstrability and definability (in 1946), and constructive functions and proofs (in 1958 and 1972). One should also mention the steady interest he showed in intuitionism through several publications in the 1930s and his 1958 paper. Thus his mathematical realism is not necessarily confined to set theory, though that is where it is most thoroughly elaborated.

---

[17] There is one earlier statement by Gödel that apparently presents a different view concerning the questions of definiteness of set-theoretical concepts. Namely, at the end of (1938) he says: "The proposition A [$V = L$] added as a new axiom seems to give a natural completion of the axioms of set theory, insofar as it determines the vague notion of an arbitrary infinite set in a definite way." In a personal communication, Martin Davis has argued: "This is not at all in the spirit of the point of view of (1947), and . . . it suggests that Gödel's 'platonism' regarding sets may have evolved more gradually than his later statements would suggest." There is currently no further evidence available which would help clarify Gödel's intentions in his 1938 remark and its relationship to his later views. [Cf. also chapter 8 in this volume.]

[18] Indeed, he proposed certain new axioms himself in unpublished manuscripts [which have subsequently been published in Gödel (1995) as (*1970a, b, and c)].

In the correspondence reproduced in Wang (1974, pp. 8–11), Gödel credits a large part of his main success, where others had failed, to his realist views; they are said to have freed him from the philosophical prejudices of the times which had shackled others. In this respect he mentions Skolem's failure to arrive at the completeness of predicate logic and Hilbert's failure to "prove" CH in contrast to his own results published in 1930 and 1940; he also mentions his belief in the objectivity of mathematical truth as having led to the incompleteness theorems of 1931. Whatever their final merits, the efficacy of Gödel's views seems in this respect to be indisputable.

A deeper examination of Gödel's ideas on the philosophy of mathematics is given in the introductory notes to a number of Gödel's published and unpublished papers in the three volumes of his *Collected Works* (1986, 1990, and 1995).

Character, Impact, and Influence of the Work

Gödel's main published papers from 1930 to 1940 were among the most outstanding contributions to logic in this century, decisively settling fundamental problems and introducing novel and powerful methods that were exploited extensively in much subsequent work. Each of these papers is marked by a sense of clear and strong purpose, careful organization, great precision—both formal and informal—and by steady and efficient progress from start to finish, with no wasted energy. Each solves a clear problem, simply formulated in terms well understood at the time (though not always previously formulated as such). Their significance, then, was in one sense prima facie evident, though their significance more generally for the foundations of mathematics would prove to be the subject of unending discussion. As he has told us, Gödel was strongly motivated by his realist philosophy of mathematics, and he credited it with much of the reason for his success in being led to the "right" results and methods.[gg] Nevertheless, philosophical questions are given bare notice in these papers. In addition, Gödel made special efforts where possible to extract results of potential mathematical (as opposed to logical or foundational) interest—for example, the compactness theorem for the first-order predicate calculus (1930), the incompleteness of axiomatic arithmetic with respect to (quantified) Diophantine problems (1931), and the existence of nonmeasurable PCA sets of reals in the universe of constructible sets (1938).[19]

Concerning Gödel's methods, one may say that many of the constructions and arguments were technically difficult for their time, or at any rate

[19]This must be moderated in two respects. It is certainly the case that Gödel himself never made any use of compactness and that we have the benefit of hindsight in assessing its mathematical value. Moreover, according to Kreisel (1980), p. 197, the existence of nonmeasurable PCA sets in L was suggested to Gödel by Stanislaw Ulam. [Gödel returned to undecidable Diophantine propositions in an unpublished lecture, which appears as (*193?) in Gödel (1995), pp. 164–175.]

too novel or unexpected to be readily absorbed (though the arguments for completeness were largely anticipated by Skolem, and those for incompleteness and completeness both seem much simpler now). But technical ingenuity is never indulged in or displayed for its own sake in Gödel's papers; rather, it is always there as a means to an end. We have considerable evidence that Gödel worked and reworked his papers many times, partly to arrive at the most efficient means of presentation.

Gödel's contributions bordered on the two fundamental technical concepts of modern logic: *truth for formal languages* and *effective computability*. With respect to the former he stated in his 1934 lectures at Princeton (and elaborated in some correspondence) that he was led to the incompleteness of arithmetic via his recognition of the undefinability of arithmetic truth in its own language, though he took care to credit Tarski for elucidating the exact concept of truth and establishing its undefinability. In the same lectures he offered a notion of general recursiveness in connection with the idea of effective computability; this was based on a modification of a definition proposed by Herbrand. In the meantime, Church was propounding his thesis, which identified the effectively computable function with the λ-definable functions. But Gödel was unconvinced by Church's thesis, since it did not rest on a direct conceptual analysis of the notion of finite algorithmic procedure. For the same reason he resisted identifying the latter with the general recursive functions in the Herbrand-Gödel sense. Indeed, in his Princeton lectures Gödel said that the notion of effectively computable function could serve just as a heuristic guide. It was only when Turing, in 1937, offered the definition in terms of his "machines" that Gödel was ready to accept such an identification, and thereafter he referred to Turing's work as having provided the "precise and unquestionably adequate definition of formal system" by his "analysis of the concept of 'mechanical procedure'" needed to give a general formulation of the incompleteness results.[hh] It is perhaps ironic that the various classes of functions (λ-definable, general recursive, Turing computable) were proved in short order to be identical, but Gödel's initial reservations were justified on philosophical grounds.

In general, Gödel shied away from new concepts as objects of study, as opposed to new concepts as tools for obtaining results. The constructible hierarchy may be offered as a case in point, concerning which Gödel says that he is only using Russell's idea of the ramified hierarchy, but with an essentially impredicative element added, namely, the use of arbitrary ordinals.[ii] Only the concept of effective functional of finite type, which he had arrived at by 1941, comes close to being a new fundamental concept (see Gödel 1958, 1972).

There is a shift in the 1940s that corresponds to Gödel's changed circumstances and interests. Prior to that time, Gödel was understandably cautious about making public his platonist ideas, contrary as they were to the "dominant philosophical prejudices" of the time.[jj] With his reputation solidly established and with the security provided by the Institute for Advanced Study, Gödel felt freer to pursue and publicly elaborate his philosophical vision.

Gödel did the major part of his logical work in isolation, though he had a certain amount of stimulating contact with Menger, von Neumann, and Bernays in the prewar period. As described in the biographical sketch above, in the 1950s the institute increasingly attracted younger logicians, many of them in the forefront of research, as well as older colleagues of the prewar generation. Some of them sought Gödel out and established lengthy scientific relations with him that were also personally comfortable and friendly. Yet Gödel never had any students, never established a school, and never collaborated with others to advance his favorite program, namely, the discovery of essentially new axioms for set theory. Nonetheless, that program was taken up by many others in the wave of work in set theory from the 1960s on. Gödel's main results proved to be absolutely basic—the sine qua non for all that followed in almost all parts of logic—and it is through the work itself that he has had his major impact and influence.

As much as anything, Gödel's achievement lay in arriving at a very clear understanding of which problems in logic could be treated in a definite mathematical way. Along with others of his generation, but always leading the way, he succeeded in establishing the subject of mathematical logic as one that could be pursued with results as decisive and significant as those in the more traditional branches of mathematics. It is for this double heritage of the content and character of his work that we are indebted to him.

Source Notes

[a]In preparing the following I have drawn on a number of sources, of which the main published ones are Christian (1980), Dawson (1983, 1984a), Kleene (1976, 1987a), Kreisel (1980), and Wang (1978, 1981). Some use has also been made of unpublished material from Gödel's *Nachlass*.

For the biographical material I have relied primarily on Kreisel (1980), pp. 151–160, Wang (1981), and Dawson (1984a). Further personal material of value has come from Quine (1979), Zemanek (1978), and Taussky-Todd (1987). I have also made use of personal impressions communicated to me by A. Raubitschek (whose father was one of Hahn's best friends and who himself knew Gödel in Princeton), and of my own impressions (from contacts with Gödel during my visit to the Institute for Advanced Study in 1959 and 1960).

Finally, I am indebted to my co-editors [J.W. Dawson, Jr., S.C. Kleene, G.H. Moore, R.M. Solovay, and J. van Heijenoort] as well as the following of my colleagues for their many useful comments which have helped appreciably to improve the presentation: J. Barwise, S. Bauer-Mengelberg, M. Beeson, G.W. Brown, M. Davis, A.B. Feferman, H. Feigl, J.E. Fenstad, R. Haller, E. Köhler, G. Kreisel, R.B. Marcus, K. Menger, G. Müller, C. Parsons, W.V. Quine, A. Raubitschek, C. Reid, J. Robinson, P.A. Schilpp, W. Sieg, L. Straus, A.S. Troelstra, and H. Wang.

[b]Following Gödel's delivery of the Gibbs lecture in 1951. The story is told by Olga Taussky-Todd in her reminiscences (1987).

[c](Dr.) Rudolf Gödel wrote up a family history in 1967, with a supplement in 1978. This was made available to Georg Kreisel along with some correspondence between Kurt Gödel and his mother; see Kreisel (1980), p. 151. Gödel also communicated information about his family to Hao Wang for the article Wang (1981).

Another useful source on this and Gödel's intellectual development is a questionnaire which had been put to Gödel in 1975 by Burke D. Grandjean, then an instructor in sociology at the University of Texas. Its purpose was to gather information on Gödel's background in connection with research that Grandjean was doing on the social and intellectual situation in Central Europe during the first third of the twentieth century. The questionnaire was found in Gödel's *Nachlass*, fully filled out, along with a covering letter to Grandjean dated 19 August 1975; however, neither was apparently ever sent. I shall refer to this several times as a source in the following, calling it "the Grandjean interview."

[d]Kreisel (1980), p. 152.

[e]Kreisel (1980), p. 153.

[f]According to Gödel in the Grandjean interview, he had already formed such views before coming to Vienna. See also Wang (1978), p. 183.

[g]The Grandjean interview.

[h]Wang (1981), p. 654. [Cf. also chapter 7 in this volume.]

[i]Wang (1981), pp. 654–655. For information on von Neumann's life and work, see Ulam (1958), Goldstine (1972), and Heims (1980).

[j]See Dawson (1985).

[k]See Grattan-Guinness (1979), Moore (1980), and Dawson (1985, 1985a). An account of Gödel's first (and perhaps only) personal meeting with Zermelo is given in Taussky-Todd (1987).

[l]Hahn's report is quoted in Christian (1980), p. 263.

[m]Kreisel (1980), p. 154.

[n]Information communicated by E. Köhler.

[o]See Montgomery (1963) and Goldstine (1972), pp. 77–79.

[p]Goldstine (1972), p. 174.

[q]The available published information about Gödel's recurrent illness during this period is slim; in this connection see Kreisel (1980), p. 154, Wang (1981), pp. 655–656, and Dawson (1984a), p. 13, as well as Taussky-Todd (1987).

[r]Wang (1981), p. 656.

[s]For more on how Gödel achieved his results on AC and GCH, see Wang (1978), p. 184, Kreisel (1980), pp. 194–198, and Dawson (1984a), p. 13. The dates given in the sources do not always square with each other. It is hoped that study of the correspondence and notes in Gödel's *Nachlass* will be of assistance in clearing this up. Already discovered is a shorthand annotation preceding Gödel's notes on the GCH in his *Arbeitsheft* 1, which has been transcribed (by C. Dawson) as "Kont. Hyp. im wesentlichen gefunden in der Nacht zum 14 und 15 Juni 1937," in other words, that Gödel had "essentially found [the proof for the consistency of] GCH during the night of 14–15 June 1937."

[t]Kreisel (1980), pp. 154–155.

[u]Feigl (1969).

[v]For accounts of these events see Kreisel (1980), pp. 155–156, and Dawson (1984a), pp. 13, 15.

[w]It is a further point of irony that Gödel was listed in the catalogues for the University of Vienna between 1941 and 1945 as "Dozent für Grundlagen der Mathematik und Logik" and under course offerings "wird nicht lesen" (information communicated by E. Köhler).

[x]Gödel's response to the Austrian Academy of Sciences is quoted in Christian (1980), p. 266; Kreisel (1980), p. 155, says that Gödel refused various honors from Austria after the war, "sometimes for mindboggling reasons."

[y]Ulam (1958), pp. 3–4, and Goldstine (1972), pp. 177–182.

[z]This is recounted (in German) in Zemanek (1978), p. 210. Zemanek locates the hearing in Washington but it was more likely Trenton. Since the story is third-hand and translated, quotations are not exact.

[aa]The Grandjean interview, Wang (1978), p. 183, and Wang (1981), pp. 658–659.

[bb]Wang (1981), p. 658. Also, Straus (1982), pp. 420-421, says that "Gödel . . . was really totally solitary and would never talk with anybody while working."

[cc]Unpublished letter from Gödel to Carl Seelig, dated 7 September 1955.

[dd]Kreisel (1980), p. 218.

[ee]See Kreisel (1980), pp. 159–160, Dawson (1984a), p. 16. Information in this paragraph on Gödel's last years also comes from an interview that Dawson had with Gödel's friend and colleague Deane Montgomery. The death of his old friend Oskar Morgenstern in mid 1977 was apparently a shock to Gödel. The quotation giving cause of death is from his death certificate, on file in the Mercer County courthouse, Trenton, New Jersey.

[ff]The Grandjean interview.

[gg]See Gödel's letters to Hao Wang, dated 7 December 1967 and 7 March 1968, quoted in Wang (1974), pp. 8–11. The significance of Gödel's convictions for his work is discussed further in Feferman (1984a) [reproduced in the next chapter].

[hh]See the note, added 28 August 1963, to (1931) and the postscript to (1934).

[ii]See (1944).

[jj]See the letters mentioned in note *gg* for Gödel's characterizations of these "prejudices," and Feferman (1984a) [in the next chapter] for a discussion of Gödel's caution in this respect.