PHIL309P
Philosophy, Politics and Economics

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Announcements

- Course website
  https://myelms.umd.edu/courses/1133211
- Reading
  - Gaus, Ch. 5
  - EP, Voting Methods (Stanford Encyclopedia of Philosophy)
  - C. List, Social Choice Theory (Stanford Encyclopedia of Philosophy)
  - M. Morreau, Arrow's Theorem (Stanford Encyclopedia of Philosophy)

Quiz 4

There are many different voting methods

Many different electoral methods: Plurality, Borda Count, Antiplurality/Veto, and k-approval; Plurality with Runoff; Single Transferable Vote (STV)/Hare; Approval Voting; Cup Rule/Voting Trees; Copeland; Banks; Slater Rule; Schwartz Rule; the Condorcet rule; Maximin/Simpson, Kemeny; Ranked Pairs/Tideman; Bucklin Method; Dodgson Method; Young's Method; Majority Judgment; Cumulative Voting; Range/Score Voting; ...

Choosing how to choose

Pragmatic considerations: Is the procedure easy to use? Is it legal? The importance of ease of use should not be underestimated: Despite its many flaws, plurality rule is, by far, the most commonly used method.

Behavioral considerations: Do the different procedures *really* lead to different outcomes in practice?

Information required from the voters: What type of information do the ballots convey? I.e., Choosing a single alternative, linearly rank all the candidates, report something about the "intensity" of preference.

Axiomatics: Characterize the different voting methods in terms of normative principles of group decision making.

Principles of group decision making

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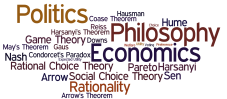

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- **Unanimity (Pareto)**: If *everyone* ranks A above B, then B should not win the election.
- **Anonymity**: The names of the voters do not matter (if two voters swap votes, then the outcome is unaffected).

Monotonicity

A candidate receiving more “support” shouldn’t maker her worse off.

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More-is-Less Paradox: If a candidate C is elected under a given a profile of rankings of the competing candidates, it is possible that, *ceteris paribus*, C may not be elected if some voter(s) raise C in their rankings.

P. Fishburn and S. Brams. *Paradoxes of Preferential Voting*. Mathematics Magazine (1983).

More-is-Less Paradox: Plurality with Runoff

# voters	6	5	4	2
	A	C	B	B
	B	A	C	A
	C	B	A	C

# voters	6	5	4	2
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More-is-Less Paradox: Plurality with Runoff

# voters	6	5	4	2
	A	C	B	B
	B	A	C	A
	C	B	A	C

Winner: *A*

# voters	6	5	4	2
	A	C	B	A
	B	A	C	B
	C	B	A	C

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	C	B	A	C

Winner: A

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# voters	6	5	4	2
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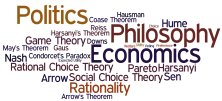
	C	B	A	C

Winner: A

# voters	6	5	4	2
	A	C	B	A
	B	A	C	B
	C	B	A	C

Winner: C

More-is-Less Paradox: Plurality with Runoff



# voters	6	5	4	2
	A	C	B	**B**
	B	A	C	**A**
	C	B	A	**C**

Winner: *A*

# voters	6	5	4	2
	A	C	B	**A**
	B	A	C	**B**
	C	B	A	**C**

Winner: *C*

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- **Twin Paradox**: A voter may obtain a less preferable outcome if his "twin" (a voter with the exact same ranking) decides to participate in the election.

Monotonicity: A candidate receiving more support shouldn't make her worse off

No-Show Paradox: A voter may obtain a more preferable outcome if he decides not to participate in an election than, *ceteris paribus*, if he decides to participate in the election.

- **Twin Paradox**: A voter may obtain a less preferable outcome if his "twin" (a voter with the exact same ranking) decides to participate in the election.
- **Truncation Paradox**: A voter may obtain a more preferable outcome if, *ceteris paribus*, he only reveals part of his ranking of the candidates.

No-Show Paradox: Plurality with Runoff

# voters	4	3	1	3
	A	B	C	C
	B	C	A	B
	C	A	B	A

# voters	2	3	1	3
	A	B	C	C
	B	C	A	B
	C	A	B	A

No-Show Paradox: Plurality with Runoff

# voters	4	3	1	3
	A	B	C	C
	B	C	A	B
	C	A	B	A

# voters	2	3	1	3
	A	B	C	C
	B	C	A	B
	C	A	B	A

No-Show Paradox: Plurality with Runoff

# voters	4	3	1	3
	A	B	C	C
	B	C	A	B
	C	A	B	A

Winner: C

# voters	2	3	1	3
	A	B	C	C
	B	C	A	B
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No-Show Paradox: Plurality with Runoff

# voters	4	3	1	3
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	B	C	A	B
	C	A	B	A

Winner: *C*

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	A	B	C	C
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	C	A	B	A

No-Show Paradox: Plurality with Runoff

# voters	4	3	1	3
	A	B	C	C
	B	C	A	B
	C	A	B	A

Winner: *C*

# voters	2	3	1	3
	A	B	C	C
	B	C	A	B
	C	A	B	A

No-Show Paradox: Plurality with Runoff

# voters	4	3	1	3
	A	B	C	C
	B	C	A	B
	C	A	B	A

Winner: *C*

# voters	2	3	1	3
	A	B	C	C
	B	C	A	B
	C	A	B	A

Winner: *B*

Twin Paradox: Plurality with Runoff

# voters	4	3	1	3
	A	B	C	C
	B	C	A	B
	C	A	B	A

Winner: *C*

# voters	2	3	1	3
	A	B	C	C
	B	C	A	B
	C	A	B	A

Winner: *B*

Failures of Monotonicity

Example: Burlington, VT 2009 Mayoral Race (rangevoting.org/Burlington.html)

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Theorem (Moulin). If there are four or more candidates, then every Condorcet consistent voting methods is susceptible to the No-Show paradox.

H. Moulin. *Condorcet's Principle Implies the No Show Paradox*. Journal of Economic Theory, 45, pgs. 53 - 64, 1988.

Spoiler Candidates: Plurality Rule

# voters	49	48	3
	A	B	C
	B	A	B
	C	C	A

Winner: A

Spoiler Candidates: Plurality Rule

# voters	49	48	3
	A	B	C
	B	A	B
	C	C	A

Winner: B

IIA

Independence of Irrelevant Alternatives: If the voters in two different electorates rank A and B in exactly the same way, then A and B should be ranked the same way in both elections.

Failure of IIA: Borda Count

# voters	3	2	2
3	A	B	C
2	B	C	A
1	C	A	B
0	X	X	X

# voters	3	2	2
	A	B	C
	B	C	X
	C	X	A
	X	A	B

Failure of IIA: Borda Count

# voters	3	2	2
3	A	B	C
2	B	C	A
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$A\ (15) >_{BC} B\ (14) >_{BC} C\ (13) >_{BC} X\ (0)$

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$A\ (15) >_{BC} B\ (14) >_{BC} C\ (13) >_{BC} X\ (0)$

# voters	3	2	2
3	A	B	C
2	B	C	X
1	C	X	A
0	X	A	B

$C\ (13) >_{BC} B\ (12) >_{BC} A\ (11) >_{BC} X\ (6)$

Voting Methods

Positional Scoring Rules: Given the rankings of the candidates provided by the voters, each candidate is assigned a score. The candidate(s) with the highest score is(are) declared the winner(s).

Examples: Borda, Plurality

Generalized Scoring Rules: Voters assign scores, or "grades", to the candidates. The candidate(s) with the "best" aggregate score is(are) declared the winner(s).

Examples: Approval Voting, Majority Judgement, Range Voting

Voting Methods

Staged Procedures: The winner(s) is(are) determined in stages. At each stage, one or more candidates are eliminated. The candidate or candidates that are never eliminated are declared the winner(s).

Examples: Plurality with Runoff, Hare, Coombs

Condorcet Consistent Methods: Voting methods that guarantee that the Condorcet winner is elected.

Examples: Copeland, Dodgson, Young

Principles

Condorcet: Elect the Condorcet winner whenever it exists.

Monotonicity: More support should never hurt a candidate.

Participation: It should never be in a voter's best interests not to vote.

Multiple-Districts: If a candidate wins in each district, then that candidate should also win when the districts are merged.

Independence: The group's ranking of A and B should only depend on the voter's rankings of A and B.

More Principles

Pareto: Never elect a candidate if another candidate is strictly preferred by all voters.

Anonymity: The outcome does not depend on the names of the voters.

Neutrality: The outcome does not depend on the names of the candidates.

Universal Domain: The voters are free to rank the candidates (or grade the candidates) in any way they want.

What are the relationships between these principles? Is there a procedure that satisfies *all* of them?

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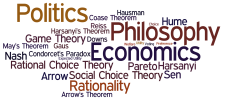
A few observations:

- Condorcet winners may not exist.
- No positional scoring method satisfies the Condorcet Principle.
- The Condorcet and Participation principles cannot be jointly satisfied.

Different Perspectives



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Finding a Compromise: Which voting method produces a ranking that comes "closest" to the "consensus" ranking?

Finding the Optimal Choice: Which voting method is most likely to yield the "correct" choice?

Proceduralist View

"[W]e could identify a set of ideals with which any collective decision-making procedure ought to comply. We might think of these as procedural ideals, and a process of collective decision making would be more or less justifiable depending on the extent to which it satisfies them...

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J. Coleman and J. Ferejohn. *Democracy and social choice*. Ethics, 97(1): 6-25, 1986..

Epistemic View

"Condorcet begins with the premise that the object of government is to make decisions that are in the best interest of society. This leads naturally to the question: what voting rules are most likely to yield good outcomes?

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(pg. 60)

H. P. Young. *Optimal Voting Rules*. The Journal of Economic Perspectives, 9:1, pgs. 51 - 64, 1995.

Which voting rule is best?

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- We need a notion of how far apart one ranking is from another, i.e., we need a notion of distance between rankings.
- Given an appropriate notion of distance, what is the definition of a compromise ranking? *mean or median*

J. Kemeny. *Mathematics Without Numbers*. Daedalus, 88, pgs. 571 - 591, 1959.

Kemeny Distance

Key idea: The ranking $A \succ B \succ C$ is *closer* to $A \succ C \succ B$ than to $B \succ C \succ A$.

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Why?

	$\{A, B\}$	$\{B, C\}$	$\{A, C\}$
$A \succ B \succ C$	$A \succ B$	$B \succ C$	$A \succ C$
$A \succ C \succ B$	$A \succ B$	$C \succ B$	$A \succ C$
$B \succ C \succ A$	$B \succ A$	$B \succ C$	$C \succ A$

Kemeny Distance

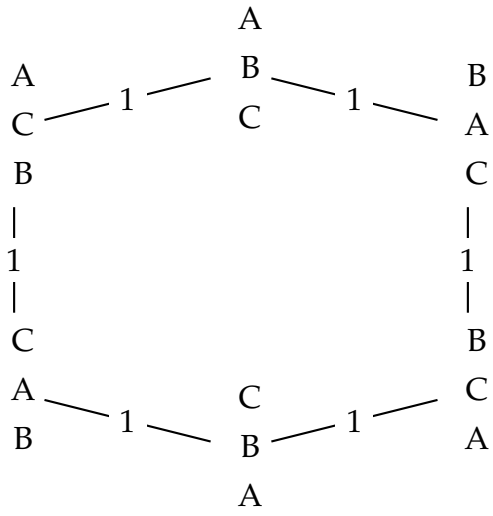
Key idea: The ranking $A \succ B \succ C$ is *closer* to $A \succ C \succ B$ than to $B \succ C \succ A$.

Why?

	$\{A, B\}$	$\{B, C\}$	$\{A, C\}$
$A \succ B \succ C$	$A \succ B$	$B \succ C$	$A \succ C$
$A \succ C \succ B$	$A \succ B$	$C \succ B$	$A \succ C$
$B \succ C \succ A$	$B \succ A$	$B \succ C$	$C \succ A$

$K(A \succ B \succ C, A \succ C \succ B) = 1$ because the rankings disagree on one pair of candidates, while $K(A \succ B \succ C, B \succ C \succ A) = 2$ because the rankings disagree on two paris of candidates.

Kemeny Distance



Mean or Median?

Mean or Median?

Let $a_1, \ldots, a_n$ be a set of numbers.

The **mean** is the number x that minimizes the sum of the square of the distance between each data point and x.

The **median** is the number x that minimizes the sum of the distances between each data point and x

Mean or Median?

# voters	21	5	4	11
	A	B	C	C
	B	C	A	B
	C	A	B	A

$K \mid K^2$	$A\ B\ C$	$A\ C\ B$	$B\ A\ C$	$B\ C\ A$	$C\ A\ B$	$C\ B\ A$
$A\ B\ C$						
$B\ C\ A$						
$C\ A\ B$						
$C\ B\ A$						

$K \mid K^2$	$A\ B\ C$	$A\ C\ B$	$B\ A\ C$	$B\ C\ A$	$C\ A\ B$	$C\ B\ A$
$A\ B\ C$	$0 \mid 0$	$1 \mid 1$	$1 \mid 1$	$2 \mid 4$	$2 \mid 4$	$3 \mid 9$
$B\ C\ A$	$2 \mid 4$	$3 \mid 9$	$1 \mid 1$	$0 \mid 0$	$2 \mid 4$	$1 \mid 1$
$C\ A\ B$	$2 \mid 4$	$1 \mid 1$	$3 \mid 9$	$2 \mid 4$	$0 \mid 0$	$1 \mid 1$
$C\ B\ A$	$3 \mid 9$	$2 \mid 4$	$2 \mid 4$	$1 \mid 1$	$1 \mid 1$	$0 \mid 0$

$n*K \mid n*K^2$	$A\ B\ C$	$A\ C\ B$	$B\ A\ C$	$B\ C\ A$	$C\ A\ B$	$C\ B\ A$
(21) $A\ B\ C$	0 | 0	21 | 21	21 | 21	42 | 84	42 | 84	63 | 189
$B\ C\ A$	2 | 4	3 | 9	1 | 1	0 | 0	2 | 4	1 | 1
$C\ A\ B$	2 | 4	1 | 1	3 | 9	2 | 4	0 | 0	1 | 1
$C\ B\ A$	3 | 9	2 | 4	2 | 4	1 | 1	1 | 1	0 | 0

$n*K \mid n*K^2$	$A\ B\ C$	$A\ C\ B$	$B\ A\ C$	$B\ C\ A$	$C\ A\ B$	$C\ B\ A$
(21) $A\ B\ C$	0 | 0	21 | 21	21 | 21	42 | 84	42 | 84	63 | 189
(5) $B\ C\ A$	10 | 20	15 | 45	5 | 5	0 | 0	10 | 20	5 | 5
(4) $C\ A\ B$	8 | 16	4 | 4	12 | 36	8 | 16	0 | 0	4 | 4
(11) $C\ B\ A$	33 | 99	22 | 44	22 | 44	11 | 11	11 | 11	0 | 0
Sum	51 | 135	62 | 114	60 | 106	61 | 111	63 | 115	71 | 198

$n*K \mid n*K^2$	$A\ B\ C$	$A\ C\ B$	$B\ A\ C$	$B\ C\ A$	$C\ A\ B$	$C\ B\ A$
(21) $A\ B\ C$	0 | 0	21 | 21	21 | 21	42 | 84	42 | 84	63 | 189
(5) $B\ C\ A$	10 | 20	15 | 45	5 | 5	0 | 0	10 | 20	5 | 5
(4) $C\ A\ B$	8 | 16	4 | 4	12 | 36	8 | 16	0 | 0	4 | 4
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Sum	51 | 135	62 | 114	60 | 106	61 | 111	63 | 115	71 | 198

# voters	21	5	4	11
	A	B	C	C
	B	C	A	B
	C	A	B	A

- Median Ranking: $A \succ B \succ C$ (Minimizes the sum of the Kemeny distances)
- Mean Ranking: $B \succ A \succ C$ (Minimizes the sum of the square of the Kemeny distances)

Axiomatics

"When a set of axioms regarding social choice can all be simultaneously satisfied, there may be several possible procedures that work, among which we have to choose.

A. Sen. *The Possibility of Social Choice*. The American Economic Review, 89:3, pgs. 349 - 378, 1999 (reprint of his Nobel lecture).

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A. Sen. *The Possibility of Social Choice*. The American Economic Review, 89:3, pgs. 349 - 378, 1999 (reprint of his Nobel lecture).

The Social Choice Model

Notation

- N is a finite set of voters (assume that $N = \{1, 2, 3, \ldots, n\}$)
- X is a (typically finite) set of alternatives, or candidates
- A relation on X is a linear order if it is transitive, irreflexive, and complete (hence, acyclic)
- $L(X)$ is the set of all linear orders over the set X
- $O(X)$ is the set of all reflexive and transitive relations over the set X

Notation

- A **profile** for the set of voters N is a sequence of (linear) orders over X, denoted $\mathbf{R} = (R_1, \ldots, R_n)$.
- $L(X)^n$ is the set of all **profiles** for n voters (similarly for $O(X)^n$)
- For a profile $\mathbf{R} = (R_1, \ldots, R_n) \in O(X)^n$, let $\mathbf{N_R}(A\ P\ B) = \{i \mid A\ P_i\ B\}$ be the set of voters that rank A above B (similarly for $\mathbf{N_R}(A\ I\ B)$ and $\mathbf{N_R}(B\ P\ A)$)

Preference Aggregation Methods

Social Welfare Function: $F : \mathcal{D} \to L(X)$, where $\mathcal{D} \subseteq L(X)^n$

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Comments

- $\mathcal{D}$ is the *domain* of the function: it is the set of all possible profiles
- Aggregation methods are *decisive*: every profile $\mathbf{R}$ in the domain is associated with exactly one ordering over the candidates
- The range of the function is $L(X)$: the social ordering is assumed to be a linear order
- Tie-breaking rules are built into the definition of a preference aggregation function

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Variants

- Social Choice Function: $F : \mathcal{D} \to \wp(X) - \emptyset$, where $\mathcal{D} \subseteq L(X)^n$ and $\wp(X)$ is the set of all subsets of X.
- Allow Ties: $F : \mathcal{D} \to O(X)$ where $O(X)$ is the set of orderings (reflexive and transitive) over X
- Allow Indifference and Ties: $F : \mathcal{D} \to O(X)$ where $O(X)$ is the set of orderings (reflexive and transitive) over X and $\mathcal{D} \subseteq O(X)^n$

Examples

$Maj(\mathbf{R}) = >_M$ where $A >_M B$ iff $|\mathbf{N_R}(A\ P\ B)| > |\mathbf{N_R}(B\ P\ A)|$

(*the problem is that* $>_M$ *may not be transitive (or complete)*)

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$Borda(\mathbf{R}) = \geq_{BC}$ where $A \geq_{BC} B$ iff the Borda score of A is greater than the Borda score for B.

(*the problem is that* $\geq_{BC}$ *may not be a linear order*)

Characterizing Majority Rule

When there are only **two** candidates A and B, then all voting methods give the same results

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When there are only two options, can we argue that majority rule is the "best" procedure?

K. May. *A Set of Independent Necessary and Sufficient Conditions for Simple Majority Decision*. Econometrica, Vol. 20 (1952).