## PHIL309P

# Philosophy, Politics and Economics 

Eric Pacuit<br>University of Maryland, College Park<br>pacuit.org<br>Politics cases maxan  Nition ine Philosophy Game The May's Theorem Gaus Nash Condorcet's Paradox kneeted<br>Rational Choice Theory. ParetoHarsany<br>ArrowSocial Choice TheorySen<br>Rationality<br>Arrow's Theorem

## Announcements

- Course website https://myelms.umd.edu/courses/1133211
- Reading: Gaus, Ch 2. (up to 2.3) Utility Theory;Reiss, Ch 3, pgs. 29-42; Gilboa dialogue.
- Weekly writing: Due Wednesday, 11.59pm.
- Office hours canceled this Wednesday.


## Preferences

 Mas semen wey Nastleana choce Theory pereteotusan Arrowsocial CholicePreferring or choosing $x$ is different that "liking" $x$ or "having a taste for $x$ ": one can prefer $x$ to $y$ but dislike both options

In utility theory, preferences are always understood as comparative: "preference" is more like "bigger" than "big"

## Representing Preferences

 wat same thern ArrowSocial Choice
Rationality

A relation $\succeq \subseteq X \times X$ is a (rational) preference relation (for a decision maker) provided

1. $\succeq$ is complete (and hence reflexive)
2. $\succeq$ is transitive

## Representing Preferences

 nes same thery ArrowSocial Choice TheorySen ${ }_{\text {Rrows }}$ Rationality

A relation $\succeq \subseteq X \times X$ is a (rational) preference relation (for a decision maker) provided

1. $\succeq$ is complete (and hence reflexive)
2. $\succeq$ is transitive

Suppose that $\succeq$ is a preference relation. Then,

- Strict preference: $x \succ y$ iff $x \succeq y$ and $y \nsucceq x$
- Indifference: $x \sim y$ iff $x \succeq y$ and $y \succeq x$
- What is the relationship between choice and preference?
- Why should preferences be complete and transitive?
- Are people's preferences complete and transitive?


## Folk Psychology

 Nashh Consorcets Paradox
Rational Choice Theory ParetoHarsany ArrowSocial Choice
Rationality

The view that human behavior can and ought to be explained by citing beliefs and desires.

Beliefs and desires are thus reasons for action.

No every reason an individual might have to perform an action also constitute the reason that explains his or her action. Rather it is the reason the individual acted on that explains the action.

## Folk Psychology

In order to infer motivations or beliefs from behavior (or other accessible forms of evidence), one must make fairly strong assumptions concerning the system of beliefs and desires people have. If individuals acted very erratically (though always on reasons!) it would be impossible to infer beliefs or desires or both both from their actions.

## Choices

 Nash oonarcets farabox ECONOMICS ArrowSocial Choice TheorySen $\underset{\text { Rrows theorem }}{\text { Rationaly }}$It is important to distinguish between mere behavior on the one hand and "action" or "choice" on the other.

## Choices

 Mens.ime weon Economics Nash Consorcets Parasox LCO Pareto Harsany Arrowsocia ChoiceIt is important to distinguish between mere behavior on the one hand and "action" or "choice" on the other.

Decisions are between beliefs and desires on the one hand and actions on the other. Whateme wisem ECOMOMICS ArowSocil chice theor owain
Aroustionality
and

Should preferences be identified with choices?
 Nash benate feyme ArrowSocial Choice TheorySen ${ }_{\text {Rrows }}$ Rationality

Should preferences be identified with choices?

The verb "to prefer" can either mean "to choose" or "to like better," and these two senses are frequently confused in economic literature. That fact that an individual chooses $A$ rather than $B$ is far from conclusive evidence that he likes $A$ better. But whether he likes $A$ better or not should be completely irrelevant to the theory of price.
(Little, 1949).

## Preferences and Choices

 wans rame therneconomics Nash onal Choice Theory ParetoHarsanyRational Cho Arrowsocial Cholice

Preferences are closely related to choices: preferences may cause and help to explain choices; preferences may be invoked to justify choices, in fortuitous circumstances, we can use preference data to make predictions about choice. But to identify the two would be a mistake.

## Preferences and Choices

- We have preferences over vastly more states of affairs than we can ever hope (or dread) to be in the position to choose.


## Preferences and Choices

 uns shemenememe Economics Arrow Social Choice
Rationality
arrows theocerem
Can't we stipulate a concept of preference that is only loosely based on our ordinary concept?

## Preferences and Choices

Can't we stipulate a concept of preference that is only loosely based on our ordinary concept?
-What about counter-preferential choice?

## Preferences and Choices

 Mas semen wey Nash Consorcets parasooxRational Choice Theory ParetoHarsany Arrow Social Choice
Rationality
arrow theosem
Can't we stipulate a concept of preference that is only loosely based on our ordinary concept?

- What about counter-preferential choice?
- Preferences must be stable over a reasonable amount of time in a way that (observed) choices aren't (needed to predict and explain choices).


## Preferences and Choices

Can't we stipulate a concept of preference that is only loosely based on our ordinary concept?

- What about counter-preferential choice?
- Preferences must be stable over a reasonable amount of time in a way that (observed) choices aren't (needed to predict and explain choices).
- Beliefs and expectations over future states of affairs are needed in addition to preferences in order to explain choices. To banish preferences understood as mental rankings because they are unobservable or subjective would mean that beliefs and expectations would have to be banished as well.

 Nashtional Choice Theory Pareto Harsanyi Arrow Sationality

Preferences will be understood as mental rankings of alternatives "all things considered".

Game tasamys rinesem Philos Hump

 Arrow Sociai Choice
Rationality

## Revealed Preference Theory

 Nashemences max ECOnOMICS Rational Choice Theory ParetoHarsany RationalityStandard economics focuses on revealed preference because economic data comes in this form. Economic data can-at best-reveal what the agent wants (or has chosen) in a particular situation. Such data do not enable the economist to distinguish between what the agent intended to choose and what he ended up choosing; what he chose and what he ought to have chosen.
(Gul and Pesendorfer, 2008)

## Sen's $\alpha$ Condition

 wans rame ther ArrowSocial Choice
Rationality

## $R$ : red wine

$W$ : white wine
L: lemonade

## Sen's $\alpha$ Condition

 waveneme weormeconomics ArrowSocial Choice
Rationality

## $R$ : red wine

$W$ : white wine
L: lemonade

## Sen's $\alpha$ Condition

 wish rame hienvems

ArrowSocial Choice
Rationality

## $R$ : red wine <br> $W$ : white wine

## Sen's $\alpha$ Condition

 waven rame thery ArrowSocial Choice
Rationality

## $R$ : red wine

$W$ : white wine

## Sen's $\alpha$ Condition

$R$ : red wine
$W$ : white wine
L: lemonade

## $R$ : red wine

W: white wine

## Sen's $\alpha$ Condition

$R$ : red wine
$W$ : white wine
$L$ : lemonade
$R$ : red wine
$W$ : white wine

If the world champion is American, then she must be a US champion too.

Observations of actual choices will only partially constrain preference attribution. That someone chooses red wine when white wine is available does not allow one to conclude that the choice of an white wine was ruled out by her preferences, only that her preferences ruled the red wine in.

## Sen's $\beta$ Condition


 ArrowSocial Choice
Rationality

## $R$ : red wine

$W$ : white wine

## $R$ : red wine

$W$ : white wine

## Sen's $\beta$ Condition


 ArrowSocial Choice
Rationality

## $R$ : red wine

W: white wine
L: lemonade

## Sen's $\beta$ Condition

## $R$ : red wine

W: white wine
L: lemonade

## Sen's $\beta$ Condition



## $R$ : red wine

## W: white wine

L: lemonade

If some American is a world champion, then all champions of America must be world champions.

## Revealed Preference Theory

 Mas semen weymeronomics Nash Consorcets paraoosRational Choice Theory ParetoHarsany
Arrow Social Choice Theory Sen Arrowsocia Choice

A decision maker's choices over a set of alternatives $X$ are rationalizable iff there is a (rational) preference relation on $X$ such that the decision maker's choices maximize the preference relation.

## Revealed Preference Theory

A decision maker's choices over a set of alternatives $X$ are rationalizable iff there is a (rational) preference relation on $X$ such that the decision maker's choices maximize the preference relation.

Revelation Theorem. A decision maker's choices satisfy Sen's $\alpha$ and $\beta$ if and only if the decision maker's choices are rationalizable.

## Choice Functions

 Nes semene mo conomics $\underset{\text { Rrrows theorem }}{\text { Ratity }}$

Suppose $X$ is a set of options. And consider $B \subseteq X$ as a choice problem. A choice function is any function where $C(B) \subseteq B$. $B$ is sometimes called a menu and $C(B)$ the set of "rational" or "desired" choices.

## Choice Functions

 Mas seme temo M Nonomics Arrow Rationality

Suppose $X$ is a set of options. And consider $B \subseteq X$ as a choice problem. A choice function is any function where $C(B) \subseteq B$. $B$ is sometimes called a menu and $C(B)$ the set of "rational" or "desired" choices.

A relation $R$ on $X$ rationalizes a choice function $C$ if for all $B$ $C(B)=\{x \in B \mid$ for all $y \in B \quad x R y\}$.

## Choice Functions

 Mas semen wey ArrowSocial Choice TheorySen

Suppose $X$ is a set of options. And consider $B \subseteq X$ as a choice problem. A choice function is any function where $C(B) \subseteq B$. $B$ is sometimes called a menu and $C(B)$ the set of "rational" or "desired" choices.

A relation $R$ on $X$ rationalizes a choice function $C$ if for all $B$ $C(B)=\{x \in B \mid$ for all $y \in B x R y\}$.

Sen's $\alpha$ : If $x \in C(A)$ and $B \subseteq A$ and $x \in B$ then $x \in C(B)$

## Choice Functions

 Mas semen wey ArrowSocial Choice
Rationality

Suppose $X$ is a set of options. And consider $B \subseteq X$ as a choice problem. A choice function is any function where $C(B) \subseteq B$. $B$ is sometimes called a menu and $C(B)$ the set of "rational" or "desired" choices.

A relation $R$ on $X$ rationalizes a choice function $C$ if for all $B$ $C(B)=\{x \in B \mid$ for all $y \in B x R y\}$.

Sen's $\alpha$ : If $x \in C(A)$ and $B \subseteq A$ and $x \in B$ then $x \in C(B)$
Sen's $\beta$ : If $x, y \in C(A), A \subseteq B$ and $y \in C(B)$ then $x \in C(B)$.

## Maximizing

A. Sen. Maximization and the Act of Choice. Econometrica, Vol. 65, No. 4, 1997, 745-779.
"The formulation of maximizing behavior in economics has often parallels the modeling of maximization in physics an related disciplines.

## Maximizing

A. Sen. Maximization and the Act of Choice. Econometrica, Vol. 65, No. 4, 1997, 745-779.
"The formulation of maximizing behavior in economics has often parallels the modeling of maximization in physics an related disciplines. But maximizing behavior differs from nonvolitional maximization because of the fundamental relevance of the choice act, which has to be placed in a central position in analyzing maximizing behavior.

## Maximizing

"The formulation of maximizing behavior in economics has often parallels the modeling of maximization in physics an related disciplines. But maximizing behavior differs from nonvolitional maximization because of the fundamental relevance of the choice act, which has to be placed in a central position in analyzing maximizing behavior. A person's preferences over comprehensive outcomes (including the choice process) have to be distinguished form the conditional preferences over culmination outcomes given the act of choice."

## Maximizing

 Mas semen wey National Chowe Theory peretertiskny Arrow Sacia ChoiceYou arrive at a garden party and can readily identify the most comfortable chair. You would be delighted if an imperious host were to assign you that chair. However, if the matter is left to your own choice, you may refuse to rush to it.

## Maximizing

 Nash
Rational Choice
Arrow Social Choice ParetoHarsany Arrowsocia Choice

You arrive at a garden party and can readily identify the most comfortable chair. You would be delighted if an imperious host were to assign you that chair. However, if the matter is left to your own choice, you may refuse to rush to it. You select a "less preferred" chair.

## Maximizing

 uns sementuen Economics NashRational Choice
Theory ParetoHarsany Arrow Racia Choice

You arrive at a garden party and can readily identify the most comfortable chair. You would be delighted if an imperious host were to assign you that chair. However, if the matter is left to your own choice, you may refuse to rush to it. You select a "less preferred" chair. Are you still a maximizer?

You arrive at a garden party and can readily identify the most comfortable chair. You would be delighted if an imperious host were to assign you that chair. However, if the matter is left to your own choice, you may refuse to rush to it. You select a "less preferred" chair. Are you still a maximizer? Quite possibly you are, since your preference ranking for choice behavior may well be defined over "comprehensive outcomes", including choice processes (in particular, who does the choosing) as well as the outcomes at culmination (the distribution of chairs). waven weme teon Economics Nash Condorcets Parasox Rational Choice' Theory ParetoHarsany ArrowSocial Choice
Rationality

Invoking someone's preferences will suffice to explain why some choices were not made (i.e. in terms of rational impermissibility) but not typically why some particular choice was made. To take up the slack, explanations must draw on factors other than preference: psychological one such as the framing of the choice problem or the saliency of particular options, or sociological ones such as the existence of norms or conventions governing choices of the relevant kind.

## Ordinal Utility Theory

## Utility Function

 mass chame ceess mand ECOMOMICS Nasationarac choice Theory, paretortarssny Arrowsocial CholiceA utility function on a set $X$ is a function $u: X \rightarrow \mathbb{R}$

## Utility Function

 Nashemana choie Theo Arrowsocial Cholice

A utility function on a set $X$ is a function $u: X \rightarrow \mathbb{R}$

A preference ordering is represented by a utility function iff $x$ is (weakly) preferred to $y$ provided $u(x) \geq u(y)$

## Utility Function

 wans rame ther Nash Condorces Choice' Theory ParetoHarsany Arrow Social ChoiceA utility function on a set $X$ is a function $u: X \rightarrow \mathbb{R}$

A preference ordering is represented by a utility function iff $x$ is (weakly) preferred to $y$ provided $u(x) \geq u(y)$

What properties does such a preference ordering have?

## Ordinal Utility Theory

 uns nemene wo conomics Arrowsocia Choice

Fact. Suppose that $X$ is finite and $\succeq$ is a complete and transitive ordering over $X$, then there is a utility function $u: X \rightarrow \mathfrak{R}$ that represents $\succeq$ (i.e., $x \succeq y$ iff $u(x) \geq u(y)$ )

## Ordinal Utility Theory

 Mas seme temo conomics Nash Consorcets parasooxRational Choice Theory ParetoHarsany
Arrow Social Choice Theory Sen $\underset{\text { Rrrows theorem }}{\text { Ratity }}$

Fact. Suppose that $X$ is finite and $\succeq$ is a complete and transitive ordering over $X$, then there is a utility function $u: X \rightarrow \mathfrak{R}$ that represents $\succeq$ (i.e., $x \succeq y$ iff $u(x) \geq u(y)$ )

Utility is defined in terms of preference (so it is an error to say that the agent prefers $x$ to $y$ because she assigns a higher utility to $x$ than to $y$ ).

## Important

All three of the utility functions represent the preference $x \succ y \succ z$

| Item | $u_{1}$ | $u_{2}$ | $u_{3}$ |
| :---: | :---: | :---: | :---: |
| $x$ | 3 | 10 | 1000 |
| $y$ | 2 | 5 | 99 |
| $z$ | 1 | 0 | 1 |

$x \succ y \succ z$ is represented by both $(3,2,1)$ and $(1000,999,1)$, so one cannot say that $y$ is "closer" to $x$ than to $z$.

$$
X=\{M, C, P, L\}
$$

$$
X=\{M, C, P, L\}
$$

## $M \subset P L$

$M P L$

C $P$ L
$M \subset P$
$M \subset L$


M P

M L

C $P$

C L

$L$

$$
X=\{M, C, P, L\}
$$

## $M \subset P L$


©

M C P
$M C L$


$$
X=\{M, C, P, L\}
$$



$$
\begin{gathered}
\succeq=\{(M, C),(C, M),(M, P),(M, L),(C, P),(C, L),(P, L), \\
(M, M),(P, P),(C, C),(L, L)\}
\end{gathered}
$$

$$
X=\{M, C, P, L\}
$$



$$
\begin{gathered}
\succeq=\{(M, C),(C, M),(M, P),(M, L),(C, P),(C, L),(P, L), \\
(M, M),(P, P),(C, C),(L, L)\}
\end{gathered}
$$

$$
X=\{M, C, P, L\}
$$



$$
\begin{gathered}
\succeq=\{(M, C),(C, M),(M, P),(M, L),(C, P),(C, L),(P, L), \\
(M, M),(P, P),(C, C),(L, L)\}
\end{gathered}
$$

$$
X=\{M, C, P, L\}
$$



$$
\begin{gathered}
\succeq=\{(M, C),(C, M),(M, P),(M, L),(C, P),(C, L),(P, L), \\
(M, M),(P, P),(C, C),(L, L)\}
\end{gathered}
$$

$$
X=\{M, C, P, L\}
$$



$$
X=\{M, C, P, L\}
$$



$$
X=\{M, C, P, L\}
$$

$M C P L$

(C) $P L$

- What is the relationship between choice and preference?
- Why should preferences be complete and transitive?
- Are people's preferences complete and transitive?
- Transitivity: Money-pump argument
- Completeness: Incommensurable options

(M)

$(M) \Longrightarrow(C,-1)$


$$
(M) \Longrightarrow(C,-1) \Longrightarrow(P,-2)
$$



$$
(M) \Longrightarrow(C,-1) \Longrightarrow(P,-2) \Longrightarrow(M,-3)
$$


$(M) \Longrightarrow(C,-1) \Longrightarrow(P,-2) \Longrightarrow(M,-3) \Longrightarrow(C,-4) \Longrightarrow \cdots$
[O]f all the axioms of utility theory, the completeness axiom is perhaps the most questionable. Like others, it is inaccurate as a description of real life; but unlike them we find it hard to accept even from the normative viewpoint.
(Aumann, 1962)

Rather than trying to provide instrumental or pragmatic justifications for the axioms of ordinal utility, it is better...to see them as constitutive of our conception of a fully rational agent....those disposed to blatantly ignore transitivity are unintelligible to us: we can't understand their pattern of actions as sensible.
[G], pg. 39

## Preference, Choice, and Utility

$\checkmark$ Representing preferences: relations, preference axioms
$\checkmark$ Revealed preference theory: WARP, Sen's $\alpha$ and $\beta$, Revelation Theorem

- Utility: Ordinal vs. cardinal utility, interval scale, ratio scale
- Expected utility theory: (probability), von Neumann-Morgenstern Theorem, Allais paradox, Ellsberg paradox, (Other issues: framing effects, state-dependent utility, etc.)
- Interpersonal comparison of utilities
- Reading: Gaus, Ch 2;Reiss, Ch 3; Briggs, Normative Expected Utility Theory.
- Mathematical background: my notes on choice, preference and utility.
- Weekly writing: Due Wednesday, 11.59pm.


## Cardinal Utility Theory


 $\underset{\text { Rrrows theorem }}{\text { Ratity }}$
$x \succ y \succ z$ is represented by both $(3,2,1)$ and $(1000,999,1)$, so one cannot say $y$ is "closer" to $x$ than to $z$.

## Cardinal Utility Theory

 Ms.amicher Nash Condorcets Paradox ECO ParetoRational Choice Theory Parsany
Arrow Social Choice Theory Sen Arrowsocia Choice
$x \succ y \succ z$ is represented by both $(3,2,1)$ and $(1000,999,1)$, so one cannot say $y$ is "closer" to $x$ than to $z$.

Key idea: Ordinal preferences over lotteries allows us to infer a cardinal scale (with some additional axioms).

John von Neumann and Oskar Morgenstern. The Theory of Games and Economic Behavior. Princeton University Press, 1944.

## Axioms of Cardinal Utility

 wavs namenemerneconomics Arrowsocial Cholice

Suppose that $X$ is a set of outcomes and consider lotteries over $X$ (i.e., probability distributions over $X$ )

## Axioms of Cardinal Utility


 Arrow Rationality

Suppose that $X$ is a set of outcomes and consider lotteries over $X$ (i.e., probability distributions over $X$ )

A compound lottery is $\alpha L+(1-\alpha) L^{\prime}$ meaning "play lottery $L$ with probability $\alpha$ and $L^{\prime}$ with probability $1-\alpha^{\prime \prime}$

## Axioms of Cardinal Utility

 Nash Condorcets Paragox
Rational Choice
Theory ArrowSocial Chaice

Suppose that $X$ is a set of outcomes and consider lotteries over $X$ (i.e., probability distributions over X)

A compound lottery is $\alpha L+(1-\alpha) L^{\prime}$ meaning "play lottery $L$ with probability $\alpha$ and $L^{\prime}$ with probability $1-\alpha^{\prime \prime}$

Running example: Suppose Ann prefers pizza (p) over taco ( $t$ ) over yogurt ( $y$ ) ( $p \succ t \succ y$ ) and consider the different lotteries where the prizes are $p, t$ and $y$.

## Continuity

 Mavs Theorem Geuss Rational Choice Theory Paretorarsany Arrowsocial CholiceContinuity: for all options $x, y$ and $z$ if $x \succeq y \succeq z$, there is some lottery $L$ with probability $p$ of getting $x$ and $(1-p)$ of getting $y$ such that the agent is indifferent between $L$ and $y$.

## Continuity

 mavs דinerem cerss Rational Choice Theory Paretorarsany Arrowsocial CholiceContinuity: for all options $x, y$ and $z$ if $x \succeq y \succeq z$, there is some lottery $L$ with probability $p$ of getting $x$ and $(1-p)$ of getting $y$ such that the agent is indifferent between $L$ and $y$.

Suppose Ann has $t$.

## Continuity

 Mas shame whem Economics NashRational Choice
Theory ParetoHarsany Arrowsocial Cholice
Continuity: for all options $x, y$ and $z$ if $x \succeq y \succeq z$, there is some lottery $L$ with probability $p$ of getting $x$ and $(1-p)$ of getting $y$ such that the agent is indifferent between $L$ and $y$.

Suppose Ann has $t$.
Consider the lottery $L=0.99$ get $y$ and 0.01 get $p$

## Continuity

 Mas shame why Nash ounal Choice Theory ParetoHarsan Arrowsocial CholiceContinuity: for all options $x, y$ and $z$ if $x \succeq y \succeq z$, there is some lottery $L$ with probability $p$ of getting $x$ and $(1-p)$ of getting $y$ such that the agent is indifferent between $L$ and $y$.

Suppose Ann has $t$.
Consider the lottery $L=0.99$ get $y$ and 0.01 get $p$ Would Ann trade $t$ for $L$ ?

## Continuity

 Nashh Consorcets Paraotox
Rational Choice Theory ParetoHarsany

Continuity: for all options $x, y$ and $z$ if $x \succeq y \succeq z$, there is some lottery $L$ with probability $p$ of getting $x$ and $(1-p)$ of getting $y$ such that the agent is indifferent between $L$ and $y$.

Suppose Ann has $t$.
Consider the lottery $L=0.99$ get $y$ and 0.01 get $p$ Would Ann trade $t$ for $L$ ?

Consider the lottery $L^{\prime}=0.99$ get $p$ and 0.01 get $y$

## Continuity

 Nash Condorcets Parasox
Rational Choice
Theory

Continuity: for all options $x, y$ and $z$ if $x \succeq y \succeq z$, there is some lottery $L$ with probability $p$ of getting $x$ and $(1-p)$ of getting $y$ such that the agent is indifferent between $L$ and $y$.

Suppose Ann has $t$.
Consider the lottery $L=0.99$ get $y$ and 0.01 get $p$ Would Ann trade $t$ for $L$ ?

Consider the lottery $L^{\prime}=0.99$ get $p$ and 0.01 get $y$ Would Ann trade $t$ for $L^{\prime}$ ?

## Continuity

Continuity: for all options $x, y$ and $z$ if $x \succeq y \succeq z$, there is some lottery $L$ with probability $p$ of getting $x$ and $(1-p)$ of getting $y$ such that the agent is indifferent between $L$ and $y$.

Suppose Ann has $t$.
Consider the lottery $L=0.99$ get $y$ and 0.01 get $p$ Would Ann trade $t$ for $L$ ?

Consider the lottery $L^{\prime}=0.99$ get $p$ and 0.01 get $y$ Would Ann trade $t$ for $L^{\prime}$ ?

Continuity says that there is must be some lottery where Ann is indifferent between keeping $t$ and playing the lottery.

## Better Prizes

Better Prizes: suppose $L_{1}$ is a lottery over $(w, x)$ and $L_{2}$ is over $(y, z)$ suppose that $L_{1}$ and $L_{2}$ have the same probability over prizes. The lotteries each have an equal prize in one position they have unequal prizes in the other position then if $L_{1}$ is the lottery with the better prize then $L_{1} \succ L_{2}$; if neither lottery has a better prize then $L_{1} \approx L_{2}$.

## Better Prizes

Better Prizes: suppose $L_{1}$ is a lottery over $(w, x)$ and $L_{2}$ is over $(y, z)$ suppose that $L_{1}$ and $L_{2}$ have the same probability over prizes. The lotteries each have an equal prize in one position they have unequal prizes in the other position then if $L_{1}$ is the lottery with the better prize then $L_{1} \succ L_{2}$; if neither lottery has a better prize then $L_{1} \approx L_{2}$.

Lottery $1\left(L_{1}\right)$ is 0.6 chance for $p$ and 0.4 chance for $y$
Lottery $2\left(L_{2}\right)$ is 0.6 chance for $t$ and 0.4 chance for $y$

## Better Prizes

Better Prizes: suppose $L_{1}$ is a lottery over $(w, x)$ and $L_{2}$ is over $(y, z)$ suppose that $L_{1}$ and $L_{2}$ have the same probability over prizes. The lotteries each have an equal prize in one position they have unequal prizes in the other position then if $L_{1}$ is the lottery with the better prize then $L_{1} \succ L_{2}$; if neither lottery has a better prize then $L_{1} \approx L_{2}$.

Lottery $1\left(L_{1}\right)$ is 0.6 chance for $p$ and 0.4 chance for $y$
Lottery $2\left(L_{2}\right)$ is 0.6 chance for $t$ and 0.4 chance for $y$
Since Ann prefers $p$ to $t$, this axiom says that Ann prefers $L_{1}$ to $L_{2}$

## Better Chances

Better Chances: Suppose $L_{1}$ and $L_{2}$ are two lotteries which have the same prizes, then if $L_{1}$ offers a better chance of the better prize, then $L_{1} \succ L_{2}$

## Better Chances


 ArrowSocial Choice
Rationality

Better Chances: Suppose $L_{1}$ and $L_{2}$ are two lotteries which have the same prizes, then if $L_{1}$ offers a better chance of the better prize, then $L_{1} \succ L_{2}$

Lottery $1\left(L_{1}\right)$ is 0.7 chance for $p$ and 0.3 chance for $y$
Lottery $2\left(L_{2}\right)$ is 0.6 chance for $p$ and 0.4 chance for $y$

## Better Chances

 Nash Condorcet's Paradox
Rational Choice Theory Pareto Harsanyi ArrowSocial Choice
Rationality

Better Chances: Suppose $L_{1}$ and $L_{2}$ are two lotteries which have the same prizes, then if $L_{1}$ offers a better chance of the better prize, then $L_{1} \succ L_{2}$

Lottery $1\left(L_{1}\right)$ is 0.7 chance for $p$ and 0.3 chance for $y$
Lottery $2\left(L_{2}\right)$ is 0.6 chance for $p$ and 0.4 chance for $y$

This axioms states that Ann must prefer $L_{1}$ to $L_{2}$

## Reduction of Compound Lotteries

 whens.ine wemmeconomics Nashional chooe Theory Arrow Racia ChoiceReduction of Compound Lotteries: If the prize of a lottery is another lottery, then this can be reduced to a simple lottery over prizes.

## Reduction of Compound Lotteries

 Nash
Rational Choice Theory ParetoHarsany Arrow Rationality

Reduction of Compound Lotteries: If the prize of a lottery is another lottery, then this can be reduced to a simple lottery over prizes.

This eliminates utility from the thrill of gambling and so the only ultimate concern is the prizes.

## Cardinal Utility Theory

Politics asas hawn fume
 Nash benaxecerimet ArrowSocial Choice
Rationality

Von Neumann-Morgenstern Theorem. If an agent satisfies the previous axioms, then the agents ordinal utility function can be turned into cardinal utility function.

## Cardinal Utility Theory

 Nash Consorcets parasoox
Rational Choice Theory ParetoHarsany
Arrow Social Choice Theory Sen Arrow Rationality

Von Neumann-Morgenstern Theorem. If an agent satisfies the previous axioms, then the agents ordinal utility function can be turned into cardinal utility function.

- Utility is unique only up to linear transformations. So, it still does not make sense to add two different agents cardinal utility functions.


## Cardinal Utility Theory

Von Neumann-Morgenstern Theorem. If an agent satisfies the previous axioms, then the agents ordinal utility function can be turned into cardinal utility function.

- Utility is unique only up to linear transformations. So, it still does not make sense to add two different agents cardinal utility functions.
- Issue with continuity: 1EUR $\succ 1$ cent $\succ$ death, but who would accept a lottery which is $p$ for 1EUR and $(1-p)$ for death??


## Cardinal Utility Theory

Von Neumann-Morgenstern Theorem. If an agent satisfies the previous axioms, then the agents ordinal utility function can be turned into cardinal utility function.

- Utility is unique only up to linear transformations. So, it still does not make sense to add two different agents cardinal utility functions.
- Issue with continuity: 1EUR $\succ 1$ cent $\succ$ death, but who would accept a lottery which is $p$ for 1EUR and $(1-p)$ for death??
- Deep issues about how to identify correct descriptions of the outcomes and options.


## Mathematical background: Relations

 wash shemenceme Economics Arrowsocia Choice

Suppose that $X$ is a set. A relation on $X$ is a set of ordered pairs from $X$ : $R \subseteq X \times X$.

## Mathematical background: Relations

 whens.eme weine Economics Arrow Rationality

Suppose that $X$ is a set. A relation on $X$ is a set of ordered pairs from $X$ : $R \subseteq X \times X$.
E.g., $X=\{a, b, c, d\}, R=\{(a, a),(b, a),(c, d),(a, c),(d, d)\}$

## Mathematical background: Relations


 Arrow Rationality

Suppose that $X$ is a set. A relation on $X$ is a set of ordered pairs from $X$ : $R \subseteq X \times X$.
E.g., $X=\{a, b, c, d\}, R=\{(a, a),(b, a),(c, d),(a, c),(d, d)\}$


## Mathematical background: Relations

 Nash Condorcets Parresox
Rational Choice Theory ParetoHarsany
ArrowS Social Choice Theory Sen Arrow Rationality

Suppose that $X$ is a set. A relation on $X$ is a set of ordered pairs from $X$ : $R \subseteq X \times X$.
E.g., $X=\{a, b, c, d\}, R=\{(a, a),(b, a),(c, d),(a, c),(d, d)\}$


$$
b R a
$$



## Mathematical background: Relations

 Nash Condorcets Parresox
Rational Choice Theory ParetoHarsany
ArrowSocial Choice Theory Sen Arrow Rationality

Suppose that $X$ is a set. A relation on $X$ is a set of ordered pairs from $X$ : $R \subseteq X \times X$.
E.g., $X=\{a, b, c, d\}, R=\{(a, a),(b, a),(c, d),(a, c),(d, d)\}$

a R a
$b R a$

$d R d$

## Mathematical background: Relations

 Nash consorcets Pararobx Theory ParetoHarsany
Rational Choice
ArrowSocial Choice TheorySen Arrow Rationality

Suppose that $X$ is a set. A relation on $X$ is a set of ordered pairs from $X$ : $R \subseteq X \times X$.
E.g., $X=\{a, b, c, d\}, R=\{(a, a),(b, a),(c, d),(a, c),(d, d)\}$


$$
\begin{aligned}
& a R a \\
& b R a \\
& c R d \\
& a R c \\
& d R d
\end{aligned}
$$

## Mathematical background: Relations

 Nash Condorrets Paradoox
Rational Choice Theory P ParetoHarsany Arrowsocial Cholice

Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.
Reflexive relation: for all $x \in X, x R x$

## Mathematical background: Relations

 mass Game theoryouns Nash Condorcets ParasooxRational Choice Theory ParetoHarsany
ArrowSocial Choice TheorySen $\underset{\text { Rrrows theorem }}{\text { Ratity }}$

Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.
Reflexive relation: for all $x \in X, x R x$
E.g., $X=\{a, b, c, d\}$


## Mathematical background: Relations

 mars sheorem GeusNash Condorceets Paradox ECOMN
EOMOM Nash Consorcets Parasoox
Rational Choice Theory ParetoHarsany Arrowsocial Rality

Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.
Irreflexive relation: for all $x \in X, x$ 双 $x$ (i.e., $(x, x) \notin R$ )

## Mathematical background: Relations

 Nash Condorcet't Paradoox
Rational Choice Theory ParetoHarsany
Arrow Social Choice TheorySen Arrow
Rations theonality

Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.
Irreflexive relation: for all $x \in X, x$ 及 $x$ (i.e., $(x, x) \notin R$ )
E.g., $X=\{a, b, c, d\}$


## Mathematical background: Relations

 wavs nemenemerneconomics Nash Condorcets ParadoxRational Choice
Theory Arrowsocial Cholice

Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.
Symmetric relation: for all $x, y \in X$, if $x R y$, then $y R x$

## Mathematical background: Relations

 mass Game theoryours Nash Consorcets ParadooxRational Choice Theory ParetoHarsany
ArrowSocial Choice TheorySen $\underset{\text { Rrrows theorem }}{\text { Ratity }}$

Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.
Symmetric relation: for all $x, y \in X$, if $x R y$, then $y R x$
E.g., $X=\{a, b, c, d\}$


\section*{Mathematical background: Relations} wavs nemenemerneconomics | Nash Consorcets Parasoox |
| :--- |
| Rational Choice Theory ParetoHarsany | Arrowsocial Cholice

Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.
Complete relation: for all $x, y \in X$, either $x R y$ or $y R x$

## Mathematical background: Relations

 mass Game theoryours Nash Condorcet't ParadoxRational Choice Theory ParetoHarsany
Arrow Social Choice Theory Sen $\underset{\text { Rrrows theorem }}{\text { Ratity }}$

Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.
Complete relation: for all $x, y \in X$, either $x R y$ or $y R x$
E.g., $X=\{a, b, c, d\}$


## Mathematical background: Relations

 mass Game theoryowns Nash Consorcet't ParadoxRational Choice Theory ParetoHarsany
Arrow Social Choice Theory Sen Arrowsocial Rality

Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.
Complete relation: for all $x, y \in X$, either $x R y$ or $y R x$
E.g., $X=\{a, b, c, d\}$


## Mathematical background: Relations

 Nash Condorcets Paradoox
Rational Choice Theory ParetoHarsany
ArrowSocial Choice TheorySen $\underset{\text { Rrrows theocem }}{\text { Ratity }}$

Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.
Complete relation: for all $x, y \in X$, either $x R y$ or $y R x$
E.g., $X=\{a, b, c, d\}$


## Mathematical background: Relations

 Nash Condorcets Paradox
Rational Choice Theory ParetoHarsany
ArrowSocial Choice TheorySen $\underset{\text { Rrrows theorem }}{\text { Ratity }}$

Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.
Complete relation: for all $x, y \in X$, either $x R y$ or $y R x$
E.g., $X=\{a, b, c, d\}$


## Mathematical background: Relations

 mass Game theoryoums NashRational Choice
ArrowSocial Choice ParetoHarsany $\underset{\text { Rrrows theorem }}{\text { Ratity }}$

Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.
Complete relation: for all $x, y \in X$, either $x R y$ or $y R x$
E.g., $X=\{a, b, c, d\}$


## Mathematical background: Relations

 Nash Condorcets Parresox
Rational Choice Theory ParetoHarsany Arrow Rationality

Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.
Transitive relation: for all $x, y, z \in X$, if $x R y$ and $y R z$, then $x R z$

## Mathematical background: Relations

 Nash Condorcet't Paradoox
Rational Choice Theory ParetoHarsany
Pat Arrow Sociationality

Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.
Transitive relation: for all $x, y, z \in X$, if $x R y$ and $y R z$, then $x R z$
E.g., $X=\{a, b, c, d\}$


## Mathematical background: Relations

 ArrowSocial Choice TheorySen Rationality
Arows theorem

Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.
Transitive relation: for all $x, y, z \in X$, if $x R y$ and $y R z$, then $x R z$
E.g., $X=\{a, b, c, d\}$


## Mathematical background: Relations

 Nash Condorcet't Paradoox
Rational Choice Theory ParetoHarsany
ArrowSocial Choice TheorySen $\underset{\text { Rrrows theorem }}{\text { Ratity }}$

Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.
Transitive relation: for all $x, y, z \in X$, if $x R y$ and $y R z$, then $x R z$
E.g., $X=\{a, b, c, d\}$


## Mathematical background: Relations

 Nash Condorcets Parrosox
Rational Choice Theory ParetoHarsany $\underset{\text { Rrrows theocem }}{\text { Ratity }}$

Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.
Transitive relation: for all $x, y, z \in X$, if $x R y$ and $y R z$, then $x R z$
E.g., $X=\{a, b, c, d\}$


## Mathematical background: Relations

 Nash Condorcets Parrosox
Rational Choice Theory ParetoHarsany Arrow Sociationality

Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.
Transitive relation: for all $x, y, z \in X$, if $x R y$ and $y R z$, then $x R z$
E.g., $X=\{a, b, c, d\}$


## Mathematical background: Relations

 Mens.ime ween Economics ArrowSocial Choice Paretoryarsan $\underset{\text { Rrows theorem }}{\text { Rationaly }}$Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.
Transitive relation: for all $x, y, z \in X$, if $x R y$ and $y R z$, then $x R z$
E.g., $X=\{a, b, c, d\}$


## Maximal elements, Cycles

 Rationac chowe thiac Arrow Racial Chality

Suppose that $R \subseteq X \times X$ is a relation.
$x \in X$ is maximal with respect to $R$ provided there is no $y \in X$ such that $y R x$.
For $Y \subseteq X$, let $\max _{R}(Y)=\{x \in Y \mid$ there is no $y \in Y$ such that $y R x\}$

## Maximal elements, Cycles

 wans rame therneconomics Nash Condorcets Paraotox Thery ParetoHarsany Arrowsocial CholiceSuppose that $R \subseteq X \times X$ is a relation.
$x \in X$ is maximal with respect to $R$ provided there is no $y \in X$ such that $y R x$.
For $Y \subseteq X$, let $\max _{R}(Y)=\{x \in Y \mid$ there is no $y \in Y$ such that $y R x\}$

A cycle is a set of distinct elements $x_{1}, \ldots, x_{n}$ such that

$$
x_{1} R x_{2} \cdots x_{n-1} R x_{n} R x_{1}
$$

$R$ is acyclic if it does not contain any cycles.

## Representing Preferences

 wavs rame therneconomics Naghtoman chioce Thear Arrow SociationalityLet $X$ be a set of options/outcomes. A decision maker's preference over $X$ is represented by a relation $\succeq \subseteq X \times X$.

## Representing Preferences

 Mays Theorem ceusis Nash conanal Choice Theory Pareto Harsanyi Arrow Sationality

Given $x, y \in X$, there are four possibilities:

## Representing Preferences

 waveneme weormeconomics Arrow Rationality

Given $x, y \in X$, there are four possibilities:

1. $x \succeq y$ and $y \nsucceq x$ : The decision maker ranks $x$ above $y$ (the decision maker strictly prefers $x$ to $y$ ).

## Representing Preferences


 $\underset{\text { Rrrows theorem }}{\text { Ratity }}$
Given $x, y \in X$, there are four possibilities:

1. $x \succeq y$ and $y \nsucceq x$ : The decision maker ranks $x$ above $y$ (the decision maker strictly prefers $x$ to $y$ ).
2. $y \succeq x$ and $x \nsucceq y$ : The decision maker ranks $y$ above $x$ (the decision maker strictly prefers $y$ to $x$ ).

## Representing Preferences


 $\underset{\text { Rrrows theorem }}{\text { Ratity }}$
Given $x, y \in X$, there are four possibilities:

1. $x \succeq y$ and $y \nsucceq x$ : The decision maker ranks $x$ above $y$ (the decision maker strictly prefers $x$ to $y$ ).
2. $y \succeq x$ and $x \nsucceq y$ : The decision maker ranks $y$ above $x$ (the decision maker strictly prefers $y$ to $x$ ).
3. $x \succeq y$ and $y \succeq x$ : The agent is indifferent between $x$ and $y$.

## Representing Preferences

 Ms.amicher Nashemana chioe thacy reatedizany Arrow RationalityGiven $x, y \in X$, there are four possibilities:

1. $x \succeq y$ and $y \nsucceq x$ : The decision maker ranks $x$ above $y$ (the decision maker strictly prefers $x$ to $y$ ).
2. $y \succeq x$ and $x \nsucceq y$ : The decision maker ranks $y$ above $x$ (the decision maker strictly prefers $y$ to $x$ ).
3. $x \succeq y$ and $y \succeq x$ : The agent is indifferent between $x$ and $y$.
4. $x \nsucceq y$ and $y \nsucceq x$ : The agent cannot compare $x$ and $y$

## Representing Preferences

 Ms.amicher Nash Arrow RationalityGiven $x, y \in X$, there are four possibilities:

1. $x \succeq y$ and $y \nsucceq x$ : The decision maker ranks $x$ above $y$ (the decision maker strictly prefers $x$ to $y$ ).
2. $y \succeq x$ and $x \nsucceq y$ : The decision maker ranks $y$ above $x$ (the decision maker strictly prefers $y$ to $x$ ).
3. $x \succeq y$ and $y \succeq x$ : The agent is indifferent between $x$ and $y$.
4. $x \nsucceq y$ and $y \nsucceq x$ : The agent cannot compare $x$ and $y$
