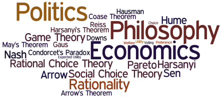
PHIL309P
Philosophy, Politics and Economics

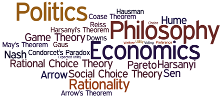
Eric Pacuit
University of Maryland, College Park
pacuit.org

Announcements



- Course website
  https://myelms.umd.edu/courses/1133211
- Problem set 1, **due on Friday**
- Online quiz 2
- Reading: Gaus, Ch 3; Reiss, Ch 4
- Weekly writing: **Due Wednesday, 11.59pm**. (Comment on the Elster article).

Subjective Expected Utility



Probability: Suppose that $W = \{w_1, \ldots, w_n\}$ is a finite set of states. A probability function on W is a function $P : W \to [0, 1]$ where $\sum_{w \in W} P(w) = 1$ (i.e., $P(w_1) + P(w_2) + \cdots + P(w_n) = 1$).

Suppose that A is an act for a set of outcomes O (i.e., $A : W \to O$) and $u : O \to \mathbb{R}$ is a **cardinal utility function**. The **expected utility** of A is:

$$\sum_{w \in W} P(w) * u(A(w))$$

Ordinal vs. Cardinal Utility

Ordinal scale: Qualitative comparisons of objects allowed, no information about differences or ratios.

Ordinal vs. Cardinal Utility

Ordinal scale: Qualitative comparisons of objects allowed, no information about differences or ratios.

Cardinal scales:

> **Interval scale**: Quantitative comparisons of objects, accurately reflects differences between objects.
>
> E.g., the difference between 75°F and 70°F is the same as the difference between 30°F and 25°F However, 70°F ($= 21.11^{\circ}$C) is **not** twice as hot as 35°F ($= 1.67^{\circ}$C). The difference between 70°F and 65°F is **not** the same as the difference between 25°C and 20°C.

Ordinal vs. Cardinal Utility

Ordinal scale: Qualitative comparisons of objects allowed, no information about differences or ratios.

Cardinal scales:

> **Interval scale**: Quantitative comparisons of objects, accurately reflects differences between objects.
>
> E.g., the difference between 75°F and 70°F is the same as the difference between 30°F and 25°F However, 70°F (= 21.11°C) is **not** twice as hot as 35°F (= 1.67°C). The difference between 70°F and 65°F is **not** the same as the difference between 25°C and 20°C.
>
> **Ratio scale**: Quantitative comparisons of objects, accurately reflects ratios between objects. E.g., 10lb is twice as much as 5lb. But, 10kg is not twice as much as 5lb.

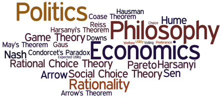
Suppose that X is a set of outcomes.

A **(simple) lottery** over X is denoted $[x_1 : p_1, x_2 : p_2, \dots, x_n : p_n]$ where for $i = 1, \dots, n$, $x_i \in X$ and $p_i \in [0, 1]$, and $\sum_i p_i = 1$.

Let $\mathcal{L}$ be the set of (simple) lotteries over X. We identify elements $x \in X$ with the lottery $[x : 1]$.

Suppose that $\succeq$ is a relation on $\mathcal{L}$.

Axioms



Preference $\succeq$ is reflexive, transitive and complete

Compound Lotteries The decision maker is indifferent between every compound lottery and the *corresponding* simple lottery.

Independence For all $L_1, L_2, L_3 \in \mathcal{L}$ and $a \in (0, 1]$, $L_1 \succ L_2$
if, and only if,
$[L_1 : a, L_3 : (1 - a)] \succ [L_2 : a, L_3 : (1 - a)]$.

Continuity For all $L_1, L_2, L_3 \in \mathcal{L}$ and $a \in (0, 1]$,
if $L_1 \succ L_2 \succ L_3$, then there exists $a \in (0, 1)$
such that $[L_1 : a, L_3 : (1 - a)] \sim L_2$

$u : \mathcal{L} \to \Re$ is linear provided for all $L = [L_1 : p_1, \ldots, L_n : p_n] \in \mathcal{L}$,

$$u(L) = \sum_{i=1}^{n} p_i u(L_i)$$

von Neumann-Morgenstern Representation Theorem A binary relation $\succeq$ on $\mathcal{L}$ satisfies Preference, Compound Lotteries, Independence and Continuity iff $\succeq$ is representable by a linear utility function $u : \mathcal{L} \to \Re$.

Moreover, $u' : \mathcal{L} \to \Re$ represents $\succeq$ iff there exists real numbers $c > 0$ and d such that $u'(\cdot) = cu(\cdot) + d$. ("$u$ is unique up to linear transformations.")

Cardinal Utility Theory

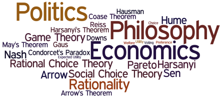
Von Neumann-Morgenstern Theorem. If an agent satisfies the previous axioms, then the agent's ordinal utility function can be turned into cardinal utility function.

Cardinal Utility Theory

Von Neumann-Morgenstern Theorem. If an agent satisfies the previous axioms, then the agent's ordinal utility function can be turned into cardinal utility function.

- Utility is unique only *up to linear transformations*. So, it still does not make sense to add two different agents cardinal utility functions.

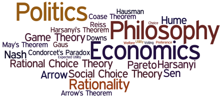
Cardinal Utility Theory



Von Neumann-Morgenstern Theorem. If an agent satisfies the previous axioms, then the agent's ordinal utility function can be turned into cardinal utility function.

- Utility is unique only *up to linear transformations*. So, it still does not make sense to add two different agents cardinal utility functions.
- Issue with continuity: 1EUR $\succ$ 1 cent $\succ$ death, but who would accept a lottery which is p for 1EUR and $(1-p)$ for death??

Cardinal Utility Theory



Von Neumann-Morgenstern Theorem. If an agent satisfies the previous axioms, then the agent's ordinal utility function can be turned into cardinal utility function.

- Utility is unique only *up to linear transformations*. So, it still does not make sense to add two different agents cardinal utility functions.
- Issue with continuity: 1EUR $\succ$ 1 cent $\succ$ death, but who would accept a lottery which is p for 1EUR and $(1-p)$ for death??
- Important issues about how to identify correct descriptions of the outcomes and options.

Objections

- The axioms are too strong. Do rational decision *have* to obey these axioms?

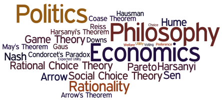
- No action guidance. Rational decision makers do not prefer an act *because* its expected utility is favorable, but can only be described as *if* they were acting from this principle.

- Utility without chance. It seems rather odd from a linguistic point of view to say that the *meaning* of utility has something to do with preferences over lotteries.

Why maximize expected utility?

Law of Large Numbers: everyone who maximizes expected utility will *almost certainly* be better off in the long run. By performing a random experiment sufficiently many times, the probability that the average outcome differs from the expected outcome can be rendered *arbitrarily* small.

Why maximize expected utility?



Law of Large Numbers: everyone who maximizes expected utility will *almost certainly* be better off in the long run. By performing a random experiment sufficiently many times, the probability that the average outcome differs from the expected outcome can be rendered *arbitrarily* small.

Gambler's Ruin: Suppose Ann and Bob start with \$1000 each and flip a fair coin.

Why maximize expected utility?

Law of Large Numbers: everyone who maximizes expected utility will *almost certainly* be better off in the long run. By performing a random experiment sufficiently many times, the probability that the average outcome differs from the expected outcome can be rendered *arbitrarily* small.

Gambler's Ruin: Suppose Ann and Bob start with $1000 each and flip a fair coin. Ann gives Bob $1 if H and Bob gives Ann $1 if T.

Why maximize expected utility?

Law of Large Numbers: everyone who maximizes expected utility will *almost certainly* be better off in the long run. By performing a random experiment sufficiently many times, the probability that the average outcome differs from the expected outcome can be rendered *arbitrarily* small.

Gambler's Ruin: Suppose Ann and Bob start with \$1000 each and flip a fair coin. Ann gives Bob \$1 if H and Bob gives Ann \$1 if T. If they flip the coin a *sufficiently* large number of times, each player is *guaranteed* to face a sequence of flips that bankrupts them.

Why maximize expected utility?

Law of Large Numbers: everyone who maximizes expected utility will *almost certainly* be better off in the long run. By performing a random experiment sufficiently many times, the probability that the average outcome differs from the expected outcome can be rendered *arbitrarily* small.

Gambler's Ruin: Suppose Ann and Bob start with \$1000 each and flip a fair coin. Ann gives Bob \$1 if *H* and Bob gives Ann \$1 if *T*. If they flip the coin a *sufficiently* large number of times, each player is *guaranteed* to face a sequence of flips that bankrupts them. The player that faces such a sequence first, will never have an opportunity to feel the effects of the Law of Large Numbers.

- Transitivity (money-pump argument)
- Completeness (very strong)
- Continuity (lotteries with extreme bads)
- Independence (Kitten example, Allais, Ellsberg, etc.)

Objections

- The axioms are too strong. Do rational decision *have* to obey these axioms?
- No action guidance. Rational decision makers do not prefer an act *because* its expected utility is favorable, but can only be described as *if* they were acting from this principle.
- Utility without chance. It seems rather odd from a linguistic point of view to say that the *meaning* of utility has something to do with preferences over lotteries.

	Bad weather (0.5)	Good weather (0.5)
Crop A	\$10,000; 10	\$30,000; 60
Crop B	\$15,000; 36	\$20,000; 50

Expected income: Crop A: \$20,000; Crop B: \$17,500

Expected utility: Crop A: 35; Crop B: 43

The farmer is risk-averse.

To explain the farmer's choice, we can cite the preferences he has over the different outcomes and the beliefs he has about the probabilities of the weather.

To explain the farmer’s choice, we can cite the preferences he has over the different outcomes and the beliefs he has about the probabilities of the weather. Most economists would say that the farmer’s preferences over the lotteries are given and basic. But this is implausible, and it prevents EUT from being a genuinely explanatory theory. It is implausible because people will have more stable and basic preferences over things they ultimately care about. The farmer in this case cares about his income and the consumption associated with it, not about playing a lottery.

If preferences over prospects are given, all an economists can say is farmer chose crop B because he preferred to do so, but isn’t there a more nuanced story that one can tell.

Allais Paradox

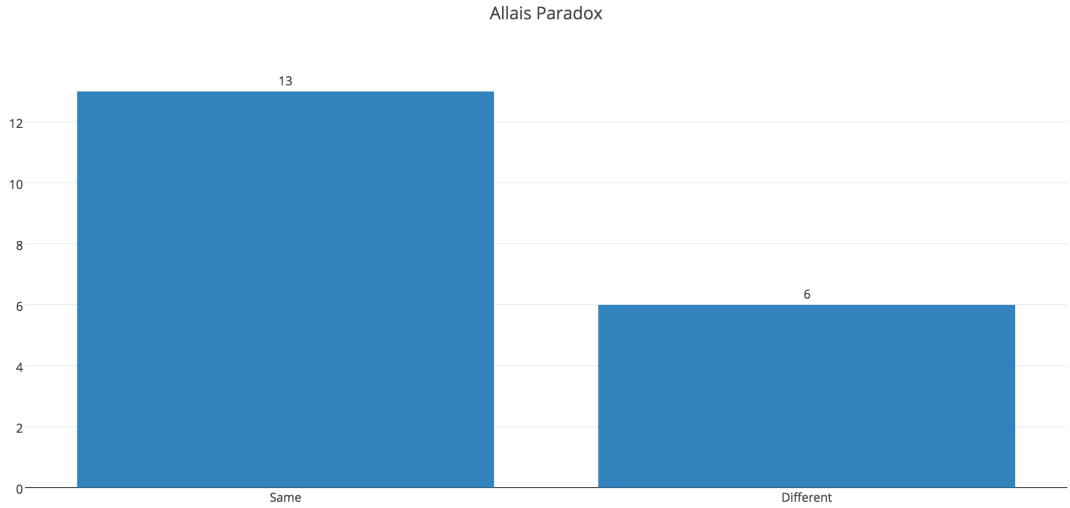
	Options	Red (1)	White (89)	Blue (10)
S_1	A	$1M$	$1M$	$1M$
	B	0	$1M$	$5M$

Allais Paradox

	Options	Red (1)	White (89)	Blue (10)
S_2	C	$1M$	0	$1M$
	D	0	0	$5M$

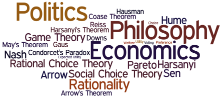
Allais Paradox

Allais Paradox



Allais Paradox

	Options	Red (1)	White (89)	Blue (10)
S_1	A	$1M$	$1M$	$1M$
	B	0	$1M$	$5M$
S_2	C	$1M$	0	$1M$
	D	0	0	$5M$



Allais Paradox

	Options	Red (1)	White (89)	Blue (10)
S_1	A	$1M$	$1M$	$1M$
	B	0	$1M$	$5M$
S_2	C	$1M$	0	$1M$
	D	0	0	$5M$

$$A \succeq B \text{ iff } C \succeq D$$

Allais Paradox

We should not conclude either

Allais Paradox

We should not conclude either

(a) The axioms of cardinal utility fail to adequately capture our understanding of rational choice, or

Allais Paradox

We should not conclude either

(a) The axioms of cardinal utility fail to adequately capture our understanding of rational choice, or

(b) those who choose A in S_1 and D is S_2 are irrational.

Allais Paradox

We should not conclude either

(a) The axioms of cardinal utility fail to adequately capture our understanding of rational choice, or

(b) those who choose A in S_1 and D is S_2 are irrational.

Rather, people's utility functions (*their rankings over outcomes*) are often far more complicated than the monetary bets would indicate....

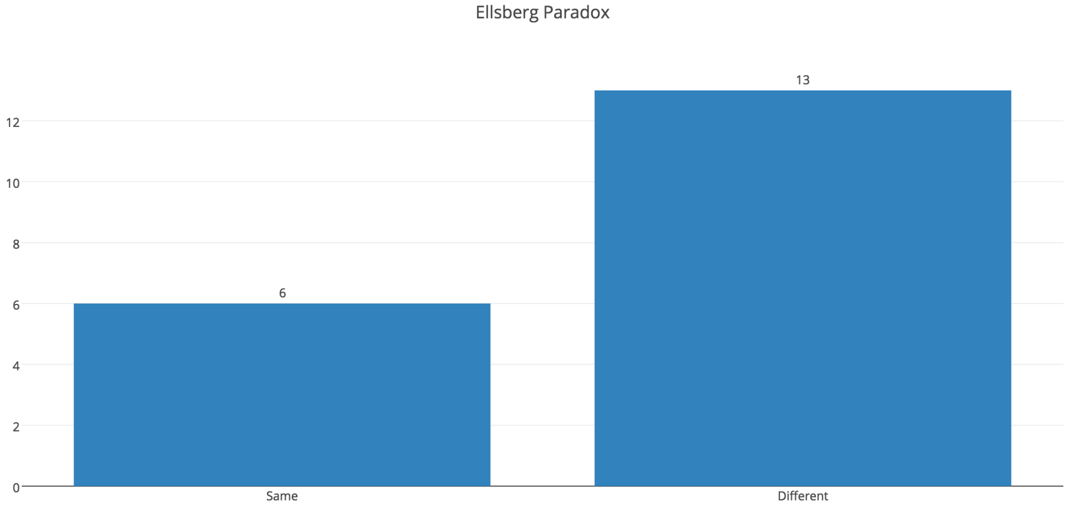
Ellsberg Paradox

	30	60	
Lotteries	Blue	Yellow	Green
L_1	$1M$	0	0
L_2	0	$1M$	0

Ellsberg Paradox

	30	60	
Lotteries	Blue	Yellow	Green
L_3	$1M$	0	$1M$
L_4	0	$1M$	$1M$

Ellsberg Paradox



Ellsberg Paradox

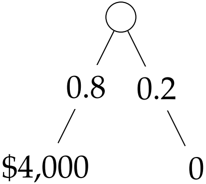
	30	60	
Lotteries	Blue	Yellow	Green
L_1	$1M$	0	0
L_2	0	$1M$	0
L_3	$1M$	0	$1M$
L_4	0	$1M$	$1M$

$$L_1 \succeq L_2 \text{ iff } L_3 \succeq L_4$$

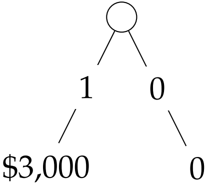
A: [\$4,000:0.80] B: [\$3,000:1]

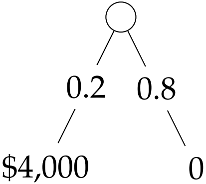
A: [\$4,000:0.80] B: [\$3,000:1]

C: [\$4,000:0.20] D: [\$3,000:0.25]

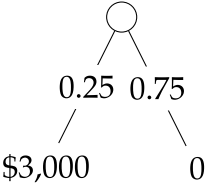


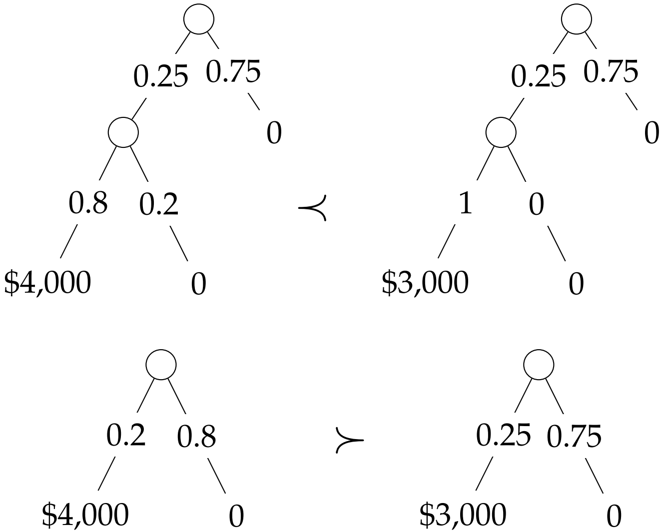
$\prec$





$\succ$




0.25
0.75
0
0.8
0.2
$4,000
0
≺
0.25
0.75
0
1
0
$3,000
0
0.2
0.8
$4,000
0
≻
0.25
0.75
$3,000
0

A: [\$6,000:0.45] B: [\$3,000:0.9]

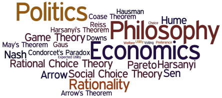
A: [\$6,000:0.45] B: [\$3,000:0.9]

C: [\$6,000:0.001] D: [\$3,000:0.002]

D. Kahneman and A. Tversky. *Prospect Theory: An Analysis of Decision under Risk*. Econometrica, Vol. 47, No. 2., pgs. . 263 - 292, 1979.

N. Barberis. *Thirty Years of Prospect Theory in Economics: A Review and Assessment*. Journal of Economic Perspectives, 27:1, pgs. 171 - 196, 2013.

Prospect Theory



Consider a gamble

$$[x_{-m} : p_{-m}; x_{-m+1} : p_{-m+1}; \ldots; x_0 : p_0; \ldots; x_{n-1} : p_{n-1}; x_n : p_n]$$

where $x_i < x_j$ for $i < j$ and $x_0 = 0$

Expected Utility

$$\sum_{i=-m}^{n} p_i U(W + x_i)$$

where W is current wealth and $U(\cdot)$ is an increasing and concave utility function.

Prospect Theory

Consider a gamble

$$(x_{-m}; p_{-m}; x_{-m+1}; p_{-m+1}; \ldots; x_0; p_0; \ldots; x_{n-1}, p_{n-1}; x_n, p_n)$$

where $x_i < x_j$ for $i < j$ and $x_0 = 0$

Cumulative Prospect Theory

$$\sum_{i=-m}^{n} \pi_i v(x_i)$$

where $v(\cdot)$ is the “value function” is an increasing function with $v(0) = 0$ and π_i are “decision weights”.

reference dependence: people derive utility from *gains and loses*, measured relative to some reference point, rather than from absolute levels of wealth.

reference dependence: people derive utility from *gains and loses*, measured relative to some reference point, rather than from absolute levels of wealth.

loss aversion: people are much more sensitive to losses—even small losses—than to gains of the same magnitude. Many people turn down a gamble $(-\$100 : \frac{1}{2}, \$110 : \frac{1}{2})$, but this is very hard to explain in classical utility theory (Rabin, 2000) *diminishing sensitivity*: people tend to be risk averse over moderate probability gains (they typically prefer a certain gain of $500 to a 50 precent chance of $1,000) and *risk seeking* over losses (they prefer a 50 precent chance of loosing $1000 to loosing $500 for sure) *diminishing sensitivity*: people tend to be risk averse over moderate probability gains (they typically prefer a certain gain of $500 to a 50 precent chance of $1,000) and *risk seeking* over losses (they prefer a 50 precent chance of loosing $1000 to loosing $500 for sure)

probability weighting: people tend to overweight the tails of a probability distribution (they tend to overweight extremely unlikely outcomes).