PHIL309P
Philosophy, Politics and Economics

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Announcements

- Course website
  https://myelms.umd.edu/courses/1133211
- Problem set 1, **due on Friday**
- Online quiz 3
- Reading: Gaus, Ch 3; Reiss, Ch 4

Subjective Expected Utility

Probability: Suppose that $W = \{w_1, \ldots, w_n\}$ is a finite set of states. A probability function on W is a function $P : W \rightarrow [0, 1]$ where $\sum_{w \in W} P(w) = 1$ (i.e., $P(w_1) + P(w_2) + \cdots + P(w_n) = 1$).

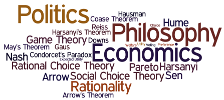
Suppose that A is an act for a set of outcomes O (i.e., $A : W \rightarrow O$) and $u : O \rightarrow \mathbb{R}$ is a **cardinal utility function**. The **expected utility** of A is:

$$\sum_{w \in W} P(w) * u(A(w))$$

Ordinal vs. Cardinal Utility

Ordinal scale: Qualitative comparisons of objects allowed, no information about differences or ratios.

Ordinal vs. Cardinal Utility



Ordinal scale: Qualitative comparisons of objects allowed, no information about differences or ratios.

Cardinal scales:

> **Interval scale**: Quantitative comparisons of objects, accurately reflects differences between objects.
>
> E.g., the difference between 75°F and 70°F is the same as the difference between 30°F and 25°F However, 70°F (= 21.11°C) is **not** twice as hot as 35°F (= 1.67°C). The difference between 70°F and 65°F is **not** the same as the difference between 25°C and 20°C.

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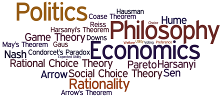

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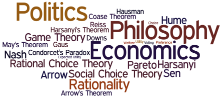
> **Ratio scale**: Quantitative comparisons of objects, accurately reflects ratios between objects. E.g., 10lb is twice as much as 5lb. But, 10kg is not twice as much as 5lb.

Suppose that X is a set of outcomes.

A **(simple) lottery** over X is denoted $[x_1 : p_1, x_2 : p_2, \ldots, x_n : p_n]$ where for $i = 1, \ldots, n$, $x_i \in X$ and $p_i \in [0, 1]$, and $\sum_i p_i = 1$.

Let $\mathcal{L}$ be the set of (simple) lotteries over X. We identify elements $x \in X$ with the lottery $[x : 1]$.

Suppose that $\succeq$ is a relation on $\mathcal{L}$.



Axioms

Preference $\succeq$ is reflexive, transitive and complete

Compound Lotteries The decision maker is indifferent between every compound lottery and the *corresponding* simple lottery.

Independence For all $L_1, L_2, L_3 \in \mathcal{L}$ and $a \in (0,1]$, $L_1 \succ L_2$
if, and only if,
$[L_1 : a, L_3 : (1-a)] \succ [L_2 : a, L_3 : (1-a)]$.

Continuity For all $L_1, L_2, L_3 \in \mathcal{L}$ and $a \in (0,1]$,
if $L_1 \succ L_2 \succ L_3$, then there exists $a \in (0,1)$
such that $[L_1 : a, L_3 : (1-a)] \sim L_2$

$u : \mathcal{L} \to \Re$ is linear provided for all $L = [L_1 : p_1, \ldots, L_n : p_n] \in \mathcal{L}$,

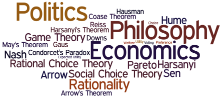
$$u(L) = \sum_{i=1}^{n} p_i u(L_i)$$

von Neumann-Morgenstern Representation Theorem A binary relation $\succeq$ on $\mathcal{L}$ satisfies Preference, Compound Lotteries, Independence and Continuity iff $\succeq$ is representable by a linear utility function $u : \mathcal{L} \to \Re$.

Moreover, $u' : \mathcal{L} \to \Re$ represents $\succeq$ iff there exists real numbers $c > 0$ and d such that $u'(\cdot) = cu(\cdot) + d$. ("$u$ is unique up to linear transformations.")

Cardinal Utility Theory

Von Neumann-Morgenstern Theorem. If an agent satisfies the previous axioms, then the agent's ordinal utility function can be turned into cardinal utility function.



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- Issue with continuity: 1EUR $\succ$ 1 cent $\succ$ death, but who would accept a lottery which is p for 1EUR and $(1-p)$ for death??
- Important issues about how to identify correct descriptions of the outcomes and options.

Why maximize expected utility?

Law of Large Numbers: everyone who maximizes expected utility will *almost certainly* be better off in the long run. By performing a random experiment sufficiently many times, the probability that the average outcome differs from the expected outcome can be rendered *arbitrarily* small.

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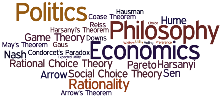

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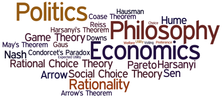

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- Transitivity (money-pump argument)
- Completeness (very strong)
- Continuity (lotteries with extreme bads)
- Independence (Kitten example, Allais, Ellsberg, etc.)

Objections

- The axioms are too strong. Do rational decision *have* to obey these axioms?

- No action guidance. Rational decision makers do not prefer an act *because* its expected utility is favorable, but can only be described as *if* they were acting from this principle.

- Utility without chance. It seems rather odd from a linguistic point of view to say that the *meaning* of utility has something to do with preferences over lotteries.

Allais Paradox

	Options	Red (1)	White (89)	Blue (10)
S_1	A	$1M$	$1M$	$1M$
	B	0	$1M$	$5M$

Allais Paradox

	Options	Red (1)	White (89)	Blue (10)
S_2	C	$1M$	0	$1M$
	D	0	0	$5M$

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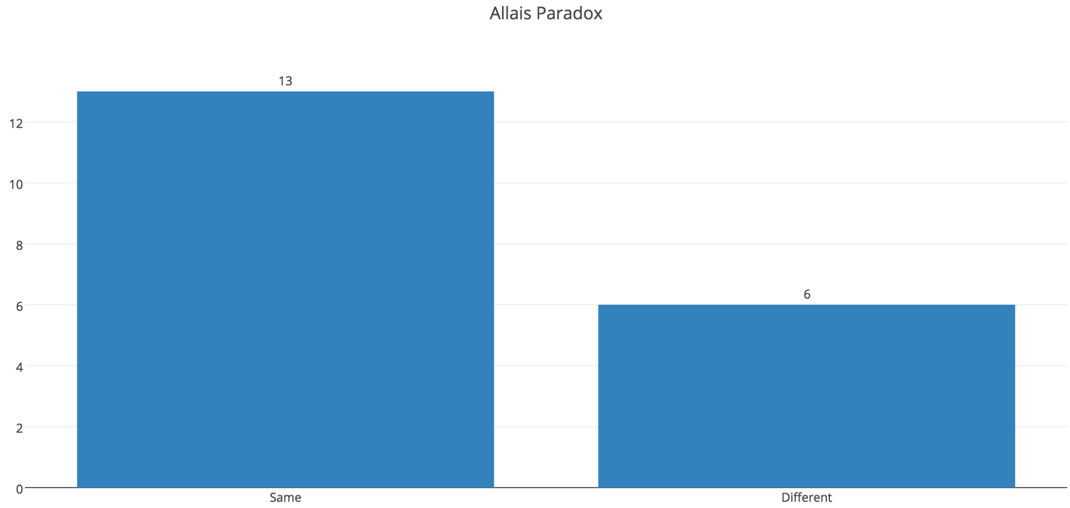

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$$A \succeq B \text{ iff } C \succeq D$$

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(a) The axioms of cardinal utility fail to adequately capture our understanding of rational choice, or

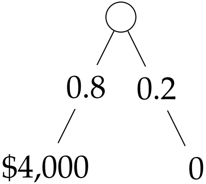
(b) those who choose A in S_1 and D is S_2 are irrational.

Rather, people's utility functions (*their rankings over outcomes*) are often far more complicated than the monetary bets would indicate....

A: [\$4,000:0.80] B: [\$3,000:1]

A: [$4,000:0.80] B: [$3,000:1]

C: [$4,000:0.20] D: [$3,000:0.25]



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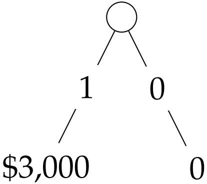

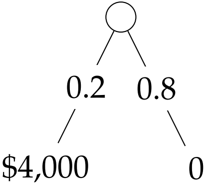

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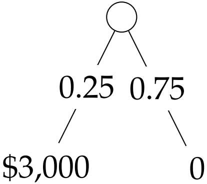

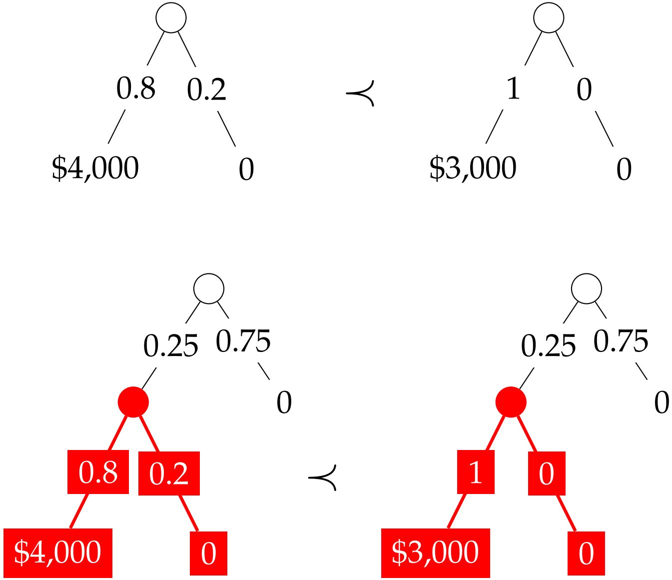
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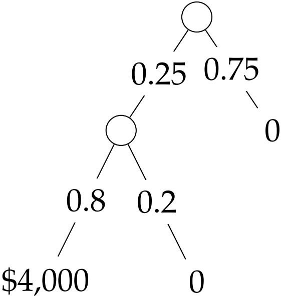

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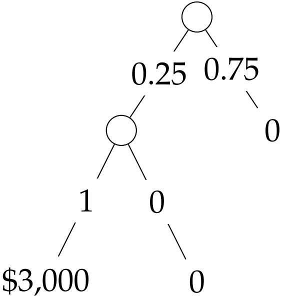

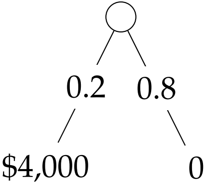

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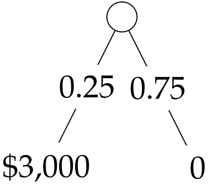
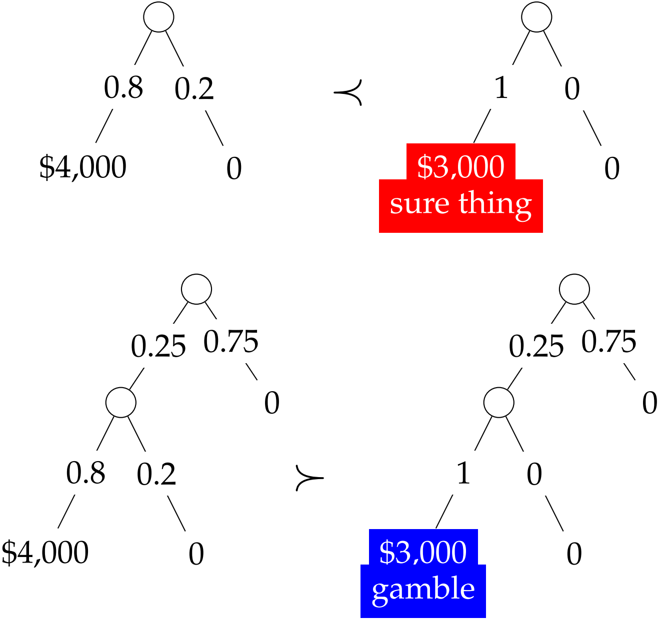


$\mathbf{0.25 * 0.8 = 0.2}$ $\mathbf{0.25 * 1 = 0.25}$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Ellsberg Paradox

	30	60	
Lotteries	Blue	Yellow	Green
L_1	$1M$	0	0
L_2	0	$1M$	0

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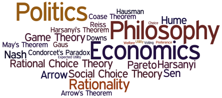

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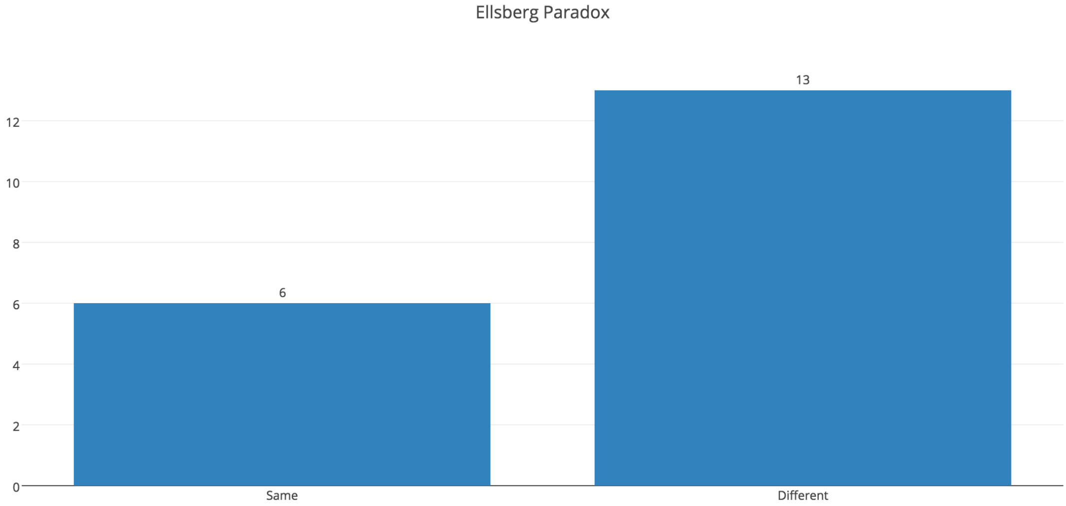

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$$L_1 \succeq L_2 \text{ iff } L_3 \succeq L_4$$

A: [\$6,000:0.45] B: [\$3,000:0.9]

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C: [\$6,000:0.001] D: [\$3,000:0.002]

Framing Matters

UMD plays Ohio State next year. Suppose that (miraculously) UMD wins the game. There are two headlines that could run in the Diamondback:

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Would you accept a gamble that offers a 10% chance to win \$95 and a 90% chance to loose \$5?

Would you pay \$5 to participate in a lottery that offers a 10% chance to win \$100 and a 90% chance to win nothing?

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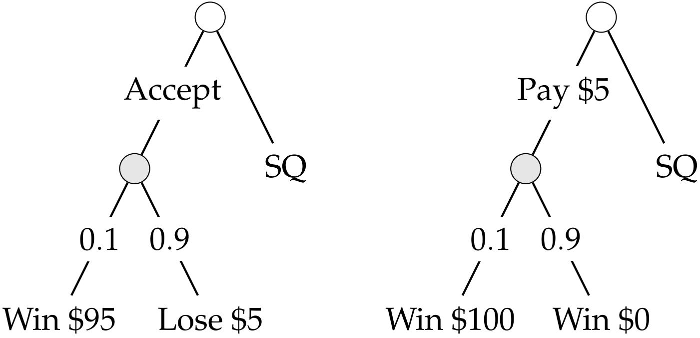

Logicophilia, a virulent virus, threatens 600 students at the University of Maryland

[Adapted from Tversky and Kahneman (1981)]

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1. You must choose between two prevention programs, resulting in:
   - A: 200 participants will be saved for sure.
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   72 % of the participants choose A over B.

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2. You must choose between two prevention programs, resulting in:

   A′: 400 will not be saved, for sure.

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   78 % of the participants choose B′ over A′.

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The Experiment:

A: 0 + 200 for sure.	**B:** (33% 600) + (66% 0).
⇒ 72 % of the participants choose A over B.	
A′: 600 - 400 for sure.	**B′:** (33% 600) + (66% 0).
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Also true of many formalisms of beliefs:

  - "Believing" A and $\vdash A \leftrightarrow B$ implies "Believing" B.

“The different choices in the two frames fit prospect theory, in which choices between gambles and sure things are resolved differently, depending on whether the outcomes are good or bad. Decision makers tend to prefer the sure thing over the gamble (they are risk averse) when the outcomes are good. They tend to reject the sure thing and accept the gamble (the are risk seeking) when both outcomes are negative. ” (Kahneman, pg. 368)

Schelling’s Example

Suppose your tax depends on your income and how many kids you have.

- The “child deduction” might be, say, 1000 per child:

$$Tax(i,k) = Base(i) - [\max(k,3) \cdot 1000]$$

Q1: Should the child deduction be larger for the rich than for the poor?

Schelling's Example

Instead of taking the "standard" household to be childless, we could lower the base tax for everyone (e.g., by 3000), and add a surcharge for households with less than 3 kids (e.g., 1000/2000/3000).

We could also let the surcharge depend on income.

$$Tax(i,k) = LowerBase(i) + [(3-k) \cdot Surcharge(i)]$$

Q2: Should the childless poor pay as large a surcharge as the childless rich?

Schelling's Example

Q1: Should the child exemption be larger for the rich than for the poor?

Q2: Should the childless poor pay as large a surcharge as the childless rich?

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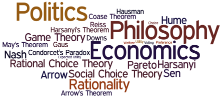

Q1: Should the child exemption be larger for the rich than for the poor?

Q2: Should the childless poor pay as large a surcharge as the childless rich?

If you answered "No" to both, then you are not endorsing a coherent policy

As Kahneman puts the point...
"The difference between the tax owed by a childless family and by a family with two children can be described as a reduction or as an increase. If you want the poor to receive at least the same benefit as the rich for having children, then you must want the poor to pay at least the same penalty as the rich for being childless. "

“The message about the nature of framing is stark: framing should not be viewed as an intervention that masks or distorts an underlying preference. At least in this instance...there is no underlying preference that is masked or distorted by the frame. Our preferences are about framed problems, and our moral intuitions are about descriptions, not substance.”

Any apparent violation of an axiom of the theory can always be interpreted as any of three things:

1. the subjects' preferences *genuinely* violate the axioms of the theory;
2. the subjects' preferences have changed during the course of the experiment;
3. the experimenter has overlooked a relevant feature of the context that affects the the subjects' preferences.

Aim of rational choice theory

- Explanation
- Prediction
- Recommendation

The Aim of Economics

The main task of the social sciences is to explain social phenomena. It is not the only task, but it is the most important one, to which others are subordinated or on which they depend. (Elster, pg. 9)

J. Elster. *Explaining Social Behavior: More Nuts and Bolts for the Social Sciences*. Cambridge University Press, 2007.

Stability Individuals' preferences are stable over the period of the investigation.

Invariance Individuals' preferences are invariant to irrelevant changes in the context of making the decision.

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Rationality is clearly an evaluative notion. A rational action is one that is commendable, and an irrational action is one that is not. One cannot consistently say that a certain choice would be irrational and at the same time that the agent ought to do it.

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Rationality is clearly an evaluative notion. A rational action is one that is commendable, and an irrational action is one that is not. One cannot consistently say that a certain choice would be irrational and at the same time that the agent ought to do it. But according to the economist's view, it is the agent's values that matter in the evaluation, not the economist's. The economist provides only some formal constraints of consistency.

The problem is that invariance is not a merely formal principle. If we left it to the agent to determine what counts as a “relevant” feature of the context, no choice would ever be irrational.

Principle of Individuation by Justifiers Outcomes should be distinguished as different if and only if they differ in a way that makes it rational to have a preference between them.

A dilemma

Either stick to the “formal axioms” of completeness, transitivity, Independence, etc. and refuse to assume the principles of stability and invariance.

A dilemma

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Either stick to the “formal axioms” of completeness, transitivity, Independence, etc. and refuse to assume the principles of stability and invariance. But then rational choice theory will be useless for all explanatory and predictive purposes because people could have fully rational preferences that constantly change or are immensely context-dependent. Alternatively an economists can assume stability and invariance but only at the expense of making rational-choice theory a substantive theory, a theory laden not just with values but with *the economist’s* values.