PHIL309P
Philosophy, Politics and Economics

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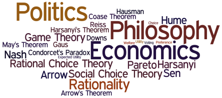
Announcements



- Course website
  https://myelms.umd.edu/courses/1133211
- Reading
  - Gaus, Ch. 5
  - EP, Voting Methods (Stanford Encyclopedia of Philosophy)
  - C. List, Social Choice Theory (Stanford Encyclopedia of Philosophy)
  - M. Morreau, Arrow's Theorem (Stanford Encyclopedia of Philosophy)
- Online videos
- Quiz 5 (Thursday, 10am)
- Problem set 2 (3/29 by midnight)

Axiomatics

"When a set of axioms regarding social choice can all be simultaneously satisfied, there may be several possible procedures that work, among which we have to choose.

A. Sen. *The Possibility of Social Choice*. The American Economic Review, 89:3, pgs. 349 - 378, 1999 (reprint of his Nobel lecture).

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"When a set of axioms regarding social choice can all be simultaneously satisfied, there may be several possible procedures that work, among which we have to choose. In order to choose between different possibilities through the use of discriminating axioms, we have to introduce *further* axioms, until only and only one possible procedure remains. This is something of an exercise in brinkmanship. We have to go on and on cutting alternative possibilities, moving—implicitly—*towards* an impossibility, but then stop just before all possibilities are eliminated, to wit, when one and only one options remains." (pg. 354)

A. Sen. *The Possibility of Social Choice*. The American Economic Review, 89:3, pgs. 349 - 378, 1999 (reprint of his Nobel lecture).

The Social Choice Model

Notation

- N is a finite set of voters (assume that $N = \{1, 2, 3, \ldots, n\}$)
- X is a (typically finite) set of alternatives, or candidates
- A relation on X is a linear order if it is transitive, irreflexive, and complete (hence, acyclic)
- $L(X)$ is the set of all linear orders over the set X
- $O(X)$ is the set of all reflexive and transitive relations over the set X

Notation

- A **profile** for the set of voters N is a sequence of (linear) orders over X, denoted $\mathbf{R} = (R_1, \ldots, R_n)$.

- $L(X)^n$ is the set of all **profiles** for n voters (similarly for $O(X)^n$)

- For a profile $\mathbf{R} = (R_1, \ldots, R_n) \in O(X)^n$, let $\mathbf{N_R}(A\ P\ B) = \{i \mid A\ P_i\ B\}$ be the set of voters that rank A above B (similarly for $\mathbf{N_R}(A\ I\ B)$ and $\mathbf{N_R}(B\ P\ A)$)

Preference Aggregation Methods

Social Welfare Function: $F : \mathcal{D} \to L(X)$, where $\mathcal{D} \subseteq L(X)^n$

Preference Aggregation Methods

Social Welfare Function: $F : \mathcal{D} \rightarrow L(X)$, where $\mathcal{D} \subseteq L(X)^n$

Comments

- $\mathcal{D}$ is the *domain* of the function: it is the set of all possible profiles
- Aggregation methods are *decisive*: every profile $\mathbf{R}$ in the domain is associated with exactly one ordering over the candidates
- The range of the function is $L(X)$: the social ordering is assumed to be a linear order
- Tie-breaking rules are built into the definition of a preference aggregation function

Preference Aggregation Methods

Social Welfare Function: $F : \mathcal{D} \rightarrow L(X)$, where $\mathcal{D} \subseteq L(X)^n$

Variants

- Social Choice Function: $F : \mathcal{D} \rightarrow \wp(X) - \emptyset$, where $\mathcal{D} \subseteq L(X)^n$ and $\wp(X)$ is the set of all subsets of X.

- Allow Ties: $F : \mathcal{D} \rightarrow O(X)$ where $O(X)$ is the set of orderings (reflexive and transitive) over X

- Allow Indifference and Ties: $F : \mathcal{D} \rightarrow O(X)$ where $O(X)$ is the set of orderings (reflexive and transitive) over X and $\mathcal{D} \subseteq O(X)^n$

Examples

$Maj(\mathbf{R}) = \ >_M$ where $A >_M B$ iff $|\mathbf{N}_{\mathbf{R}}(A\ P\ B)| > |\mathbf{N}_{\mathbf{R}}(B\ P\ A)|$

(the problem is that $>_M$ may not be transitive (or complete))

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$Borda(\mathbf{R}) \ = \ \geq_{BC}$ where $A \geq_{BC} B$ iff the Borda score of A is greater than the Borda score for B.

(*the problem is that* $\geq_{BC}$ *may not be a linear order*)

Characterizing Majority Rule

When there are only **two** candidates A and B, then all voting methods give the same results

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Majority Rule: A is ranked above (below) B if more (fewer) voters rank A above B than B above A, otherwise A and B are tied.

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Majority Rule: A is ranked above (below) B if more (fewer) voters rank A above B than B above A, otherwise A and B are tied.

When there are only two options, can we argue that majority rule is the "best" procedure?

K. May. *A Set of Independent Necessary and Sufficient Conditions for Simple Majority Decision*. Econometrica, Vol. 20 (1952).

May's Theorem: Details

Let $N = \{1, 2, 3, \ldots, n\}$ be the set of n voters and $X = \{A, B\}$ the set of candidates.

Social Welfare Function: $F : O(X)^n \rightarrow O(X)$, where $O(X)$ is the set of orderings over X
(*there are only three possibilities: $A\ P\ B$, $A\ I\ B$, or $B\ P\ A$*)

$$F_{Maj}(\mathbf{R}) = \begin{cases} A\ P\ B & \text{if } |\mathbf{N_R}(A\ P\ B)| > |\mathbf{N_R}(B\ P\ A)| \\ A\ I\ B & \text{if } |\mathbf{N_R}(A\ P\ B)| = |\mathbf{N_R}(B\ P\ A)| \\ B\ P\ A & \text{if } |\mathbf{N_R}(B\ P\ A)| > |\mathbf{N_R}(A\ P\ B)| \end{cases}$$

May's Theorem: Details

Let $N = \{1, 2, 3, \ldots, n\}$ be the set of n voters and $X = \{A, B\}$ the set of candidates.

Social Welfare Function: $F : \{1, 0, -1\}^n \to \{1, 0, -1\}$,

where 1 means $A\ P\ B$, 0 means $A\ I\ B$, and -1 means $B\ P\ A$

$$F_{Maj}(\mathbf{v}) = \begin{cases} 1 & \text{if } |\mathbf{N}_{\mathbf{v}}(1)| > |\mathbf{N}_{\mathbf{v}}(-1)| \\ 0 & \text{if } |\mathbf{N}_{\mathbf{v}}(1)| = |\mathbf{N}_{\mathbf{v}}(-1)| \\ -1 & \text{if } |\mathbf{N}_{\mathbf{v}}(-1)| > |\mathbf{N}_{\mathbf{v}}(1)| \end{cases}$$

Warm-up Exercise

Suppose that there are two voters and two candidates. How many social choice functions are there?

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Suppose that there are two voters and two candidates. How many social choice functions are there? $19,683$

- There are three possible rankings for 2 candidates.
- When there are two voters there are $3^2 = 9$ possible profiles:

$$\{(1,1),(1,0),(1,-1),(0,1),(0,0),(0,-1),(-1,1),(-1,0),(-1,-1)\}$$

- Since there are 9 profiles and 3 rankings, there are $3^9 = 19,683$ possible preference aggregation functions.

May’s Theorem: Details

- **Unanimity**: unanimously supported alternatives must be the social outcome.

- **Anonymity**: all voters should be treated equally.

- **Neutrality**: all candidates should be treated equally.

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  If $\mathbf{v} = (v_1, \ldots, v_n)$ with for all $i \in N$, $v_i = x$ then $F(\mathbf{v}) = x$ (for $x \in \{1, 0, -1\}$).

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  $F(v_1, \ldots, v_n) = F(v_{\pi(1)}, v_{\pi(2)}, \ldots, v_{\pi(n)})$ where $v_i \in \{1, 0, -1\}$ and π is a permutation of the voters.

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- **Neutrality**: all candidates should be treated equally.

  $F(-\mathbf{v}) = -F(\mathbf{v})$ where $-\mathbf{v} = (-v_1, \ldots, -v_n)$.

May's Theorem: Details

- **Positive Responsiveness** (Monotonicity): unidirectional shift in the voters' opinions should help the alternative toward which this shift occurs

If $F(\mathbf{v}) = 0$ or $F(\mathbf{v}) = 1$ and $\mathbf{v} \prec \mathbf{v}'$, then $F(\mathbf{v}') = 1$
where $\mathbf{v} \prec \mathbf{v}'$ means for all $i \in N$ $v_i \leq v'_i$ and there is some $i \in N$ with $v_i < v'_i$.

Warm-up Exercise

Suppose that there are two voters and two candidates. How many social choice functions are there that satisfy anonymity?

Anonymity: all voters should be treated equally.

$F(v_1, v_2, \ldots, v_n) = F(v_{\pi(1)}, v_{\pi(2)}, \ldots, v_{\pi(n)})$ where π is a permutation of the voters.

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Suppose that there are two voters and two candidates. How many social choice functions are there that satisfy anonymity? 729

Anonymity: all voters should be treated equally.

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- Imposing anonymity reduces the number of preference aggregation functions.
- If F satisfies anonymity, then $F(1, 0) = F(0, 1)$, $F(1, -1) = F(-1, 1)$ and $F(-1, 0) = F(0, -1)$.
- This means that there are essentially 6 elements of the domain. So, there are $3^6 = 729$ preference aggregation functions.

May’s Theorem: Details

May’s Theorem (1952) A social decision method F satisfies unanimity, neutrality, anonymity and positive responsiveness iff F is majority rule.

Proof Idea

If $(1, 0, -1)$ is assigned 1 or -1 then

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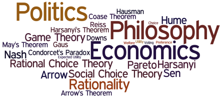

If $(1, 0, -1)$ is assigned 1 or -1 then

✓ Anonymity implies $(-1, 0, 1)$ is assigned 1 or -1

Proof Idea

If $(1, 0, -1)$ is assigned 1 or -1 then

- ✓ Anonymity implies $(-1, 0, 1)$ is assigned 1 or -1
- ✓ Neutrality implies $(1, 0, -1)$ is assigned -1 or 1
  Contradiction.

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Contradiction.

Other characterizations

G. Asan and R. Sanver. *Another Characterization of the Majority Rule*. Economics Letters, 75 (3), 409-413, 2002.

E. Maskin. *Majority rule, social welfare functions and game forms*. in *Choice, Welfare and Development*, The Clarendon Press, pgs. 100 - 109, 1995.

G. Woeginger. *A new characterization of the majority rule*. Economic Letters, 81, pgs. 89 - 94, 2003.

Can May's Theorem be generalized to more than 2 candidates?

Can May's Theorem be generalized to more than 2 candidates? No!

Arrow's Theorem

K. Arrow. *Social Choice and Individual Values*. John Wiley & Sons, 1951.

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Let X be a finite set with *at least three elements* and N a finite set of n voters.

Social Welfare Function: $F : \mathcal{D} \rightarrow O(X)$ where $\mathcal{D} \subseteq O(X)^n$

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Reminders:

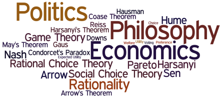
- $O(X)$ is the set of transitive and complete relations on X
- For $R \in O(X)$, let P_R denote the strict subrelation and I_R the indifference subrelation:
  - $A \; P_R \; B$ iff $A \; R \; B$ and not $B \; R \; A$
  - $A \; I_R \; B$ iff $A \; R \; B$ and $B \; R \; A$

Unanimity

$F : \mathcal{D} \to O(X)$

If each agent ranks A above B, then so does the social ranking.

Unanimity



$F : \mathcal{D} \to O(X)$

If each agent ranks A above B, then so does the social ranking.

For all profiles $\mathbf{R} = (R_1, \ldots, R_n) \in \mathcal{D}$:

$$\text{If for each } i \in N,\ A\ P_i\ B \text{ then } A\ P_{F(\mathbf{R})}\ B$$

Universal Domain

$F : \mathcal{D} \to O(X)$

Voter's are free to choose any preference they want.

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Voter's are free to choose any preference they want.

The domain of F is the set of *all* profiles, i.e., $\mathcal{D} = O(X)^n$.

Independence of Irrelevant Alternatives

$F : \mathcal{D} \to O(X)$

The social ranking (higher, lower, or indifferent) of two alternatives A and B depends only the relative rankings of A and B for each voter.

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For all profiles $\mathbf{R} = (R_1, \ldots, R_n)$ and $\mathbf{R}' = (R'_1, \ldots, R'_n)$:

$$\text{If } R_{i\{A,B\}} = R'_{i\{A,B\}} \text{ for all } i \in N \text{, then } F(\mathbf{R})_{\{A,B\}} \text{ iff } F(\mathbf{R}')_{\{A,B\}}.$$

where $R_{\{X,Y\}} = R \cap \{X, Y\} \times \{X, Y\}$

IIA For all profiles $\mathbf{R} = (R_1, \ldots, R_n)$ and $\mathbf{R}' = (R'_1, \ldots, R'_n)$:

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IIA* For all profiles $\mathbf{R} = (R_1, \ldots, R_n)$ and $\mathbf{R}' = (R'_1, \ldots, R'_n)$:

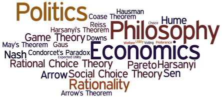
$$\text{If } A\ R_i\ B \text{ iff } A\ R'_i\ B \text{ for all } i \in N, \text{ then } A\ F(\mathbf{R})\ B \text{ iff } A\ F(\mathbf{R}')\ B.$$

Dictatorship

$F : \mathcal{D} \to O(X)$

A voter $d \in N$ is a **dictator** if society strictly prefers A over B *whenever* d strictly prefers A over B.

Dictatorship



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There is a $d \in N$ such that for each profile $\mathbf{R} = (R_1, \ldots, R_d, \ldots, R_n)$, if $A\ P_d\ B$, then $A\ P_{F(\mathbf{R})}\ B$

M. Morreau. *Arrow's Theorem*. Stanford Encyclopedia of Philosophy, 2014.

Arrow’s Theorem

Theorem (Arrow, 1951). Suppose that there are at least three candidates and finitely many voters. Any social welfare function that satisfies universal domain, independence of irrelevant alternatives and unanimity is a dictatorship.

Arrow's Theorem

D. Campbell and J. Kelly. *Impossibility Theorems in the Arrovian Framework*. Handbook of Social Choice and Welfare Volume 1, pgs. 35 - 94, 2002.

W. Gaertner. *A Primer in Social Choice Theory*. Oxford University Press, 2006.

J. Geanakoplos. *Three Brief Proofs of Arrow's Impossibility Theorem*. Economic Theory, **26**, 2005.

P. Suppes. *The pre-history of Kenneth Arrow's social choice and individual values*. Social Choice and Welfare, 25, pgs. 319 - 326, 2005.

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Weakening IIA

Given a profile and a set of candidates $S \subseteq X$, let $\mathbf{R}|_S$ denote the restriction of the profile to candidates in S.

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Binary Independence: For all profiles $\mathbf{R}, \mathbf{R}'$ and candidates $A, B \in X$:

$$\text{If } \mathbf{R}|_{\{A,B\}} = \mathbf{R}'|_{\{A,B\}}, \text{ then } F(\mathbf{R})|_{\{A,B\}} = F(\mathbf{R}')|_{\{A,B\}}$$

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m**-Ary Independence**: For all profiles $\mathbf{R}, \mathbf{R}'$ and for all $S \subseteq X$ with $|S| = m$:

$$\text{If } \mathbf{R}|_S = \mathbf{R}'|_S, \text{ then } F(\mathbf{R})|_S = F(\mathbf{R}')|_S$$

Weakening IIA

Theorem. (Blau) Suppose that $m = 2, \ldots, |X| - 1$. If a social welfare function F satisfies m-ary independence, then it also satisfies binary independence.

J. Blau. *Arrow's theorem with weak independence*. Economica, 38, pgs. 413 - 420, 1971.

S. Cato. *Independence of Irrelevant Alternatives Revisited*. Theory and Decision, 2013.

Arrow’s Theorem

Theorem (Arrow, 1951). Suppose that there are at least three candidates and finitely many voters. Any social welfare function that satisfies universal domain, independence of irrelevant alternatives and unanimity is a dictatorship.

Weakening Unanimity

$F : \mathcal{D} \to O(X)$

Dictatorial: there is a $d \in N$ such that for all $A, B \in X$ and all profiles $\mathbf{R}$: if $A\ P_d\ B$, then $A\ P_{F(\mathbf{R})}\ B$

Inversely Dictatorial: there is a $d \in N$ such that for all $A, B \in X$ and all profiles $\mathbf{R}$: if $A\ P_d\ B$, then $B\ P_{F(\mathbf{R})}\ A$

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Null: For all $A, B \in X$ and for all $\mathbf{R} \in \mathcal{D}$: $A \; I_{F(\mathbf{R})} \; B$

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Null: For all $A, B \in X$ and for all $\mathbf{R} \in \mathcal{D}$: $A\ I_{F(\mathbf{R})}\ B$

Non-Imposition: For all $A, B \in X$, there is a $\mathbf{R} \in \mathcal{D}$ such that $A\ F(\mathbf{R})\ B$

Weakening Unanimity

Theorem (Wilson) Suppose that N is a finite set. If a social welfare function satisfies universal domain, independence of irrelevant alternatives and non-imposition, then it is either null, dictatorial or inversely dictatorial.

R. Wilson. *Social Choice Theory without the Pareto principle*. Journal of Economic Theory, 5, pgs. 478 - 486, 1972.

Y. Murakami. *Logic and Social Choice*. Routledge, 1968.

S. Cato. *Social choice without the Pareto principle: A comprehensive analysis*. Social Choice and Welfare, 39, pgs. 869 - 889, 2012.

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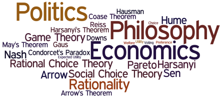
Theorem (Arrow, 1951). Suppose that there are at least three candidates and finitely many voters. Any social welfare function that satisfies universal domain, independence of irrelevant alternatives and unanimity is a dictatorship.

Social Choice Functions



$F : \mathcal{D} \rightarrow \wp(X) - \emptyset$

Resolute: For all profiles $\mathbf{R} \in \mathcal{D}$, $|F(\mathbf{R})| = 1$

Non-Imposed: For all candidates $A \in X$, there is a $\mathbf{R} \in \mathcal{D}$ such that $F(\mathbf{R}) = \{A\}$.

Monotonicity: For all profiles $\mathbf{R}$ and $\mathbf{R}'$, if $A \in F(\mathbf{R})$ and for all $i \in N$, $\mathbf{N}_{\mathbf{R}}(A\ P_i\ B) \subseteq \mathbf{N}_{\mathbf{R}'}(A\ P_i'\ B)$ for all $B \in X - \{A\}$, then $A \in F(\mathbf{R}')$.

Dictator: A voter d is a dictator if for all $\mathbf{R} \in \mathcal{D}$, $F(\mathbf{R}) = \{A\}$, where A is d's top choice.

Social Choice Functions

Muller-Satterthwaite Theorem. Suppose that there are more than three alternatives and finitely many voters. Every resolute social choice function $F : L(X)^n \to X$ that is monotonic and non-imposed is a dictatorship.

E. Muller and M.A. Satterthwaite. *The Equivalence of Strong Positive Association and Strategy-Proofness*. Journal of Economic Theory, 14(2), pgs. 412 - 418, 1977.

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Theorem (Arrow, 1951). Suppose that there are at least three candidates and finitely many voters. Any social welfare function that satisfies universal domain, independence of irrelevant alternatives and unanimity is a dictatorship.