PHIL309P
Philosophy, Politics and Economics

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Announcements

- Course website
  https://myelms.umd.edu/courses/1133211
- Reading
  - Gaus, Ch. 5
  - EP, Voting Methods (Stanford Encyclopedia of Philosophy)
  - C. List, Social Choice Theory (Stanford Encyclopedia of Philosophy)
  - M. Morreau, Arrow's Theorem (Stanford Encyclopedia of Philosophy)

Voting Situations

# voters	3	5	7	6
best	A	A	B	C
↑	B	C	D	B
↑	C	B	C	D
worst	D	D	A	A

- 21 voters and 4 candidates: Ann (A), Bob (B), Charles (C) and Dora (D)

Voting Situations

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- 21 voters and 4 candidates: Ann (A), Bob (B), Charles (C) and Dora (D)
- Each voter ranks the candidates from best (at the top of the list) to worst (at the bottom of the list) resulting in the 4 voting blocks given in the above table

Voting Situations

# voters	3	5	7	6
best	A	A	B	C
	B	C	D	B
	C	B	C	D
worst	D	D	A	A

Who *should* win the election?

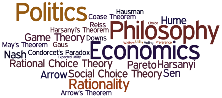
Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
	B	C	D	B
	C	B	C	D
worst	D	D	A	A

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
↑	B	C	D	B
	C	B	C	D
worst	D	D	A	A

- **Candidate** *A*: More people (8) rank *A* first than any other candidate



Which candidate *should* be chosen?

# voters	3	5	**7**	**6**
best	A	A	B	C
	B	C	D	B
	C	B	C	D
worst	D	D	**A**	**A**

- **Candidate** *A*: More people rank *A* first than any other candidate
- **Candidate** *A* **should** ***not*** **win**: more than half rank *A* last

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
	B	C	D	B
	C	B	C	D
worst	D	D	A	A

- **Candidate** *A*: More people rank *A* first than any other candidate
- **Candidate** *D* **should** ***not*** **win**

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
	B	C	D	B
	C	B	C	D
worst	D	D	A	A

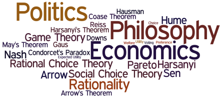
- **Candidate** *A*: More people rank *A* first than any other candidate
- **Candidate** *D* **should *not* win**: *everyone* ranks *B* higher than *D*

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
	B	C	D	B
	C	B	C	D
worst	D	D	A	A

- Which of B or C should win?

Which candidate *should* be chosen?



VS.

Marquis de Condorcet (1743 - 1794)

Jean-Charles de Borda (1733 -1799)

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
	B	C	D	B
	C	B	C	D
worst	D	D	A	A

- **Candidate** C **should win:** *C* beats every other candidate in head-to-head elections (*C* is the *Condorcet winner*)

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
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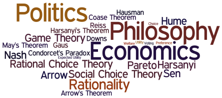

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Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	**B**	C
	B	C	D	**B**
	C	**B**	C	D
worst	D	D	A	A

- **Candidate** C **should win**: C beats every other candidate in head-to-head elections (C is the *Condorcet winner*)
- **Candidate** B **should win**: Taking into account the *entire* ordering, B has the most "support" (B is the *Borda winner*)

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	**B**	C
	B	C	D	**B**
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- *B* gets 13 (vs. *A*)

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
	B	C	D	B
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worst	D	D	A	A

- **Candidate C should win**: C beats every other candidate in head-to-head elections (C is the *Condorcet winner*)
- **Candidate B should win**: Taking into account the *entire* ordering, B has the most "support" (B is the *Borda winner*)

- B gets 13 (vs. A) + 10 (vs. C)

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
	B	C	D	B
	C	B	C	D
worst	D	D	A	A

- **Candidate** *C* **should win**: *C* beats every other candidate in head-to-head elections (*C* is the *Condorcet winner*)
- **Candidate** *B* **should win**: Taking into account the *entire* ordering, *B* has the most "support" (*B* is the *Borda winner*)

- *B* gets 13 (vs. *A*) + 10 (vs. *C*) + 21 (vs. *D*) = 44 points

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
↑	B	C	D	B
	C	B	C	D
worst	D	D	A	A

- **Candidate C should win**: C beats every other candidate in head-to-head elections (C is the *Condorcet winner*)
- **Candidate B should win**: Taking into account the *entire* ordering, B has the most "support" (B is the *Borda winner*)

- C get 13 (vs. A) + 11 (vs. B) + 14 (vs. D) = 38 points

Which candidate *should* be chosen?

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best	A	A	B	C
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Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
	B	C	D	B
	C	B	C	D
worst	D	D	A	A

- Candidate A should *not* win: more than half rank A last
- Candidate D should *not* win: *everyone* ranks B higher than D
- Candidate C: C beats every other candidate in head-to-head elections (C is the *Condorcet winner*)
- Candidate B: Taking into account the *entire* ordering, B has the most "support" (B is the *Borda winner*)

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
	B	C	D	B
	C	B	C	D
worst	D	D	A	A

Conclusion: *there are many ways to answer the above question!*

- ...ns (C is the *Condorcet winner*)
- Candidate *B*: Taking into account the *entire* ordering, *B* has the most "support" (*B* is the *Borda winner*)

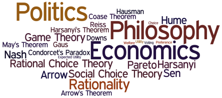
The Condorcet Paradox

Recall Condorcet's Idea

# voters	3	5	7	6
best	A	A	B	**C**
	B	**C**	D	B
	C	B	**C**	D
worst	D	D	A	A

- Candidate C should win since C beats every other candidate in head-to-head elections.

Recall Condorcet's Idea



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# voters	3	5	7	6
best	A	A	**B**	C
	B	C	D	**B**
	C	**B**	C	D
worst	D	D	A	A

- Candidate C should win since C beats every other candidate in head-to-head elections. *B* is ranked second

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# voters	3	5	7	6
best	A	A	**B**	C
↑	**B**	C	D	**B**
	C	**B**	C	D
worst	D	D	A	A

- Candidate C should win since C beats every other candidate in head-to-head elections. *B* is ranked second

Recall Condorcet's Idea

# voters	3	5	7	6
best	A	A	B	C
	B	C	D	B
	C	B	C	D
worst	D	D	A	A

- Candidate C should win since C beats every other candidate in head-to-head elections. B is ranked second, D is ranked third, and A is ranked last.

$$C >_M B >_M D >_M A$$

The Majority Relation

Suppose that X and Y are candidates and P_i represents voter i's *strict preference*.

$\mathbf{N}(X\ P\ Y) = |\{i \mid X\ P_i\ Y\}|$
"the number of voters that rank X strictly above Y"

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X is a **Condorcet winner** if X beats every other candidate in an head-to-head election: there is no candidate Y such that $Y >_M X$

X is a **Condorcet loser** if X loses to every other candidate in an head-to-head elections: there is no candidate Y such that, $X >_M Y$

The Problem

Voter 1	Voter 2	Voter 3
A	C	B
B	A	C
C	B	A

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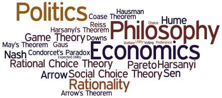
Voter 1	Voter 2	Voter 3
A	C	B
B	A	C
C	B	A

- Does the group prefer *A* over *B*?

The Problem



Voter 1	Voter 2	Voter 3
A	C	B
B	A	C
C	B	A

- Does the group prefer A over B? Yes

The Problem

Voter 1	Voter 2	Voter 3
A	C	B
B	A	C
C	B	A

- Does the group prefer A over B? Yes
- Does the group prefer B over C? Yes

The Problem

Voter 1	Voter 2	Voter 3
A	C	B
B	A	C
C	B	A

- Does the group prefer A over B? Yes
- Does the group prefer B over C? Yes
- Does the group prefer A over C? No

The Problem

Voter 1	Voter 2	Voter 3
A	C	B
B	A	C
C	B	A

The majority relation $>_M$ is **not** transitive!

There is a **Condorcet cycle**: $A >_M B >_M C >_M A$

How bad is this?

- Final decisions are extremely sensitive to institutional features such as who can set the agenda, arbitrary time limits place on deliberation, who is permitted to make motions, etc.

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- How *likely* is a Condorcet cycle?

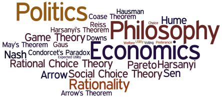
Should we select a Condorcet winner (when one exists)?

Is the Condorcet winner the “best” choice?

# voters	47	47	3	3
	A	B	C	C
	C	C	A	B
	B	A	B	A

C is the Condorcet winner

Is the Condorcet winner the “best” choice?



# voters	47	47	3	3
	A	B	C	C
	C	C	A	B
	B	A	B	A

C is the Condorcet winner; however, it seems that supporters of the main rivals *A* and *B* would rather see *C* win than their candidate’s principal opponent, but this does not mean that there is “positive support” for *C*.

Condorcet's Other Paradox

# voters	30	1	29	10	10	1
	A	A	B	B	C	C
	B	C	A	C	A	B
	C	B	C	A	B	A

Condorcet's Other Paradox

# voters	30	1	29	10	10	1
2	A	A	B	B	C	C
1	B	C	A	C	A	B
0	C	B	C	A	B	A

$$BS(A) = 2 \times 31 + 1 \times 39 + 0 \times 11 = 101$$
$$BS(B) = 2 \times 39 + 1 \times 31 + 0 \times 11 = 109$$
$$BS(C) = 2 \times 11 + 1 \times 11 + 0 \times 59 = 33$$

$$B >_{BC} A >_{BC} C$$

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# voters	30	1	29	10	10	1
	A	A	B	B	C	C
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$B >_{BC} A >_{BC} C$ $\qquad$ $A >_{M} B >_{M} C$

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Condorcet's Other Paradox

# voters	30	1	29	10	10	1
s_2	A	A	B	B	C	C
s_1	B	C	A	C	A	B
s_0	C	B	C	A	B	A

Condorcet's Other Paradox: No *scoring rule* will work...

$B >_{BC} A >_{BC} C$ $\qquad$ $A >_M B >_M C$

Condorcet's Other Paradox

# voters	30	1	29	10	10	1
s_2	A	A	B	B	C	C
s_1	B	C	A	C	A	B
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Condorcet's Other Paradox: No *scoring rule* will work...

$Score(A) = s_2 \times 31 + s_1 \times 39 + s_0 \times 11$

$Score(B) = s_2 \times 39 + s_1 \times 31 + s_0 \times 11$

$B >_{BC} A >_{BC} C$ $\qquad$ $A >_M B >_M C$

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# voters	30	1	29	10	10	1
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Condorcet's Other Paradox: No *scoring rule* will work...

$Score(A) = s_2 \times 31 + s_1 \times 39 + s_0 \times 11$

$Score(B) = s_2 \times 39 + s_1 \times 31 + s_0 \times 11$

$Score(A) > Score(B) \Rightarrow 31s_2 + 39s_1 > 39s_2 + 31s_1 \Rightarrow s_1 > s_2$

$B >_{BC} A >_{BC} C$ $\qquad$ $A >_M B >_M C$

Condorcet's Other Paradox

# voters	30	1	29	10	10	1
s_2	A	A	B	B	C	C
s_1	B	C	A	C	A	B
s_0	C	B	C	A	B	A

Theorem (Fishburn 1974). For all $m \geq 3$, there is some voting situation with a Condorcet winner such that every scoring rule will have at least $m - 2$ candidates with a greater score than the Condorcet winner.

P. Fishburn. *Paradoxes of Voting*. The American Political Science Review, 68:2, pgs. 537 - 546, 1974.

# voters	30	1	29	10	10	1
	A	A	B	B	C	C
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$B >_{BC} A >_{BC} C$ $\qquad$ $A >_{M} B >_{M} C$

Condorcet Triples

G_1	G_2	G_3
A	B	C
B	C	A
C	A	B

G_1	G_2	G_3
A	C	B
C	B	A
B	A	C

If $G_1 = G_2 = G_3$, then this group of voters "cancel out" each other's votes

Saari’s argument

# voters	30	1	29	10	10	1
	A	A	B	B	C	C
	B	C	A	C	A	B
	C	B	C	A	B	A

Saari's argument

# voters	30	1	29	10	10	1
	A	A	B	B	C	C
	B	C	A	C	A	B
	C	B	C	A	B	A

10	10	10
A	B	C
B	C	A
C	A	B

Saari's argument

# voters	20	1	29	0	0	1
	A	A	B	B	C	C
	B	C	A	C	A	B
	C	B	C	A	B	A

10	10	10
A	B	C
B	C	A
C	A	B

1	1	1
A	C	B
C	B	A
B	A	C

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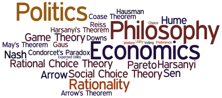

# voters	20	0	28	0	0	0
	A		B			
	B		A			
	C		C			

10	10	10
A	B	C
B	C	A
C	A	B

1	1	1
A	C	B
C	B	A
B	A	C

There are many different voting methods

Many different electoral methods: Plurality, Borda Count, Antiplurality/Veto, and k-approval; Plurality with Runoff; Single Transferable Vote (STV)/Hare; Approval Voting; Cup Rule/Voting Trees; Copeland; Banks; Slater Rule; Schwartz Rule; the Condorcet rule; Maximin/Simpson, Kemeny; Ranked Pairs/Tideman; Bucklin Method; Dodgson Method; Young's Method; Majority Judgment; Cumulative Voting; Range/Score Voting; …

Choosing how to choose

Pragmatic considerations: Is the procedure easy to use? Is it legal? The importance of ease of use should not be underestimated: Despite its many flaws, plurality rule is, by far, the most commonly used method.

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Information required from the voters: What type of information do the ballots convey? I.e., Choosing a single alternative, linearly rank all the candidates, report something about the "intensity" of preference.

Choosing how to choose

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Voting Methods

Positional Scoring Rules: Given the rankings of the candidates provided by the voters, each candidate is assigned a score. The candidate(s) with the highest score is(are) declared the winner(s).

Examples: Borda, Plurality

Generalized Scoring Rules: Voters assign scores, or "grades", to the candidates. The candidate(s) with the "best" aggregate score is(are) declared the winner(s).

Examples: Approval Voting, Majority Judgement, Range Voting

Voting Methods

Staged Procedures: The winner(s) is(are) determined in stages. At each stage, one or more candidates are eliminated. The candidate or candidates that are never eliminated are declared the winner(s).

Examples: Plurality with Runoff, Hare, Coombs

Condorcet Consistent Methods: Voting methods that guarantee that the Condorcet winner is elected.

Examples: Copeland, Dodgson, Young

Voting Methods Tutorial