PHIL309P
Philosophy, Politics and Economics

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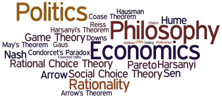
Announcements



- Course website
  https://myelms.umd.edu/courses/1133211
- Online quiz 4
- Reading: Gaus, Ch 4; **Reiss, Ch 4**

- Strategic form and extensive form games.
- Zero-sum vs. non-zero-sum games
- Best response
- Nash equilibrium
- Mixed strategies
- Repeated games (infinite horizon vs. finite horizon)

In zero-sum games

- There exists a mixed strategy Nash equilibrium
- There may be more than one Nash equilibria
- Security strategies are always a Nash equilibrium
- Components of Nash equilibria are interchangeable: If σ and σ' are Nash equilibria in a 2-player game, then (σ_1, σ_2') is also a Nash equilbiria.

Rock-Paper-Scissors

		Bob		
		R	P	S
Ann	R	0,0	-1,1	1,-1
	P	1,-1	0,0	-1,1
	S	-1,1	1,-1	0,0

The unique Nash equilbrium is $([\frac{1}{3}:R,\frac{1}{3}:P,\frac{1}{3}:S],[\frac{1}{3}:R,\frac{1}{3}:P,\frac{1}{3}:S])$

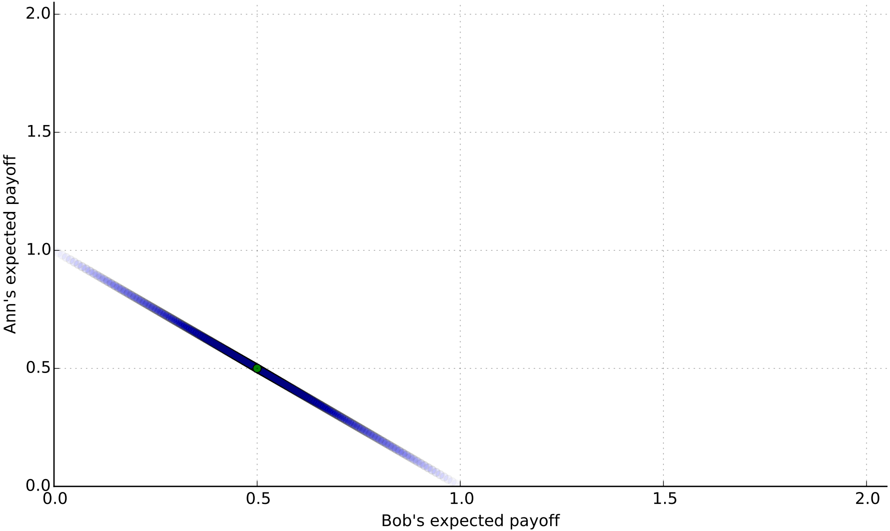
Prisoner's Dilemma

		Bob	
		C	D
Ann	C	$1, 0$	$0, 1$
	D	$1, 0$	$0, 1$

Prisoner’s Dilemma

		Bob	
		C	D
Ann	C	1, 0	0, 1
	D	0, 1	1, 0

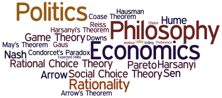
$([0.5 : U, 0.5 : D], [0.5 : L, 0.5 : R])$ is the unique Nash equilibrium



2.0
1.5
1.0
0.5
0.0
Ann's expected payoff
0.0
0.5
1.0
1.5
2.0
Bob's expected payoff

What happens in non-zero-sum games?

Prisoner's Dilemma

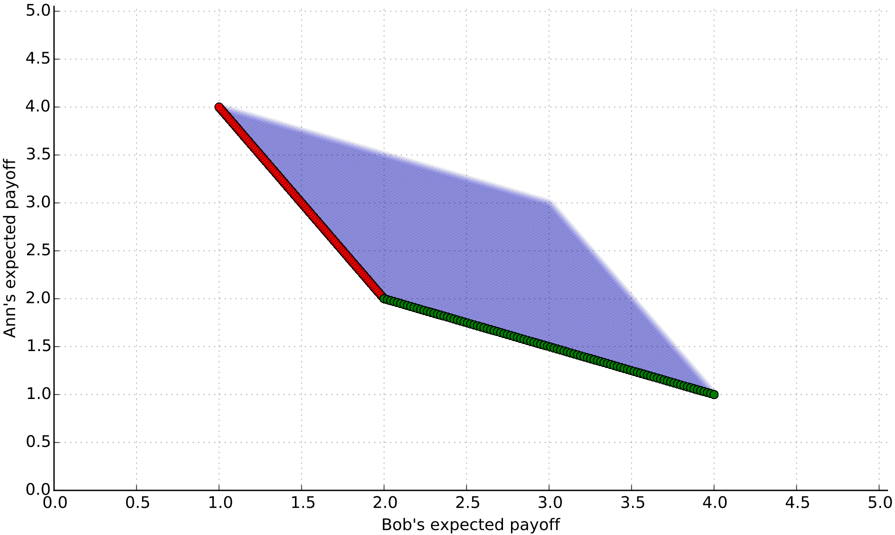


		Bob	
		C	D
Ann	C	$3, 3$	$1, 4$
	D	$4, 1$	$2, 2$

Prisoner's Dilemma

		Bob	
		C	D
Ann	C	$3, 3$	$1, 4$
	D	$4, 1$	$2, 2$

(D, D) is the unique Nash equilibrium


Ann's expected payoff
Bob's expected payoff
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0

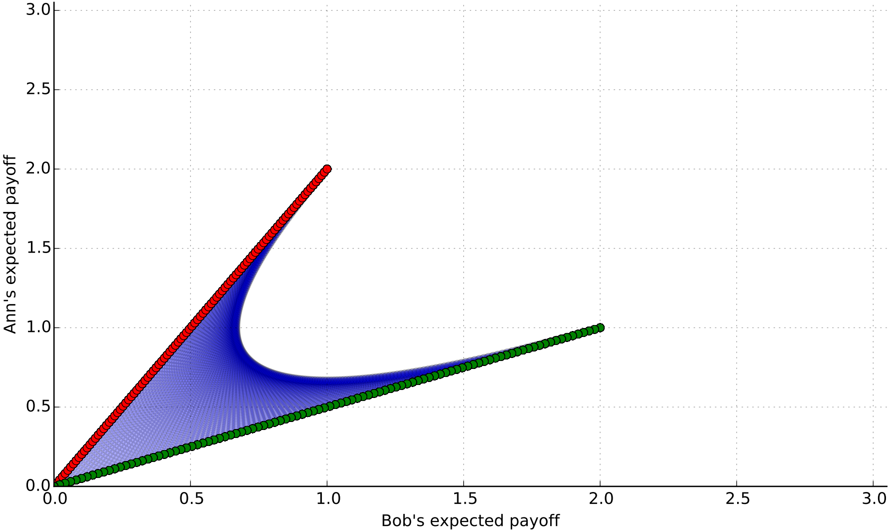
Battle of the Sexes

		Bob	
		L	R
Ann	U	$2, 1$	$0, 0$
	D	$0, 0$	$1, 2$

Battle of the Sexes

		Bob	
		L	R
Ann	U	$2, 1$	$0, 0$
	D	$0, 0$	$1, 2$

(U, L) and (D, R) are pure strategy Nash equilibria


3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Ann's expected payoff
Bob's expected payoff

Mixed Strategies

"We are reluctant to believe that our decisions are made at random. We prefer to be able to point to a reason for each action we take. Outside of Las Vegas we do not spin roulettes."

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- *Harsanyi's purification theorem*: A player's mixed strategy is thought of as a plan of action which is dependent on private information which is not specified in the model. Although the player's behavior appears to be random, it is actually deterministic.

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- *Harsanyi's purification theorem*: A player's mixed strategy is thought of as a plan of action which is dependent on private information which is not specified in the model. Although the player's behavior appears to be random, it is actually deterministic.
- Mixed strategies are beliefs held by all *other* players concerning a player's actions.

Why play Nash equilibrium?

Self-Enforcing Agreements: Nash equilibria are recommended by being the only strategy combinations on which the players could make self-enforcing agreements, i.e., agreements that each has reason to respect, even without external enforcement mechanisms.

M. Risse. *What is rational about Nash equilibria?*. Synthese, 124:3, pgs. 361 - 384, 2000.

		Bob	
		C	D
Ann	C	$3, 3$	$1, 4$
	D	$4, 1$	$2, 2$

Can Ann and Bob **agree** to play U, L?

Stag-Hunt

Ann		Bob	
		S	H
	S	$3, 3$	$0, 2$
	H	$2, 0$	$1, 1$

Stag-Hunt

		Bob	
		S	H
Ann	S	$3, 3$	$0, 2$
	H	$2, 0$	$1, 1$

(S, S) and (H, H) are Nash equilibria

Stag-Hunt

		Bob	
		S	H
Ann	S	$3, 3$	$0, 2$
	H	$2, 0$	$1, 1$

(S, S) is Pareto-superior, but (H, H) is less risky

Bob

Ann	L	C	R
T	4, 6	5, 4	0, 0
M	5, 7	4, 8	0, 0
B	0, 0	0, 0	1, 1

			Bob	
		L	C	R
Ann	T	$4, 6$	$5, 4$	$0, 0$
	M	$5, 7$	$4, 8$	$0, 0$
	B	$0, 0$	$0, 0$	$1, 1$

(B, R) is a Nash equilibrium, but it is **not self-enforcing**

Bob

Ann	L	R
U	0, 0	4, 2
D	2, 4	3, 3

		Bob	
		L	R
Ann	U	$0, 0$	$\mathbf{4}, 2$
	D	$2, \mathbf{4}$	$3, 3$

(D,R) is self-enforcing, but **not a Nash equilibrium**

Self-Enforcing Agreements: Nash equilibria are recommended by being the only strategy combinations on which the players could make self-enforcing agreements, i.e., agreements that each has reason to respect, even without external enforcement mechanisms.

- Not all Nash equilibria are “equally” self-enforcing
- There are Nash equilibria that are not self-enforcing
- There are self-enforcing outcomes that are not Nash equilibria

Playing a Nash equilibrium is *required* by the players rationality and *common knowledge* thereof.

Zero-sum games

"Let us now imagine that there exists a complete theory of the zero-sum two-person game which tells a player what to do, and which is absolutely convincing. If the players knew such a theory then each player would have to assume that his strategy has been "found out" by his opponent. The opponent knows the theory, and he knows that the player would be unwise not to follow it... a satisfactory theory can exist only if we are able to harmonize the two extremes...strategies of player 1 'found out' or of player 2 'found out.' " (pg. 148)

J. von Neumann and O. Morgenstern. *Theory of Games and Economic Behavior*. Princeton University Press, 1944.

"Von Neumann and Morgenstern are assuming that the *payoff matrix* is common knowledge to the players, but presumably the players' subjective probabilities might be private. Then each player might quite reasonably act to maximize subjective expected utility, believing that he will *not* be found out, with the result *not* being a Nash equilibrium."

(Skyrms, pg. 14)

		Bob	
		L	R
Ann	U	1,4	4,1
	D	2,3	3,2

		Bob	
		L	R
Ann	U	1,4	4,1
	D	2,3	3,2

- Suppose that Ann believes Bob will play L with probability $1/4$, *for whatever reason*. Then,

$$1 \times 0.25 + 4 \times 0.75 = 3.25 \geq 2 \times 0.25 + 3 \times 0.75 = 2.75$$

		Bob	
		L	R
Ann	U	1,4	4,1
	D	2,3	3,2

- Suppose that Ann believes Bob will play L with probability $1/4$, *for whatever reason*. Then,

$$1 \times 0.25 + 4 \times 0.75 = 3.25 \geq 2 \times 0.25 + 3 \times 0.75 = 2.75$$

- But, L is maximizes expected utility no matter what belief Bob may have:

$$p + 3 = 4 \times p + 3 \times (1 - p) \geq 1 \times p + 2 \times (1 - p) = 2 - p$$

Bob

Ann	L	C	R
T	3, 2	0, 0	2, 3
M	0, 0	1, 1	0, 0
B	2, 3	0, 0	3, 2

Bob

Ann		L	C	R
	T	$3, 2$	$0, 0$	$2, 3$
	M	$0, 0$	$1, 1$	$0, 0$
	B	$2, 3$	$0, 0$	$3, 2$

(M, C) is the unique Nash equilibrium

Bob

Ann	L	C	R
T	$3, 2$	$0, 0$	$2, 3$
M	$0, 0$	$1, 1$	$0, 0$
B	$2, 3$	$0, 0$	$3, 2$

T, L, B and R are **rationalizable**

Bob

Ann

	L	C	R
T	3, 2	0, 0	2, 3
M	0, 0	1, 1	0, 0
B	2, 3	0, 0	3, 2

T, L, B and R are **rationalizable**

Bob

Ann	L	C	R
T	1, 1	2, 0	-2, 1
M	0, 2	1, 1	2, 1
B	1, -2	1, 2	1, 1

		Bob		
		L	C	R
Ann	T	1, 1	2, 0	-2, 1
	M	0, 2	1, 1	2, 1
	B	1, -2	1, 2	1, 1

(T, L) is the unique pure-strategy Nash equilibrium

Ann \ Bob	L	C	R
T	1, 1	2, 0	-2, 1
M	0, 2	1, 1	2, 1
B	1, -2	1, 2	1, 1

(T, L) is the unique pure-strategy Nash equilibrium

Bob

Ann	L	C	R
T	1, 1	2, 0	-2, 1
M	0, 2	1, 1	2, 1
B	1, -2	1, 2	1, 1

Why not play B and R?

Playing a Nash equilibrium is *required* by the players rationality and *common knowledge* thereof.

- Players need not be *certain* of the other players' beliefs
- Strategies that are not an equilibrium may be *rationalizable*
- Sometimes considerations of riskiness trump the Nash equilibrium

“Rationality has a clear interpretation in individual decision making, but it does not transfer comfortably to interactive decisions, because interactive decision makers cannot maximize expected utility without strong assumptions about how the other participant(s) will behave. In game theory, common knowledge and rationality assumptions have therefore been introduced, but under these assumptions, rationality does not appear to be characteristic of social interaction in general.” (pg. 152, Colman)

A. Colman. *Cooperation, psychological game theory, and limitations of rationality in social interaction*. Behavioral and Brain Sciences, 26, pgs. 139 - 198, 2003.