PHIL309P
Philosophy, Politics and Economics

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Announcements

- Course website
  https://myelms.umd.edu/courses/1133211
- Online quiz 3
- Reading: Gaus, Ch 4; **Reiss, Ch 4**

Strategic Games

A **strategic game** is a tuple $\langle N, \{A_i\}_{i\in N}, \{\succeq_i\}_{i\in N}\rangle$ where

- N is a finite set of **players**

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Strategic Games

A **strategic game** is a tuple $\langle N, \{A_i\}_{i\in N}, \{\succeq_i\}_{i\in N}\rangle$ where

- N is a finite set of **players**
- for each $i \in N$, A_i is a nonempty set of **actions**
- for each $i \in N$, $\succeq_i$ is a **preference relation** on $A = \Pi_{i\in N} A_i$
  (Often $\succeq_i$ are represented by **utility functions** $u_i : A \to \mathbb{R}$)

Strategic Games: Comments on Preferences

- Preferences may be over a set of consequences C. Assume $g : A \to C$ and $\{\succeq_i^* \mid i \in N\}$ a set of preferences on C. Then for $a, b \in A$,

$$a \succeq_i b \text{ iff } g(a) \succeq_i^* g(b)$$

- Consequences may be affected by exogenous random variable whose realization is not known before choosing actions. Let Ω be a set of states, then define $g : A \times \Omega \to C$. Where $g(a|\cdot)$ is interpreted as a *lottery*.

- Often $\succeq_i$ are represented by **utility functions** $u_i : A \to \mathbb{R}$

Strategic Games: Example

		Column	
		r	l
Row	u		
	d		

- $N = \{Row, Column\}$
- $A_{Row} = \{u, d\}$, $A_{Column} = \{r, l\}$
- $(u, r) \succeq_{Row} (d, l) \succeq_{Row} (u, l) \sim_{Row} (d, r)$
  $(u, r) \succeq_{Column} (d, l) \succeq_{Column} (u, l) \sim_{Column} (d, r)$

Strategic Games: Example

$$
\begin{array}{cc|c|c|}
 & & \multicolumn{2}{c}{\text{Column}} \\
 & & \text{r} & \text{l} \\
\hline
\text{Row} & \text{u} & (2,2) & (0,0) \\
\hline
 & \text{d} & (0,0) & (1,1) \\
\hline
\end{array}
$$

- $N = \{Row, Column\}$
- $A_{Row} = \{u, d\}$, $A_{Column} = \{r, l\}$
- $u_{Row} : A_{Row} \times A_{Column} \to \{0, 1, 2\}$, $u_{Column} : A_{Row} \times A_{Column} \to \{0, 1, 2\}$ with $u_{Row}(u, r) = u_{Column}(u, r) = 2$, $u_{Row}(d, l) = u_{Column}(d, l) = 2$, and $u_x(u, l) = u_x(d, r) = 0$ for $x \in N$.

Nash Equilibrium

Let $\langle N, \{A_i\}_{i \in N}, \{\succeq_i\}_{i \in N} \rangle$ be a strategic game

For $a_{-i} \in A_{-i}$, let

$$B_i(a_{-i}) = \{a_i \in A_i \mid (a_{-i}, a_i) \succeq_i (a_{-i}, a_i') \; \forall \; a_i' \in A_i\}$$

B_i is the **best-response** function.

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B_i is the **best-response** function.

$a^* \in A$ is a **Nash equilibrium** iff $a_i^* \in B_i(a_{-i}^*)$ for all $i \in N$.

Example: Bach or Stravinsky?

	b_c	s_c
b_r	2,1	0,0
s_r	0,0	1,2

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$$N = \{r, c\} \quad A_r = \{b_r, s_r\}, A_c = \{b_c, s_c\}$$

$$B_r(b_c) = \{b_r\} \qquad\qquad B_r(s_c) = \{s_r\}$$

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$$B_c(b_r) = \{b_c\} \qquad B_c(s_r) = \{s_c\}$$

(b_r, b_c) is a Nash Equilibrium $\qquad$ (s_r, s_c) is a Nash Equilibrium

Battle of the Sexes

		Bob	
		B	S
Ann	B	$2, 1$	$0, 0$
	S	$0, 0$	$1, 2$

Battle of the Sexes

Ann \ Bob	B	M
B	$2,1$	$0,0$
S	$0,0$	$1,2$

(B,B) (S,S), and $([2/3:B,1/3:S],[1/3:B,2/3:S])$ are Nash equilibria.

Kevin Quealy. *Lessons From Game Theory: What Keeps Kasich in the Race?*. New York Times, Feb. 24, 2016.

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"The Republican establishment has a problem. It is headed for a car crash. With Jeb Bush out of the Republican presidential race, the two remaining mainstream candidates Marco Rubio and John Kasich are living out an issue studied for decades in game theory. Game theorists might call the G.O.P. predicament an anti-coordination game or even a volunteers dilemma. But most of us might call it by a more familiar name: chicken."

Chicken

Kasich \ Rubio	*D*	*S*
D	-4,-4	-1,1
S	1,-1	-2,-2

Chicken

Rubio

Kasich	D	S
D	-4,-4	-1,1
S	1,-1	-2,-2

(D, S) and (S, D) are Nash equilibria. If both choose their components of these equilibria, we may end up at (D, D).

Chicken

Kasich \ Rubio	D	S
D	-4,-4	-1,1
S	1,-1	-2,-2

(D, S) and (S, D) are Nash equilibria. Their security strategies are (S, S).

Part of the reason this dilemma exists in the first place is that mainstream Republicans lack the unity or influence to **compel any cooperation**....If establishment Republicans had a clear, unimpeachable leader who was not a participant in the race, that person might be able to compel a candidate to drop out and support whomever the party determined to be strongest, allowing candidates who quit to save face by saying they did it for "the good of the party."

Second, this is a game that’s played just once. The chance to be your partys nominee for president comes along only every four or eight years, even for the very luckiest candidates. If the candidates lived in a universe in which they could run for president hundreds of times, they might agree that, on average, their shared interests were better served by cooperating....

Second, this is a game that's played just once. The chance to be your partys nominee for president comes along only every four or eight years, even for the very luckiest candidates. If the candidates lived in a universe in which they could run for president hundreds of times, they might agree that, on average, their shared interests were better served by cooperating....But this is not an **iterated dilemma**. It's a one-time-only dilemma with a tremendous payoff for the winner. As much as Mr. Kasich might think about his legacy, the good of the party or even his own chances in 2020 or 2024, the future is very far away.

In an arbitrary (finite) games (that are not zero-sum)

- There exists a mixed strategy Nash equilibrium
- Security strategies are not necessarily a Nash equilibrium
- There may be more than on Nash equilibrium
- Components of Nash equilibrium are not interchangeable.
- Why *should* players play a Nash equilibrium?

Prisoner's Dilemma

		Bob	
		C	D
Ann	C		
	D		

Prisoner's Dilemma

		Bob	
		C	D
Ann	C	3	1
	D	4	2

Ann's preferences

Prisoner's Dilemma

		Bob	
		C	D
Ann	C	3	4
	D	1	2

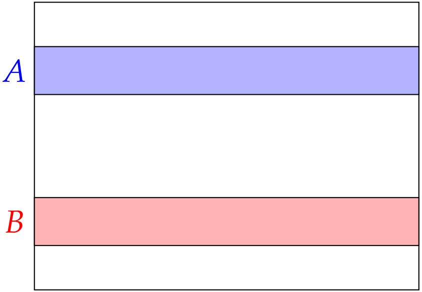
Bob's preferences

Prisoner's Dilemma

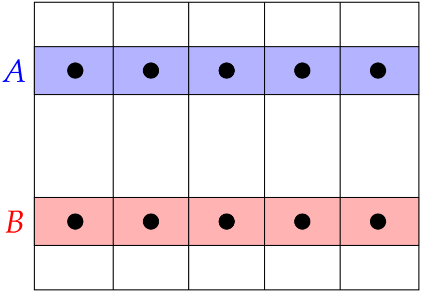
		Bob	
		C	D
Ann	C	3,3	1,4
	D	4,1	2,2

What should Ann (Bob) do?

Dominance Reasoning



Dominance Reasoning



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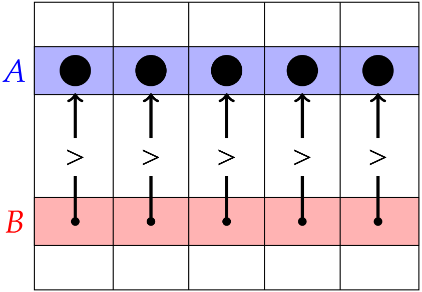

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Dominance reasoning is appropriate only when probability of outcome is *independent of choice*.

A nasty nephew wants inheritance from his rich Aunt. The nephew wants the inheritance, but other things being equal, does not want to apologize. Does dominance give the nephew a reason to not apologize? *Whether or not the nephew is cut from the will may depend on whether or not he apologizes*.

Prisoner's Dilemma

Ann \ Bob	C	D
C	3,3	1,4
D	4,1	2,2

What should Ann (Bob) do?

Prisoner's Dilemma

		Bob	
		C	D
Ann	C	3,3	1,4
	D	4,1	2,2

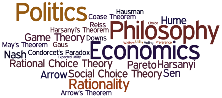
What should Ann (Bob) do? *Dominance reasoning*

Prisoner’s Dilemma

		Bob	
		C	*D*
Ann	*C*	3,3	1,4
	D	4,1	2,2

What should Ann (Bob) do? *Dominance reasoning*

Prisoner's Dilemma



		Bob	
		C	D
Ann	C	3,3	1,4
	D	4,1	2,2

What should Ann (Bob) do? *Dominance reasoning* is not **Pareto**!

Prisoner's Dilemma

Ann \ Bob	C	D
C	3	2.5
D	2.5	2

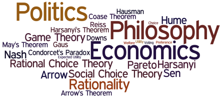
What should Ann (Bob) do? *Think as a group*!

Prisoner’s Dilemma

		Bob	
		C	D
Ann	C	3,3	1,4
	D	4,1	2,2

What should Ann (Bob) do? *Play against your mirror image*!

Prisoner's Dilemma



		Bob	
		C	D
Ann	C	3,3	1,4
	D	4,1	2,2

What should Ann (Bob) do? *Play against your mirror image*!

Prisoner's Dilemma

Ann \ Bob	C	D
C	ϵ,ϵ	$1,4$
D	$4,1$	$2,2$

What should Ann (Bob) do? *Change the game...*

Nozick: Symbolic Utility

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R. Nozick. *The Nature of Rationality*. Princeton University Press, 1993.

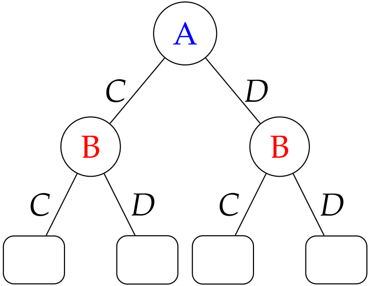
Ann \ Bob	C	D
C	3,3	1,4
D	4,1	2,2

Prisoner's Dilemma

What should/will Ann (Bob) do?

		Bob	
		C	D
Ann	C	3,3	1,4
	D	4,1	2,2

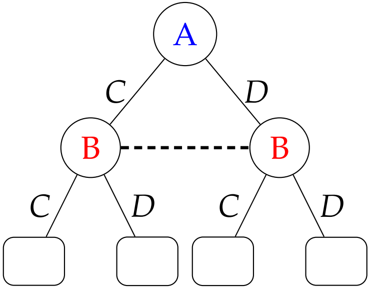
Prisoner's Dilemma



What should/will Ann (Bob) do?

		Bob	
		C	D
Ann	C	3,3	1,4
	D	4,1	2,2

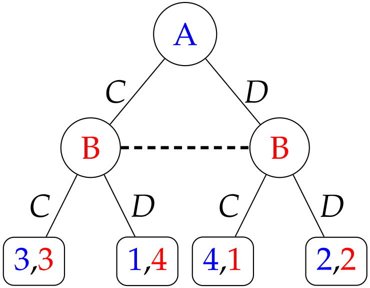
Prisoner's Dilemma



What should/will Ann (Bob) do?

		Bob C	Bob D
Ann	C	3,3	1,4
Ann	D	4,1	2,2

Prisoner's Dilemma



What should/will Ann (Bob) do?

		Bob	
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Ann	C	3,3	1,4
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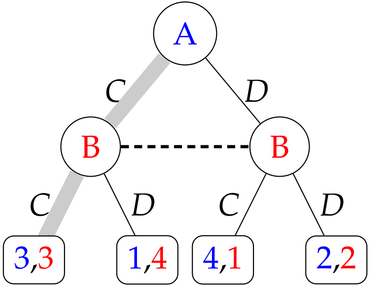
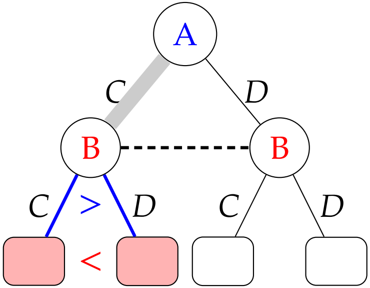
Prisoner's Dilemma



What should/will Ann (Bob) do?

		Bob	
		C	D
Ann	C	3,3	1,4
	D	4,1	2,2

Prisoner's Dilemma



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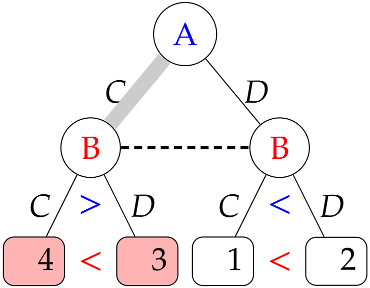
		Bob	
		C	D
Ann	C	3,3	1,4
	D	4,1	2,2

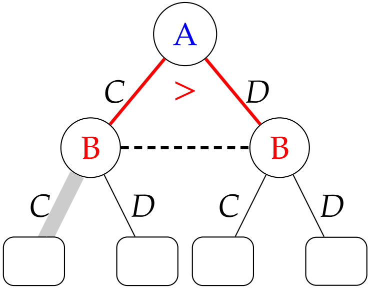
Prisoner's Dilemma



What should/will Ann (Bob) do?

		Bob	
		C	D
Ann	C	3,3	1,4
	D	4,1	2,2

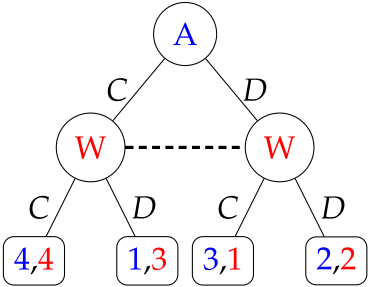
Prisoner's Dilemma



What should/will Ann (Bob) do?

		Bob	
		C	D
Ann	C	3,3	1,4
	D	4,1	2,2

Prisoner's Dilemma



What should/will Ann (Bob) do?

		Bob	
		C	*D*
Ann	*C*	3,3	1,4
	D	4,1	2,2

Prisoner's Dilemma

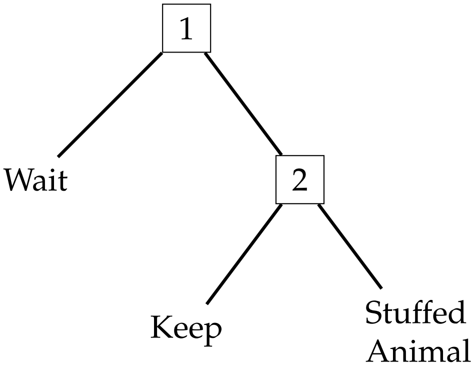
		Bob	
		C	*D*
Ann	*C*	4,4	1,3
	D	3,1	2,2

Assurance Game

What should/will Ann (Bob) do? *Change the game* (eg., Symbolic Utilities)

The difference between a standard Assurance Game and Nozick's symbolic solution to the Prisoner's Dilemma is not, as Nozick would have it, that some payoffs are relevant but are not included in the game, as if there is some extra utility lurking some-where outside the matrix.

We can have two games that have identical payoffs yet the nature of their decision trees can differ.



1. I want you to decide if you *really* want the stuffed animal. If you decide that you want something now and want to use your money, then you can have the stuffed animal.
2. You can have the stuffed animal (but you must use some of your own money).

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“Game theorists think it just plain wrong to claim that the Prisoners’ Dilemma embodies the essence of the problem of human cooperation. On the contrary, it represents a situation in which the dice are as loaded against the emergence of cooperation as they could possibly be. If the great game of life played by the human species were the Prisoner’s Dilemma, we wouldn’t have evolved as social animals!

"Game theorists think it just plain wrong to claim that the Prisoners' Dilemma embodies the essence of the problem of human cooperation. On the contrary, it represents a situation in which the dice are as loaded against the emergence of cooperation as they could possibly be. If the great game of life played by the human species were the Prisoner's Dilemma, we wouldn't have evolved as social animals! .... No paradox of rationality exists. Rational players don't cooperate in the Prisoners' Dilemma, because the conditions necessary for rational cooperation are absent in this game." (pg. 63)

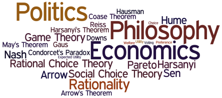
K. Binmore. *Natural Justice*. Oxford University Press, 2005.

Nozick's intuition is right. Just because the payoffs are the same—the games look the same in their strategic form—they may nevertheless be different games in their extensive form....In a game, everything of normative relevance for choice—"even the structure of the decision tree itself"—is part of the consequence domain. The utility at the terminal nodes sums up all the normatively relevant considerations. (G, pp. 115, 116)

Split or Steal

Given a pot of money (say 1,000 pounds), contestants are asked to ”Split” or ”Steal”. If both choose “Split”, the pot is split. If both choose “Steal”, they go home with nothing. If only one chooses “Steal”, then that person goes home with the money.

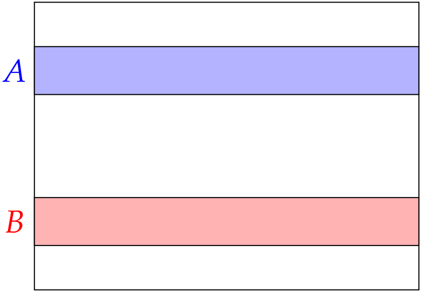
Split or Steal



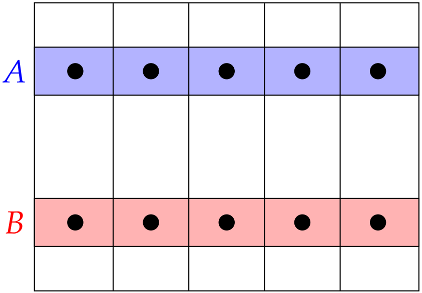
		Contestent 2	
		Split	*Steal*
Contestant 1	*Split*	500,500	0,1000
	Steal	1000,0	0,0

What would you do?

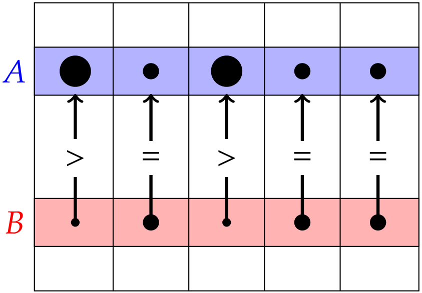
Weak Dominance



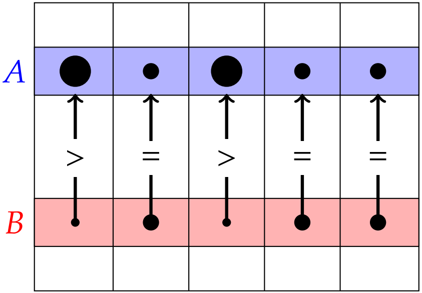
Weak Dominance



Weak Dominance



Weak Dominance



Contestent 2

Contestant 1	*Split*	*Steal*
Split	500,500	0,1000
Steal	1000,0	0,0

What would you do?

Kasich-Rubio Game

Second, this is a game that's played just once. The chance to be your partys nominee for president comes along only every four or eight years, even for the very luckiest candidates. If the candidates lived in a universe in which they could run for president hundreds of times, they might agree that, on average, their shared interests were better served by cooperating.... But this is not an **iterated dilemma**. It's a one-time-only dilemma with a tremendous payoff for the winner. As much as Mr. Kasich might think about his legacy, the good of the party or even his own chances in 2020 or 2024, the future is very far away.

Iterated Prisoner’s Dilemma

	C	D
C	3,3	0,4
D	4,0	1,1

	C	D
C	3,3	0,4
D	4,0	1,1

	C	D
C	3,3	0,4
D	4,0	1,1

	C	D
C	3,3	0,4
D	4,0	1,1

$\cdots$

Iterated Prisoner's Dilemma

	C	D
C	3,3	0,4
D	4,0	1,1

	C	D
C	3,3	0,4
D	4,0	1,1

	C	D
C	3,3	0,4
D	4,0	1,1

	C	D
C	3,3	0,4
D	4,0	1,1

Iterated Prisoner's Dilemma

	C	D
C	3,3	0,4
D	4,0	1,1

	C	D
C	3,3	0,4
D	4,0	1,1

	C	D
C	3,3	0,4
D	4,0	1,1

	C	D
C	3,3	0,4
D	4,0	1,1

Iterated Prisoner's Dilemma

	C	D
C	3,3	0,4
D	4,0	1,1

	C	D
C	3,3	0,4
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	C	D
C	3,3	0,4
D	4,0	1,1

	C	D
C	3,3	0,4
D	4,0	1,1

Iterated Prisoner's Dilemma

	C	D
C	3,3	0,4
D	4,0	1,1

	C	D
C	3,3	0,4
D	4,0	1,1

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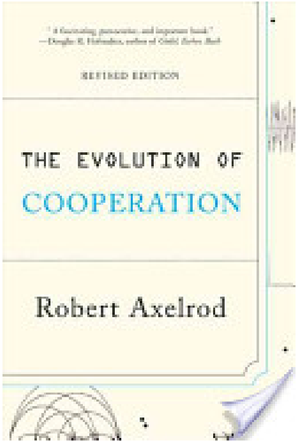
...


REVISED EDITION
THE EVOLUTION OF
COOPERATION
Robert Axelrod

Strategies

- Periodic: All-C, All-D, CD, CCD, CDD, CCDD, …
- Random
- Memory: Tit-for-Tat, Two-Tit-for-Tat, …

	C	D
C	3,3	0,4
D	4,0	1,1

	C	D
C	3,3	0,4
D	4,0	1,1

	C	D
C	3,3	0,4
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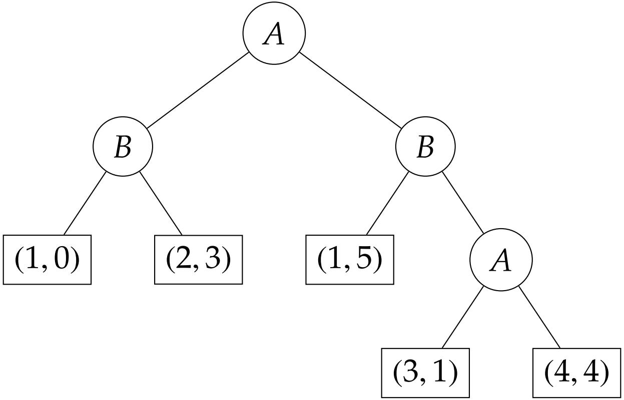
	C	D
C	3,3	0,4
D	4,0	1,1

…

Additional Reading

- S. Kuhn, Prisoner's Dilemma, Stanford Encyclopedia of Philosophy, plato.stanford.edu/entries/prisoner-dilemma/
- W. Poundstone, Prisoner's Dilemma, Anchor, 1993
- Online Game Theory Course (M. Jackson, K. Leyton-Brown and Y. Shoham): game-theory-class.org

Backward Induction



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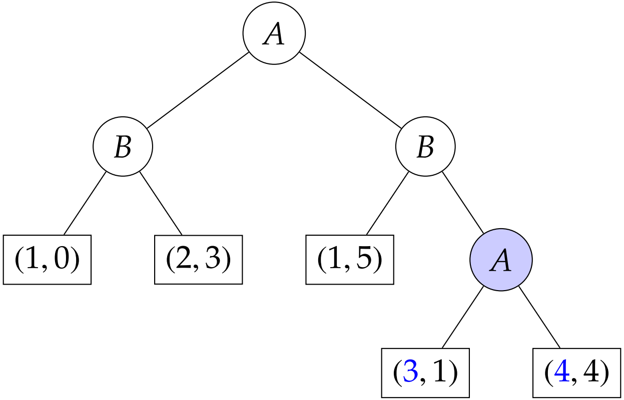

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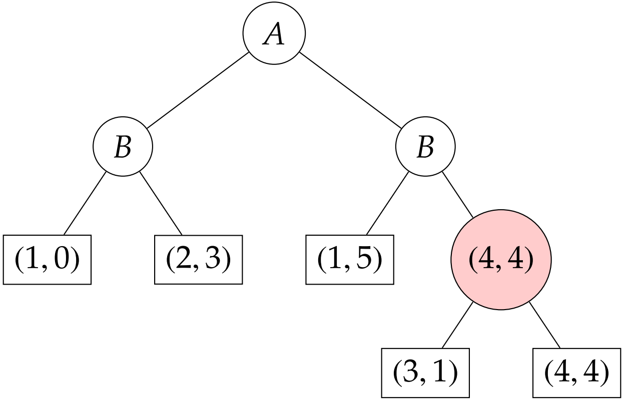

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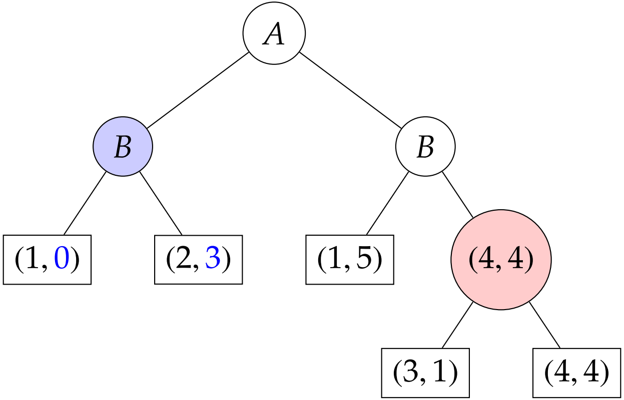

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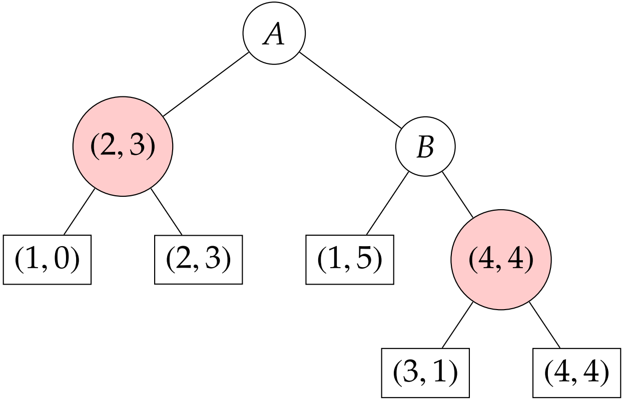

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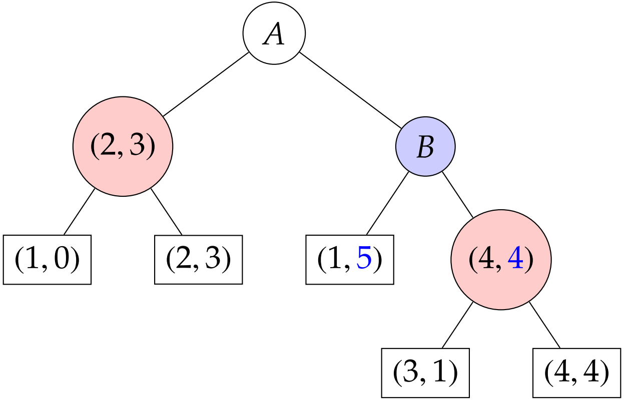

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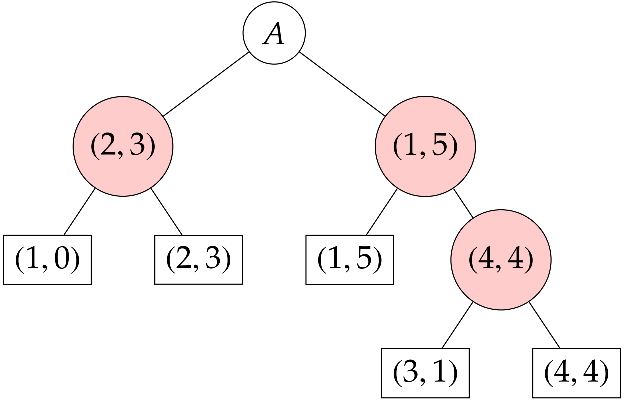

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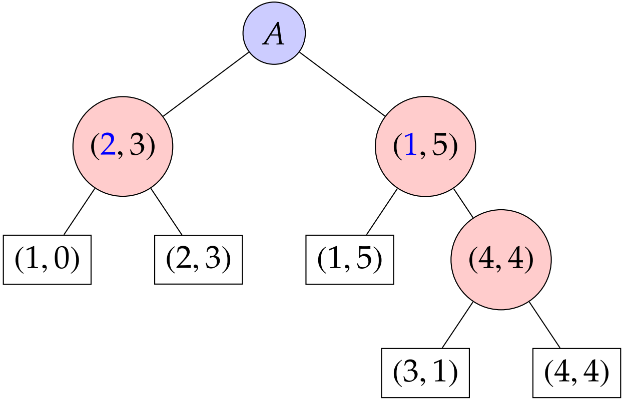

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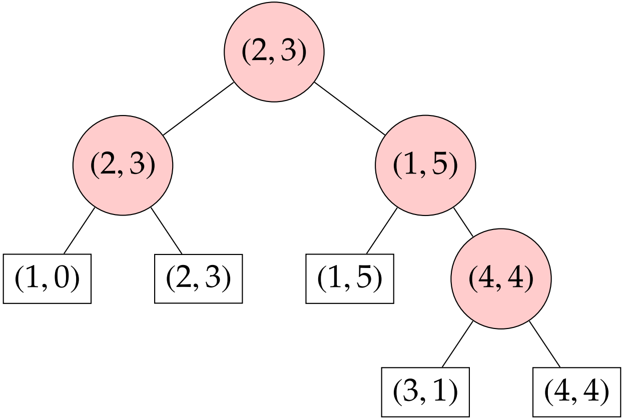

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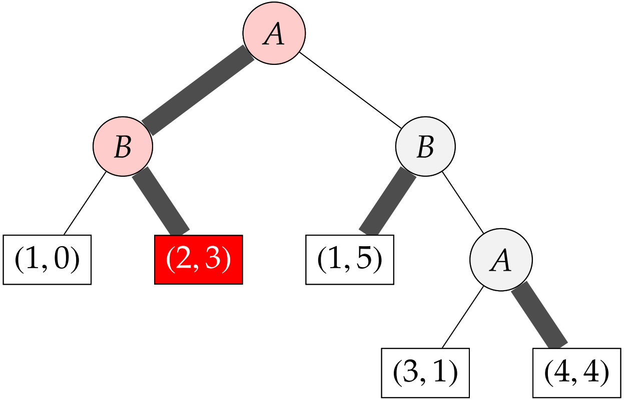

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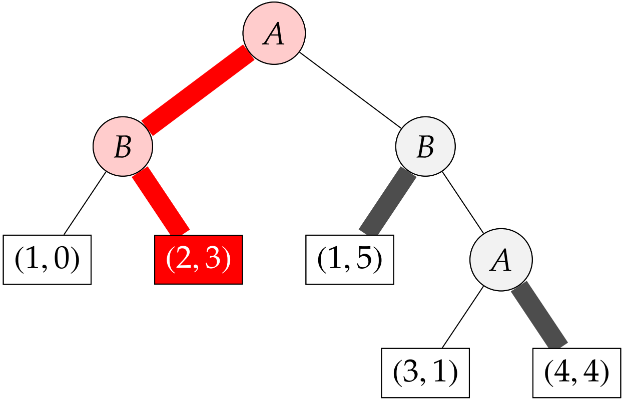
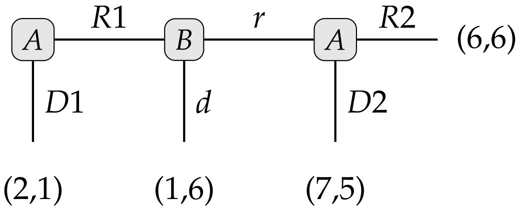


BI Puzzle



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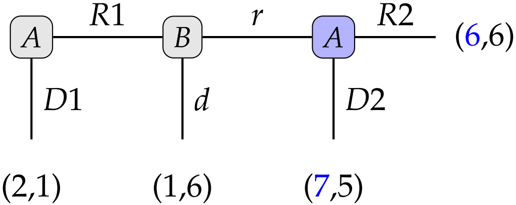

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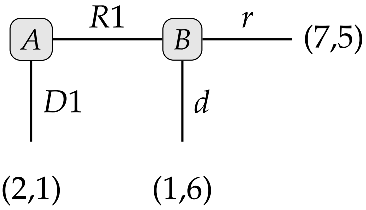

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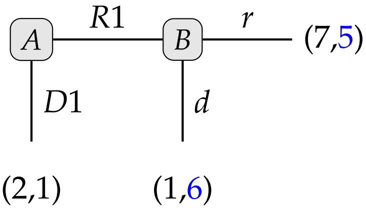

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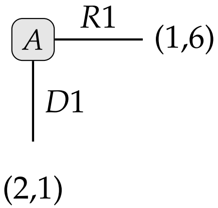

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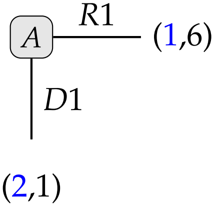

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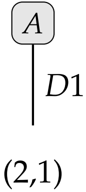

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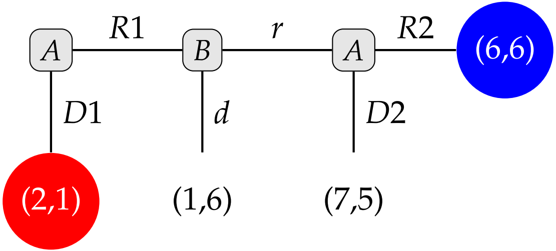

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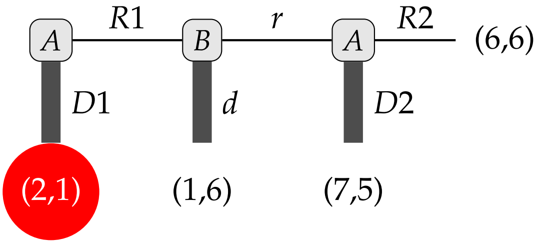
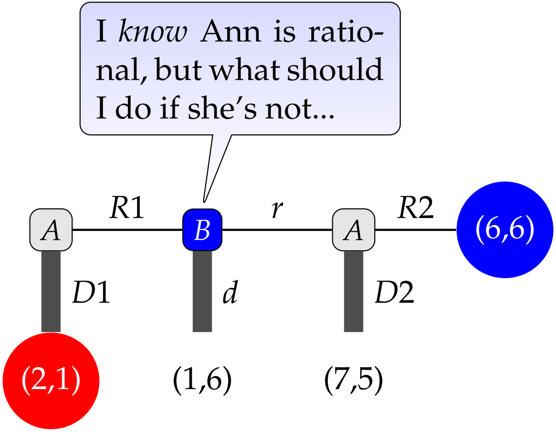
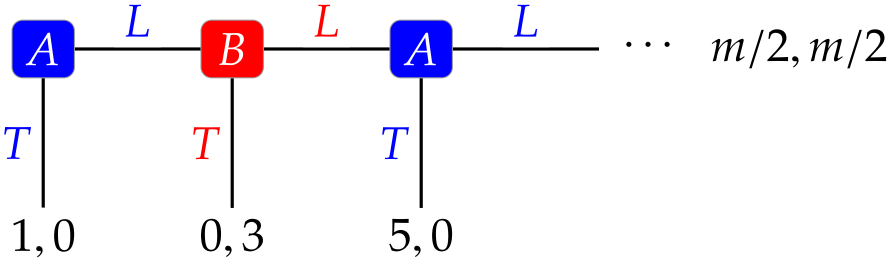


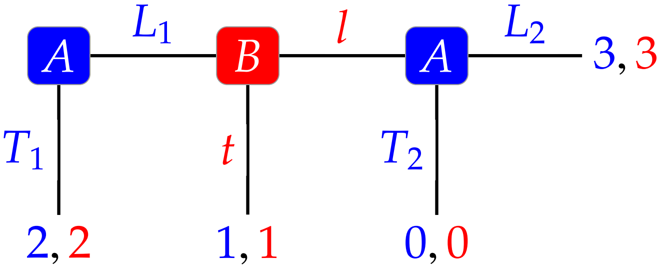
BI Puzzle?

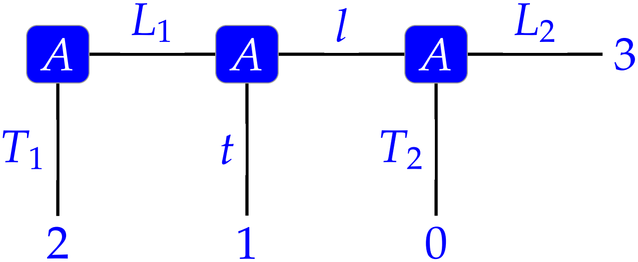

I *know* Ann is rational, but what should I do if she's not...
A
R1
B
r
A
R2
(6,6)
D1
d
D2
(2,1)
(1,6)
(7,5)

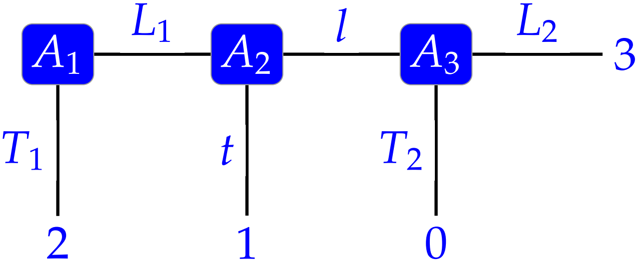
Experimentally, 92% of participants choose to continue at the first node. This is perhaps attributed to a social norm of reciprocity - If player 1 continues at the first node, it is more likely that player 2 will also play continue at the second node. Given this behavior, the optimal choice (the one that yields the highest payoff) is actually for player 1 to play continue: Given the distribution of actual play in the laboratory, the ones who play stop are actually making a mistake!

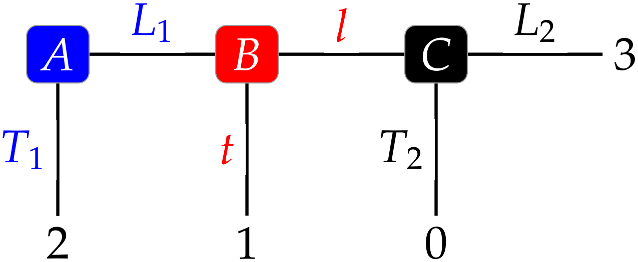
McKelvey and Palfrey. *An experimental study of the centipede game*. Games and Economic Behavior, 1992.

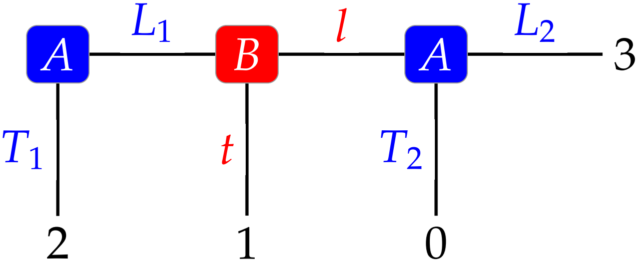

A
L
B
L
A
L
⋯ $m/2, m/2$
T
T
T
1, 0
0, 3
5, 0


A
L1
B
l
A
L2
3, 3
T1
t
T2
2, 2
1, 1
0, 0


A
A
A
L_1
l
L_2
3
T_1
t
T_2
2
1
0


A_1
A_2
A_3
L_1
l
L_2
3
T_1
t
T_2
2
1
0


A
B
C
L_1
l
L_2
3
T_1
t
T_2
2
1
0


A
B
A
L_1
l
L_2
3
T_1
t
T_2
2
1
0

Another Example: Pure Coordination

		Bob	
		L	R
Ann	U	1,1	0,0
	D	0,0	1,1

Another Example: Hi-Low

		Bob	
		L	R
Ann	U	3,3	0,0
	D	0,0	1,1

Focal Points

> *"There are these two broad empirical facts about Hi-Lo games, people almost always choose A [Hi] and people with common knowledge of each other's rationality think it is obviously rational to choose A [Hi]."*
> [Bacharach, *Beyond Individual Choice*, 2006, pg. 42]

See also chapter 2 of:

C.F. Camerer. *Behavioral Game Theory*. Princeton UP, 2003.

N. Bardsley, J. Mehta, C. Starmer and R. Sugden. *The Nature of Salience Revisited: Cognitive Hierarchy Theory* versus *Team Reasoning*. Economic Journal.

Focal Points

‘primary salience’: players’ psychological propensities to play particular strategies by default, when there are no other reasons for choice.

pickers: choose between labels without any incentive to choose one rather than the other

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guessers: guess how pickers have behaved

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coordinators: try to coordinate their choices

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guessers: guess how pickers have behaved

coordinators: try to coordinate their choices

labels vs. *options*

$\{water, beer, sherry, whisky, wine\}$

$$\{water, beer, sherry, whisky, wine\}$$

Task 1: pick an option

{**water**, *beer*, *sherry*, *whisky*, *wine*}

Task 1: pick an option

$\{\textbf{water}, \textit{beer}, \textit{sherry}, \textit{whisky}, \textit{wine}\}$

Task 1: pick an option
Task 2: guess what your opponent picked

$\{\textbf{water}, \textit{beer}, \textit{sherry}, \textit{whisky}, \textit{wine}\}$

Task 1: pick an option
Task 2: guess what your opponent picked
Task 3: try to coordinate with your (unknown) partner

{**water**, *beer*, *sherry*, *whisky*, *wine*}

Task 1: pick an option
Task 2: guess what your opponent picked
Task 3: try to coordinate with your (unknown) partner

	pick	guess	coordinate
water	20	15	38
beer	13	26	11
sherry	4	1	0
whisky	6	6	5
wine	10	4	2

"The basic intellectual premise, or working hypothesis, for rational players in this game seems to be the premise that some rule must be used if success is to exceed coincidence, and that the best rule to be found, whatever its rationalization, is consequently a rational rule." (Thomas Schelling)

Let $G = \langle \{S_i\}_{i \in N}, \{u_i\}_{i \in N} \rangle$ be a finite strategic game.

$\Sigma_i = \{p \mid p : S_i \to [0,1] \text{ and } \sum_{s_i \in S_i} p(s_i) = 1\}$

The **mixed extension** of G is the game $\langle \{\Sigma_i\}_{i \in N}, \{U_i\}_{i \in N} \rangle$ where for $\sigma \in \Sigma = \Sigma_1 \times \cdots \times \Sigma_n$:

$$U_i(\sigma) = \sum_{(s_1,\ldots,s_n) \in S} \sigma_1(s_1)\sigma_2(s_2)\cdots\sigma_n(s_n)u_i(s_1,\ldots,s_n)$$

Theorem. Suppose that σ is a Nash equilibrium in mixed strategies for a game $G = \langle \{S_i\}_{i \in N}, \{u_i\}_{i \in N} \rangle$. Suppose that $s_i, s_i^* \in S_i$ are two pure strategies such that $\sigma_i(s_i) > 0$ and $\sigma_i(s_i^*) > 0$, then

$$U_i(s_i, \sigma_{-i}) = U_i(s_i^*, \sigma_{-i})$$

Theorem (Nash). Every finite game G has a Nash equilibrium in mixed strategies (i.e., there is a Nash equilibrium in the mixed extension G).

Not all equilibrium are created equal...

Perfect equilibrium

		Bob L	Bob R
Ann	U	1,1	0,0
Ann	D	0,0	0,0

Perfect equilibrium

		Bob	
		L	R
Ann	U	1,1	0,0
	D	0,0	0,0

Perfect equilibrium

		Bob	
		L	R
Ann	U	1,1	0,0
	D	0,0	0,0

Isn't (U, L) more "reasonable" than (D, R)?

Perfect equilibrium

Ann \ Bob	L	R
U	1,1	0,0
D	0,0	0,0

Completely mixed strategy: a mixed strategy in which every strategy gets some positive probability

Perfect equilibrium

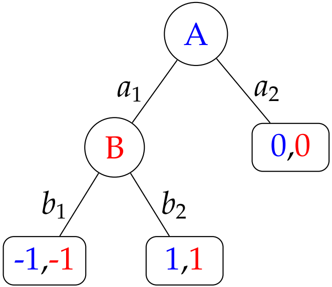
		Bob	
		L	R
Ann	U	1,1	0,0
	D	0,0	0,0

Completely mixed strategy: a mixed strategy in which every strategy gets some positive probability

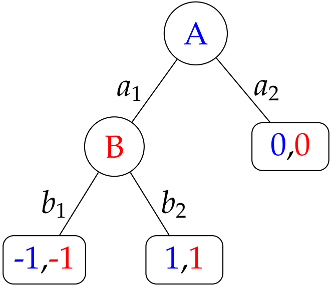
ϵ-perfect equilibrium: a completely mixed strategy profile in which any pure strategy that is not a best reply receives probability less than ϵ

Prefect equilibrium: the mixed strategy profile that is the limit as ϵ goes to 0 of ϵ-prefect equilibria.

Normal form vs. Extensive form

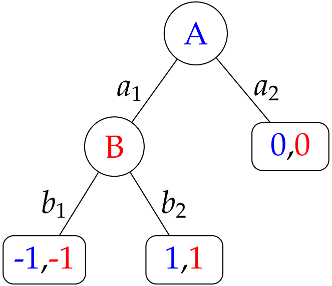


Normal form vs. Extensive form



	b_1 if a_1	b_2 if a_1
a_1	-1,-1	1,1
a_2	0,0	0,0

Normal form vs. Extensive form



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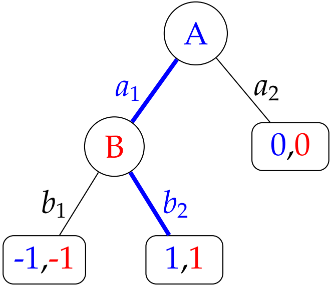

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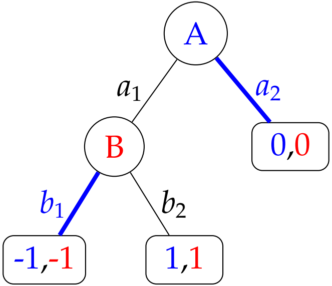

	b_1 if a_1	b_2 if a_1
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(Cf. the various notions of *sequential equilibrium*)

Who is game theory about?

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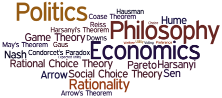

1. **Classical view**: idealized world with *perfectly rational agents*

2. **Humanistic view**: real people in interactive situations

L. Samuelson. *Comments on Game Theory*. Game Theory: 5 Questions, Automatic Press, 2007.

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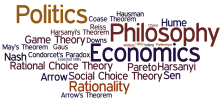


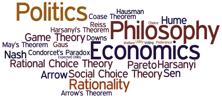
1. **Classical view**: idealized world with *perfectly rational agents*

   - The game itself it taken to be a literal description of the strategic interaction

"We adhere to the classical point of view that the game under consideration fully describes the real situation — that any (pre) commitment possibilities, any repetitive aspect, any probabilities of error, or any possibility of jointly observing some random event, have already been modeled in the game tree." (pg. 1005)

E. Kohlberg and J.-F. Mertens. *On the strategic stability of equilibria*. Econometrica, 54, pgs. 1003 - 1038, 1986.

L. Samuelson. *Comments on Game Theory*. Game Theory: 5 Questions, Automatic Press, 2007.



Who is game theory about?

1. **Classical view**: idealized world with *perfectly rational agents*
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    - Any appropriate concept of equilibrium should be an *implication* of the information provided in the modeled interpreted through an assumption of perfect rationality.
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    - Any appropriate concept of equilibrium should be an *implication* of the information provided in the modeled interpreted through an assumption of perfect rationality.
2. **Humanistic view**: real people in interactive situations
    - the mathematical structures are *models* of interactive situations
    - the appropriate notion of equilibrium is part of the specification of the model

L. Samuelson. *Comments on Game Theory*. Game Theory: 5 Questions, Automatic Press, 2007.