PHIL309P
Philosophy, Politics and Economics

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Announcements

- Course website
  https://myelms.umd.edu/courses/1133211
- Problem set 1
- Online quiz 2
- Reading: Gaus, Ch 2; Reiss, Ch 3; Briggs SEP article.
- Weekly writing: **Due Wednesday, 11.59pm**.

Decision Problems

In many circumstances the decision maker doesn't get to choose outcomes directly, but rather chooses an instrument that affects what outcome actually occurs.

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In many circumstances the decision maker doesn't get to choose outcomes directly, but rather chooses an instrument that affects what outcome actually occurs.

Choice under

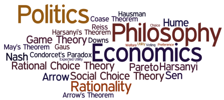
- *certainty*: highly confident about the relationship between actions and outcomes
- *risk*: clear sense of possibilities and their likelihoods
- *uncertainty*: the relationship between actions and outcomes is so imprecise that it is not possible to assign likelihoods

Decision Problems

A

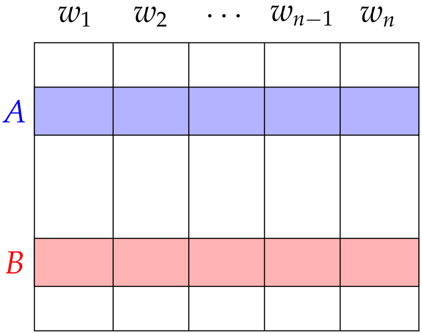
B

Decision Problems



	w_1	w_2	$\cdots$	w_{n-1}	w_n
A					
B					

Decision Problems



An **act** is a function $A : W \to O$

Making an omelet

States: {the sixth egg is good, the sixth egg is rotten}

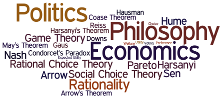
Consequences: { six-egg omelet, no omelet and five good eggs destroyed, six-egg omelet and a cup to wash....}

Acts: { break egg into bowl, break egg into a cup, throw egg away}

Making an omelet

	Good egg (s_1)	Bad egg (s_2)
Break into a bowl (A_1)	six egg omelet (o_1)	no omelet and five good eggs destroyed (o_2)
Break into a cup (A_2)	six egg omelet and a cup to wash (o_3)	five egg omelet and a cup to wash (o_4)
Throw away (A_3)	five egg omelet and one good egg destroyed (o_5)	five egg omelet (o_6)

Making an omelet



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$A_1(s_1) = o_1$ $\quad\quad$ $A_1(s_2) = o_2$

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$A_1(s_1) = o_1$ $A_1(s_2) = o_2$

Making an omelet

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$$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2$$

Making an omelet

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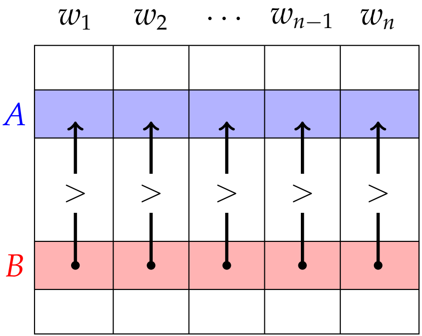
$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2$

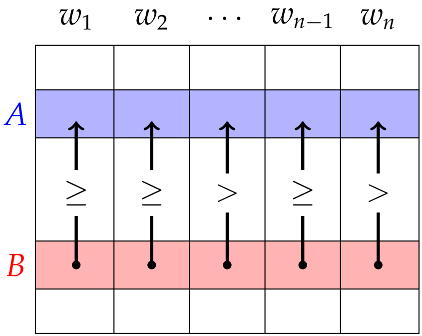
How should A_1, A_2 and A_3 be ranked?

Strict Dominance



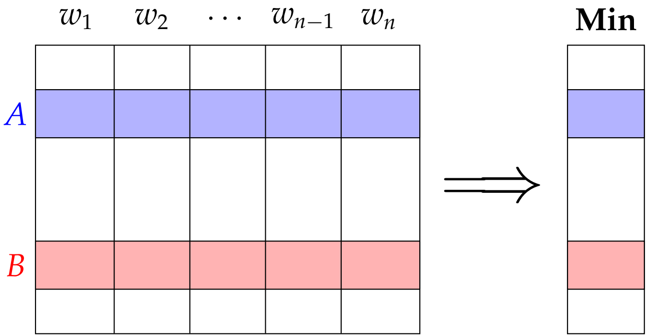
$$\forall\, w \in W, u(A(w)) > u(B(w))$$

Weak Dominance



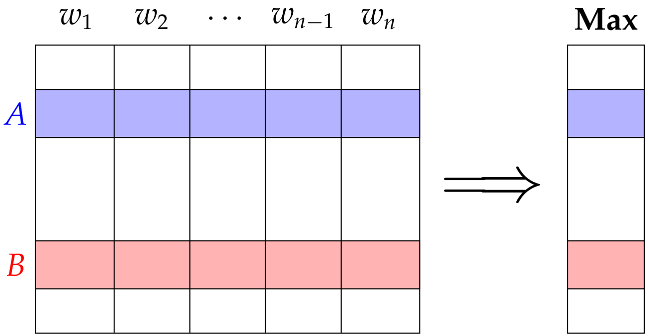
$$\forall\, w \in W, u(A(w)) \geq u(B(w)) \text{ and } \exists\, w \in W, u(A(w)) > u(B(w))$$

MaxMin (Security)

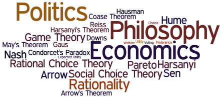


$$\min(\{u(A(w)) \mid w \in W\})$$

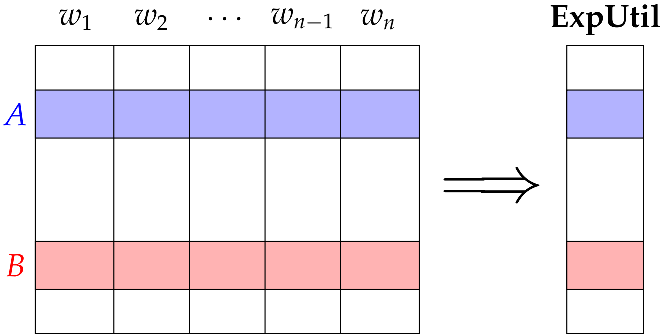
MaxMax



$$\max(\{u(A(w)) \mid w \in W\})$$



Maximize (Subjective) Expected Utility



$$\sum_{w \in W} P_A(w) * u(A(w))$$

Subjective Expected Utility

Probability: Suppose that $W = \{w_1, \ldots, w_n\}$ is a finite set of states. A probability function on W is a function $P : W \to [0, 1]$ where $\sum_{w \in W} P(w) = 1$ (i.e., $P(w_1) + P(w_2) + \cdots + P(w_n) = 1$).

Suppose that A is an act for a set of outcomes O (i.e., $A : W \to O$). The **expected utility** of A is:

$$\sum_{w \in W} P(w) * u(A(w))$$

Making an omelet

	Good egg (s_1)	Bad egg (s_2)
Break into a bowl (A_1)	six egg omelet (o_1)	no omelet and five good eggs destroyed (o_2)
Break into a cup (A_2)	six egg omelet and a cup to wash (o_3)	five egg omelet and a cup to wash (o_4)
Throw away (A_3)	five egg omelet and one good egg destroyed (o_5)	five egg omelet (o_6)

Making an omelet

	Good egg (s_1) **0.8**	Bad egg (s_2) **0.2**
Break into a bowl (A_1)	six egg omelet (o_1) **6**	no omelet and five good eggs destroyed (o_2) **1**
Break into a cup (A_2)	six egg omelet and a cup to wash (o_3) **4**	five egg omelet and a cup to wash (o_4) **3**
Throw away (A_3)	five egg omelet and one good egg destroyed (o_5) **2**	five egg omelet (o_6) **5**

$$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2 \qquad P(s_1) = 0.8, P(s_2) = 0.2$$

$$u(o_1) = 6, u(o_6) = 5, u(o_3) = 4, u(o_4) = 3, u(o_5) = 2, u(o_2) = 1$$

Making an omelet

	Good egg (s_1) **0.8**	Bad egg (s_2) **0.2**
Break into a bowl (A_1)	six egg omelet (o_1) **6**	no omelet and five good eggs destroyed (o_2) **1**
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$$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2 \qquad P(s_1) = 0.8, P(s_2) = 0.2$$

$$EU(A_1) = P(s_1) * u(A_1(s_1)) + P(s_2) * u(A_1(s_2)) = 0.8 * 6 + 0.2 * 1 = 5.0$$

Making an omelet

	Good egg (s_1) **0.8**	Bad egg (s_2) **0.2**
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$$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2 \qquad P(s_1) = 0.8, P(s_2) = 0.2$$

$$EU(A_2) = P(s_1) * u(A_2(s_1)) + P(s_2) * u(A_2(s_2)) = 0.8 * 4 + 0.2 * 3 = 3.8$$

Making an omelet

	Good egg (s_1) **0.8**	Bad egg (s_2) **0.2**
Break into a bowl (A_1)	six egg omelet (o_1) **6**	no omelet and five good eggs destroyed (o_2) **1**
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$$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2 \qquad P(s_1) = 0.8, P(s_2) = 0.2$$

$$EU(A_3) = P(s_1) * u(A_3(s_1)) + P(s_2) * u(A_3(s_2)) = 0.8 * 2 + 0.2 * 5 = 2.6$$

Making an omelet

	Good egg (s_1) **0.8**	Bad egg (s_2) **0.2**
Break into a bowl (A_1)	six egg omelet (o_1) **6**	no omelet and five good eggs destroyed (o_2) **1**
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$$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2 \qquad P(s_1) = 0.8, P(s_2) = 0.2$$

$$EU(A_1) = 5 > EU(A_2) = 3.8 > EU(A_3) = 2.6$$

Making an omelet

	Good egg (s_1) **0.8**	Bad egg (s_2) **0.2**
Break into a bowl (A_1)	six egg omelet (o_1) **9**	no omelet and five good eggs destroyed (o_2) **0**
Break into a cup (A_2)	six egg omelet and a cup to wash (o_3) **8**	five egg omelet and a cup to wash (o_4) **7**
Throw away (A_3)	five egg omelet and one good egg destroyed (o_5) **1**	five egg omelet (o_6) **9.5**

$$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2 \qquad P(s_1) = 0.8, P(s_2) = 0.2$$

$$u(o_1) = 9, u(o_6) = 9.5, u(o_3) = 8, u(o_4) = 7, u(o_5) = 1, u(o_2) = 0$$

Making an omelet

	Good egg (s_1) **0.8**	Bad egg (s_2) **0.2**
Break into a bowl (A_1)	six egg omelet (o_1) **9**	no omelet and five good eggs destroyed (o_2) **0**
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Throw away (A_3)	five egg omelet and one good egg destroyed (o_5) **1**	five egg omelet (o_6) **9.5**

$$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2 \qquad P(s_1) = 0.8, P(s_2) = 0.2$$

$$EU(A_2) = 7.8 > EU(A_1) = 7.2 > EU(A_3) = 2.7$$

Cardinal Utility Theory

$$u : X \to \mathbb{R}$$

Which comparisons are meaningful?

1. $u(x)$ and $u(y)$? (ordinal utility)
2. $u(x) - u(y)$ and $u(a) - u(b)$?
3. $u(x)$ and $2 * u(z)$?

Cardinal Utility Theory

$x \succ y \succ z$ is represented by both $(3, 2, 1)$ and $(1000, 999, 1)$, so we cannot say y whether is "closer" to x than to z.

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$x \succ y \succ z$ is represented by both $(3, 2, 1)$ and $(1000, 999, 1)$, so we cannot say y whether is "closer" to x than to z.

Key idea: Ordinal preferences over *lotteries* allows us to infer a cardinal scale (with some additional axioms).

John von Neumann and Oskar Morgenstern. *The Theory of Games and Economic Behavior*. Princeton University Press, 1944.

A Choice

R

B

W

S

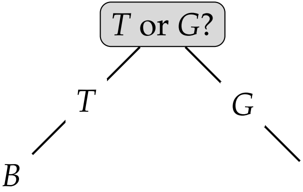
A Choice

R

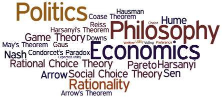
B

W

S



A Choice

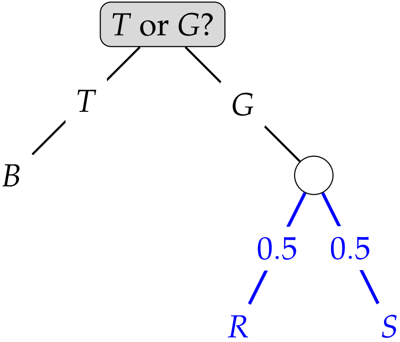


R

B

W

S



A Choice

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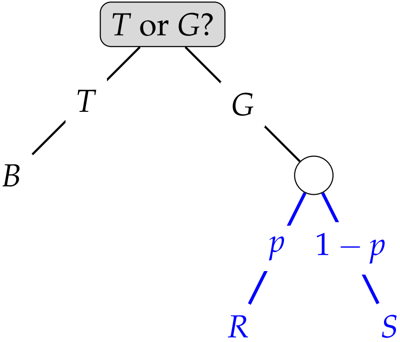

A Choice

R

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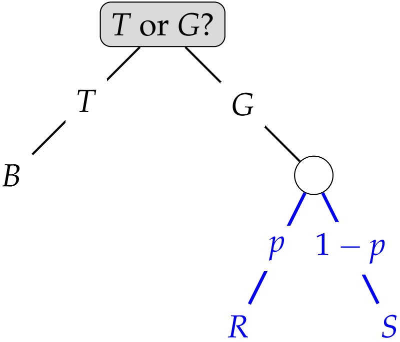


$$[1 : B] \sim [p : R, 1-p : S]$$

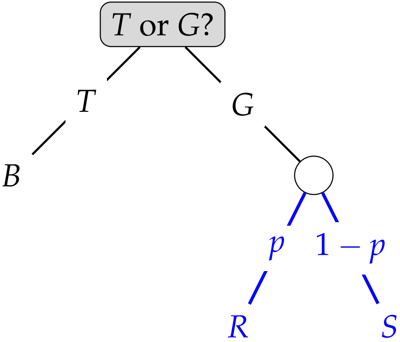
A Choice

R

B

W

S



$$1 * u(B) = p * u(R) + (1 - p) * u(S)$$

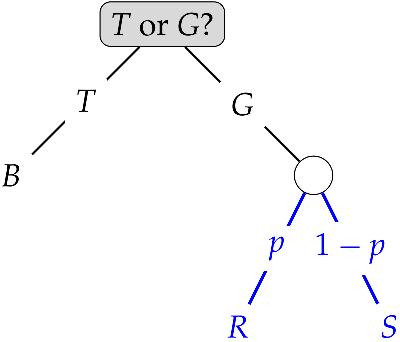
A Choice

R

B

W

S



$$u(B) = p * 1 + (1 - p) * 0 = p$$