PHIL309P
Philosophy, Politics and Economics

Eric Pacuit
University of Maryland, College Park
pacuit.org

Announcements

- Course website
  https://myelms.umd.edu/courses/1133211
- Online quiz 3
- Reading: Gaus, Ch 4; Reiss, Ch 4

Taking Stock

- Preferences (transitivity, completeness)
- Ordinal vs. cardinal utilities
- Subjected expected utility
- Payoff is not the same as utility (von Neumann-Morgenstern utilities)
- Rational choice models should be applied with care (attitudes towards risk, attitudes toward ambiguity, act-state dependence, . . .)

From Decisions to Games, I

Commenting on the difference between Robin Crusoe's maximization problem and the maximization problem faced by participants in a social economy, von Neumann and Morgenstern write:

"Every participant can determine the variables which describe his own actions but not those of the others. Nevertheless those "alien" variables cannot, from his point of view, be described by statistical assumptions.

From Decisions to Games, I

Commenting on the difference between Robin Crusoe's maximization problem and the maximization problem faced by participants in a social economy, von Neumann and Morgenstern write:

"Every participant can determine the variables which describe his own actions but not those of the others. Nevertheless those "alien" variables cannot, from his point of view, be described by statistical assumptions. This is because the others are guided, just as he himself, by rational principles—whatever that may mean—and no *modus procedendi* can be correct which does not attempt to understand those principles and the interactions of the conflicting interests of all participants."

(vNM, pg. 11)

Game Situations

1. a group of *self-interested* agents (players) involved in some interdependent decision problem

Game Situations

	Bob	
	L	R
	1	0

1. a group of *self-interested* agents (players) involved in some interdependent decision problem

Game Situations

	Bob	
	L	R
	1	0
	0	1

1. a group of *self-interested* agents (players) involved in some interdependent decision problem

Game Situations

		Bob	
		L	R
Ann	U	1 1	0 0
	D	0 0	1 1

1. a group of *self-interested* agents (players) involved in some interdependent decision problem

Game Situations

Ann \ Bob	L	R
U	1,1	0,0
D	0,0	1,1

1. a group of *self-interested* agents (players) involved in some interdependent decision problem

Game Situations

Ann \ Bob	L	R
U	1,1	0,0
D	0,0	1,1

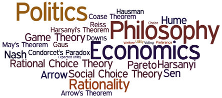
1. a group of *self-interested* agents (players) involved in some interdependent decision problem

Just Enough Game Theory

A **game** is a mathematical model of a strategic interaction that includes

- the actions the players *can* take
- the players' interests (i.e., preferences),
- the "structure" of the decision problem

Just Enough Game Theory



A **game** is a mathematical model of a strategic interaction that includes

- the actions the players *can* take
- the players' interests (i.e., preferences),
- the "structure" of the decision problem

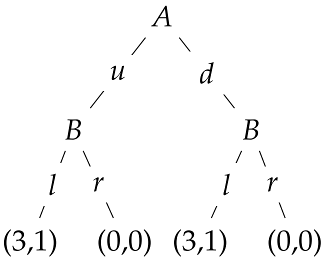
It does not specify the actions that the players do take.

Games

		B	
		l	r
A	u	3, 1	0, 0
	d	0, 0	1, 3

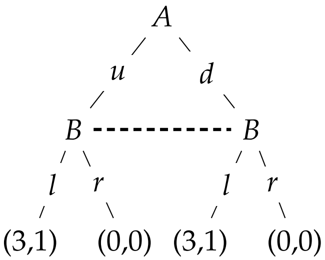
Games

		B	
		l	r
A	u	3, 1	0, 0
	d	0, 0	1, 3



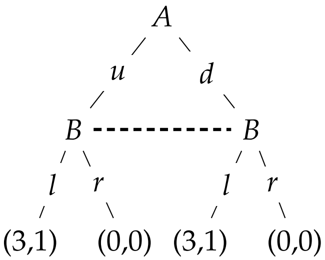
Games

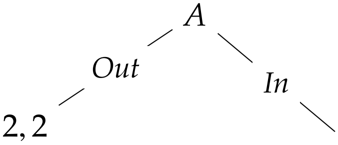
		B	
		l	r
A	u	3, 1	0, 0
	d	0, 0	1, 3



Games

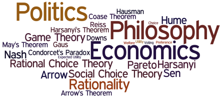
		B	
		l	r
A	u	3, 1	0, 0
	d	0, 0	1, 3





		B	
		l	r
A	u	3, 1	0, 0
	d	0, 0	1, 3

From Decisions to Games, II

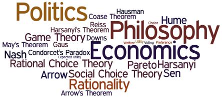


"*[T]he* fundamental insight of game theory [is] that a rational player must take into account that the players reason about each other in deciding how to play."

R. Aumann and J. Dreze. *Rational Expectations in Games*. American Economic Review, 98, pp. 72-86, 2008.

The Guessing Game

Guess a number between 1 & 100.
The closest to 2/3 of the average wins.



The Guessing Game

Guess a number between 1 & 100.
The closest to 2/3 of the average wins.

What number should you guess?

The Guessing Game

Guess a number between 1 & 100.
The closest to 2/3 of the average wins.

What number should you guess? 100

The Guessing Game

Guess a number between 1 & 100.
The closest to 2/3 of the average wins.

What number should you guess? ~~100~~, 99

The Guessing Game

Guess a number between 1 & 100.
The closest to 2/3 of the average wins.

What number should you guess? ~~100~~, ~~99~~, . . . , 67

The Guessing Game

Guess a number between 1 & 100.
The closest to 2/3 of the average wins.

What number should you guess? ~~100~~, ~~99~~, . . . , ~~67~~, . . . ,
2, 1

The Guessing Game

Guess a number between 1 & 100.
The closest to 2/3 of the average wins.

What number should you guess? ~~100~~, ~~99~~, . . . , ~~67~~, . . . , ~~2~~, 1

Solution Concept

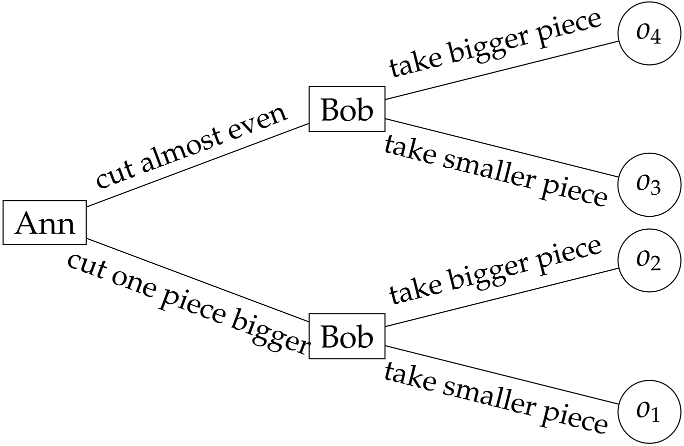
A **solution concept** is a systematic description of the outcomes that may emerge in a family of games.

This is the starting point for most of game theory and includes many variants: Nash equilibrium, backwards induction, or iterated dominance of various kinds.

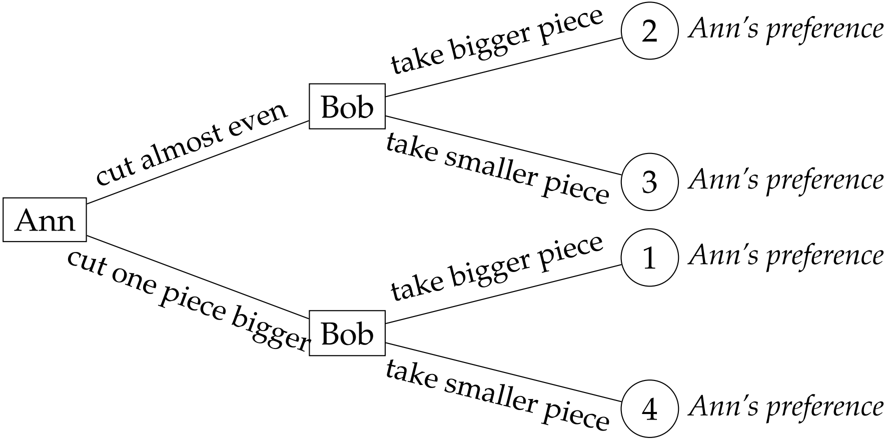
These are usually thought of as the embodiment of “rational behavior” in some way and used to analyze game situations.

Suppose there are two players Ann and Bob dividing a cake. Suppose that Ann cuts the cake and then Bob chooses the first piece. (Suppose they *only* care about the size of the piece). Ann cannot cut the cake exactly evenly, so one piece is always larger than the other.

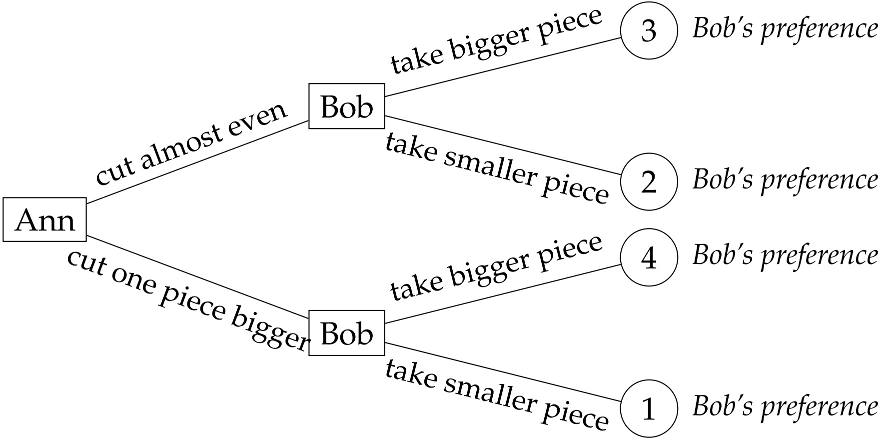
Suppose there are two players Ann and Bob dividing a cake. Suppose that Ann cuts the cake and then Bob chooses the first piece. (Suppose they *only* care about the size of the piece). Ann cannot cut the cake exactly evenly, so one piece is always larger than the other.

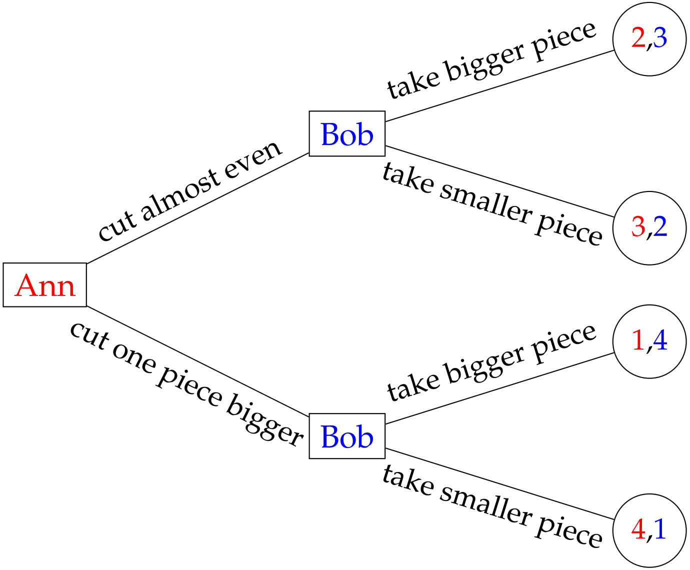


Suppose there are two players Ann and Bob dividing a cake. Suppose that Ann cuts the cake and then Bob chooses the first piece. (Suppose they *only* care about the size of the piece). Ann cannot cut the cake exactly evenly, so one piece is always larger than the other.



Suppose there are two players Ann and Bob dividing a cake. Suppose that Ann cuts the cake and then Bob chooses the first piece. (Suppose they *only* care about the size of the piece). Ann cannot cut the cake exactly evenly, so one piece is always larger than the other.




Ann
cut almost even
cut one piece bigger
Bob
take bigger piece
take smaller piece
2,3
3,2
Bob
take bigger piece
take smaller piece
1,4
4,1

		Bob	
		TB	TS
Ann	CB	1,4	4,1
	CE	2,3	3,2

What should Ann *do*?

Ann \ Bob	TB	TS
CB	1,4	4,1
CE	2,3	3,2

What should Ann *do*? *Bob best choice in Ann's worst choice*

		Bob	
Ann	*TB*	*TS*	
CB	1,4	4,1	1
CE	2,3	3,2	2

What should Ann *do*? *maximize over each row and choose the maximum value*

		Bob	
		TB	*TS*
Ann	*CB*	1,4	4,1
	CE	2,3	3,2
		3	1

What should Bob *do*? *minimize over each column and choose the maximum value*

Zero-Sum Games

Von Neumann Minmax Theorem. In any finite, two-player, zero-sum game, there is always at least one minmax solution.

Let $G = \langle \{S_i\}_{i \in N}, \{u_i\}_{i \in N} \rangle$ be a finite strategic game (each S_i is finite and the set of players N is finite).

A **strategy profile** is an element $\sigma \in S = S_1 \times \cdots \times S_n$

σ is a **Nash equilibrium** provided for all i, for all $s_i \in S_i$,

$$u_i(\sigma) \geq u_i(s_i, \sigma_{-i})$$

Zero-Sum Games

Ann \ Bob	L	R
U	1,4	4,1
D	2,3	3,2

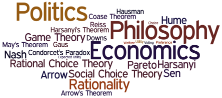
The profile of security strategies (D, L) is a Nash equilbirium

Matching Pennies

		Bob	
		H	T
Ann	H	1,-1	-1, 1
	T	-1,1	1,-1

There are no pure strategy Nash equilibria.

Mixed Strategies



Ann \ Bob	H	T
H	1,-1	-1, 1
T	-1,1	1,-1

A **mixed strategy** is a probability distribution over the set of pure strategies. For instance:

- $[1/2 : H, 1/2 : T]$
- $[1/3 : H, 2/3 : T]$
- ...

Matching Pennies

Bob

Ann		H	T
	H	1,-1	-1, 1
	T	-1,1	1,-1

The mixed strategy $([1/2 : H, 1/2 : T], [1/2 : H, 1/2 : T])$ is the only Nash equilibrium.

Theorem (Von Neumann). For every two-player zero- sum game with finite strategy sets S_1 and S_2, there is a number v, called the **value** of the game such that:

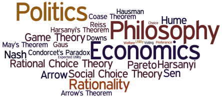
1. $v = \max_{p \in \Delta(S_1)} \min_{q \in \Delta(S_2)} U_1(p, q) = \min_{q \in \Delta(S_2)} \max_{p \in \Delta(S_1)} U_1(p, q)$
2. The set of mixed Nash equilibria is nonempty. A mixed strategy profile (p, q) is a Nash equilibrium if and only if

$$p \in \operatorname{argmax}_{p \in \Delta(S_1)} \min_{q \in \Delta(S_2)} U_1(p, q)$$

$$q \in \operatorname{argmax}_{q \in \Delta(S_2)} \min_{p \in \Delta(S_1)} U_1(p, q)$$

3. For all mixed Nash equilibria (p, q), $U_1(p, q) = v$

Prisoner’s Dilemma



Two people commit a crime.

Prisoner’s Dilemma

Two people commit a crime. The are arrested by the police, who are quite sure they are guilty but cannot prove it without at least one of them confessing.

Prisoner's Dilemma

Two people commit a crime. The are arrested by the police, who are quite sure they are guilty but cannot prove it without at least one of them confessing. The police offer the following deal. Each one of them can confess and get credit for it.

Prisoner's Dilemma

Two people commit a crime. The are arrested by the police, who are quite sure they are guilty but cannot prove it without at least one of them confessing. The police offer the following deal. Each one of them can confess and get credit for it. If only one confesses, he becomes a state witness and not only is he not punished, he gets a reward.

Prisoner's Dilemma

Two people commit a crime. The are arrested by the police, who are quite sure they are guilty but cannot prove it without at least one of them confessing. The police offer the following deal. Each one of them can confess and get credit for it. If only one confesses, he becomes a state witness and not only is he not punished, he gets a reward. If both confess, they will be punished but will get reduced sentences for helping the police.

Prisoner's Dilemma

Two people commit a crime. The are arrested by the police, who are quite sure they are guilty but cannot prove it without at least one of them confessing. The police offer the following deal. Each one of them can confess and get credit for it. If only one confesses, he becomes a state witness and not only is he not punished, he gets a reward. If both confess, they will be punished but will get reduced sentences for helping the police. If neither confesses, the police honestly admit that there is no way to convict them, and they are set free.

Prisoner's Dilemma

Two options: Cooperate with each other by not confessing (C), Defect by confessing (D)

Prisoner's Dilemma

Two options: Cooperate with each other by not confessing (C), Defect by confessing (D)

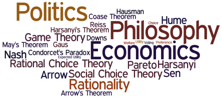
Possible outcomes:

Prisoner's Dilemma

Two options: Cooperate with each other by not confessing (C), Defect by confessing (D)

Possible outcomes: Both cooperate (C, C),

Prisoner's Dilemma



Two options: Cooperate with each other by not confessing (C), Defect by confessing (D)

Possible outcomes: Both cooperate (C, C), I cooperate but my partner doesn't (C, D),

Prisoner's Dilemma

Two options: Cooperate with each other by not confessing (C), Defect by confessing (D)

Possible outcomes: Both cooperate (C, C), I cooperate but my partner doesn't (C, D), My partner cooperates but I don't (D, C),

Prisoner's Dilemma

Two options: Cooperate with each other by not confessing (C), Defect by confessing (D)

Possible outcomes: Both cooperate (C, C), I cooperate but my partner doesn't (C, D), My partner cooperates but I don't (D, C), both defect (D, D).

Prisoner's Dilemma

		Bob	
		C	D
Ann	C		
	D		

Prisoner's Dilemma

Bob

Ann	C	D
C	3	1
D	4	2

Ann's preferences

Prisoner's Dilemma

Ann \ Bob	C	D
C	3	4
D	1	2

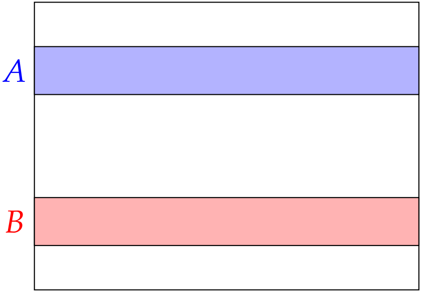
Bob's preferences

Prisoner's Dilemma

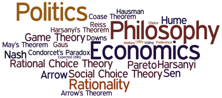
Ann \ Bob	C	D
C	3,3	1,4
D	4,1	2,2

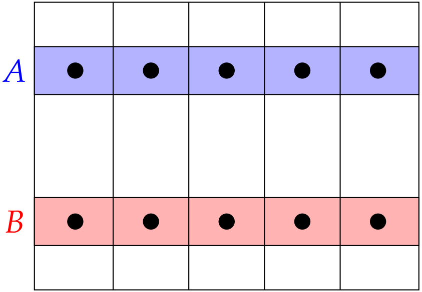
What should Ann (Bob) do?

Dominance Reasoning

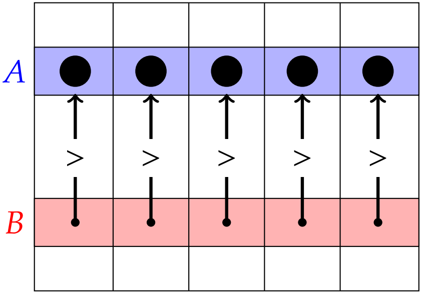


Dominance Reasoning





Dominance Reasoning



Dominance reasoning is appropriate only when probability of outcome is *independent of choice*.

Dominance reasoning is appropriate only when probability of outcome is *independent of choice*.

A nasty nephew wants inheritance from his rich Aunt.

Dominance reasoning is appropriate only when probability of outcome is *independent of choice*.

A nasty nephew wants inheritance from his rich Aunt. The nephew wants the inheritance, but other things being equal, does not want to apologize.

Dominance reasoning is appropriate only when probability of outcome is *independent of choice*.

A nasty nephew wants inheritance from his rich Aunt. The nephew wants the inheritance, but other things being equal, does not want to apologize. Does dominance give the nephew a reason to not apologize?

Dominance reasoning is appropriate only when probability of outcome is *independent of choice*.

A nasty nephew wants inheritance from his rich Aunt. The nephew wants the inheritance, but other things being equal, does not want to apologize. Does dominance give the nephew a reason to not apologize? *Whether or not the nephew is cut from the will may depend on whether or not he apologizes*.

Prisoner's Dilemma

Ann \ Bob	C	D
C	3,3	1,4
D	4,1	2,2

What should Ann (Bob) do?

Prisoner's Dilemma

		Bob	
		C	D
Ann	C	3,3	1,4
	D	4,1	2,2

What should Ann (Bob) do? *Dominance reasoning*

Prisoner’s Dilemma

Ann \ Bob	*C*	*D*
C	3,3	1,4
D	4,1	2,2

What should Ann (Bob) do? *Dominance reasoning*

Prisoner's Dilemma

		Bob	
		C	*D*
Ann	*C*	3,3	1,4
	D	4,1	2,2

What should Ann (Bob) do? *Dominance reasoning* is not **Pareto**!

Prisoner's Dilemma

		Bob	
		C	D
Ann	C	3	2.5
	D	2.5	2

What should Ann (Bob) do? *Think as a group*!

Prisoner's Dilemma

		Bob: C	Bob: D
Ann	C	3,3	1,4
	D	4,1	2,2

What should Ann (Bob) do? *Play against your mirror image*!

Prisoner's Dilemma

Ann \ Bob	C	D
C	3,3	1,4
D	4,1	2,2

What should Ann (Bob) do? *Play against your mirror image*!

Prisoner's Dilemma

Ann \ Bob	C	D
C	ϵ,ϵ	1,4
D	4,1	2,2

What should Ann (Bob) do? *Change the game...*