PHIL309P
Philosophy, Politics and Economics

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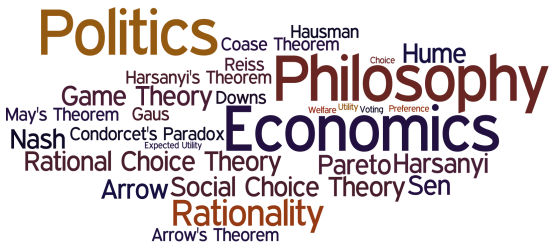

Announcements

- Course website
  https://myelms.umd.edu/courses/1133211
- Reading
  - Gaus, Ch. 5
  - EP, Voting Methods (Stanford Encyclopedia of Philosophy)
  - C. List, Social Choice Theory (Stanford Encyclopedia of Philosophy)
  - M. Morreau, Arrow's Theorem (Stanford Encyclopedia of Philosophy)
- Online videos
- Quiz

Arrow's Theorem

Theorem (Arrow, 1951). Suppose that there are at least three candidates and finitely many voters. Any social welfare function that satisfies universal domain, independence of irrelevant alternatives and unanimity is a dictatorship.

- Infinitely many voters.
- Domain restrictions.
- Richer ballots.

Approval Voting: Each voter selects a subset of candidates. The candidate with the most "approvals" wins the election.

S. Brams and P. Fishburn. *Approval Voting*. Birkhauser, 1983.

J.-F. Laslier and M. R. Sanver (eds.). *Handbook of Approval Voting*. Studies in Social Choice and Welfare, 2010.

Under Approval Voting (AV), voters are asked to select the candidates that the voter *approves*.

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Under ranking voting procedures (such as Borda Count), voters are asked to (linearly) rank the candidates.

The two pieces of information are related, but not derivable from each other

Approving of a candidate is not (necessarily) the same as simply ranking the candidate first.

Why Approval Voting?

www.electology.org/approval-voting

S. Brams and P. Fishburn. *Going from Theory to Practice: The Mixed Success of Approval Voting*. Handbook of Approval Voting, pgs. 19-37, 2010.

Example

Voters	A	B	C	D
1	1	0	1	1
2	0	1	1	0
3	0	1	0	0
4	0	0	0	0
5	1	1	1	1

Example

Voters	A	B	C	D
1	1	0	1	1
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4	0	0	0	0
5	1	1	1	1

1	2	3	4	5
A	B	D	D	A
B	C	B	C	B
C	A	C	B	D
D	D	A	A	C

Example

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1	2	3	4	5
A	B	D	D	A
B	C	B	C	B
C	A	C	B	D
D	D	A	A	C

An AV ballot is **sincere** if, given the lowest-ranked candidate that a voter approves of, he or she also approves of all candidates ranked higher.

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1	2	3	4	5
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C	A	C	B	D
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Approval Voting is more flexible

# voters	2	2	1
	A	B	C
	D	D	A
	B	A	B
	C	C	D

The Condorcet winner is A.

Approval Voting is more flexible

There is no fixed rule that always elects a unique Condorcet winner.

# voters	2	2	1
	A	B	C
	D	D	A
	B	A	B
	C	C	D

The Condorcet winner is A.
Vote-for-1 elects $\{A, B\}$

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# voters	2	2	1
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	C	C	D

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There is no fixed rule that always elects a unique Condorcet winner.

# voters	2	2	1
	A	B	C
	D	D	A
	B	A	B
	C	C	D

The Condorcet winner is A.
Vote-for-1 elects $\{A, B\}$, vote-for-2 elects $\{D\}$, vote-for-3 elects $\{A, B\}$.

Approval Voting is more flexible

AV may elect the Condorcet winner

# voters	2	2	1
	A	B	C
	D	D	A
	B	A	B
	C	C	D

The Condorcet winner is A.
$(\{A\}, \{B\}, \{C, A\})$ elects A under AV.

Possible Failure of Unanimity

# voters	1	1	1
	A	C	D
	B	A	A
	C	B	B
	D	D	C

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# voters	1	1	1
	A	C	D
	B	A	A
	C	B	B
	D	D	C

Approval Winners: A, B

Indeterminate or Responsive?

# voters	6	5	4
	A	B	C
	C	C	B
	B	A	A

Plurality winner: A, Borda and Condorcet winner: C.

Indeterminate or Responsive?

# voters	6	5	4
	A	B	C
	C	C	B
	B	A	A

Plurality winner: A, Borda and Condorcet winner: C.
Any combination of A, B and C can be an AV winner (or AV winners).

Generalizing Approval Voting

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Ask the voters to provide both a linear ranking of the candidates and the candidates that they approve.

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Ask the voters to provide both a linear ranking of the candidates and the candidates that they approve.

Make the ballots more expressive: Dis&Approval voting, RangeVoting, Majority Judgement

Grading

In many group decision situations, people use measures or grades from a **common language of evaluation** to evaluate candidates or alternatives:

- in figure skating, diving and gymnastics competitions;
- in piano, flute and orchestra competitions;
- in classifying wines at wine competitions;
- in ranking university students;
- in classifying hotels and restaurants, e.g., the Michelin $*$

Voting by Grading: Questions

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Voting by Grading: Questions

- What grading language should be used? (e.g., $A - F$, $0 - 10$, $* - ****$)
- How should we *aggregate* the grades? (e.g., Average or Median)
- Should there be a "no opinion" option?

Voting by Grading: Examples

Approval Voting: voters can assign a single grade "approve" to the candidates

Dis&Approval Voting: voters can approve or disapprove of the candidates

Majority Judgement, **Score Voting**: voters can assign any grade from a fixed set of grades to the candidates

Score Voting/Range Voting

Fixe a common grading language consisting of, for example, the integers $\{0, 1, 2, \ldots, 10\}$

The candidate with the largest *average* grade is declared the winner.

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Suppose A's grades are $\{7, 7, 8, 8, 9, 9, 9, 10\}$. The average grade is 8.375

Suppose B's grades are $\{9, 9, 9, 9, 9, 10, 10, 10\}$. The average grade is 9.375

So, Score Vote (Range Vote) ranks B above candidate A.

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www.electology.org/score-voting and rangevoting.org

Majority Judgement

Fix a common grading language. For example, $\{0, 1, 2, \ldots, 10\}$

The candidate with the largest median grade is declared the winner.

The *median* grade is the grade that is in the middle of the list when the grades are ordered (If there is an even number of judges, then the median grade is the lowest grade in the middle interval.)

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Suppose that A's grades are $\{6, 6, 7, 7, \mathbf{7}, 8, 9, 10, 10\}$: The median grade is 7.

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Suppose that A's grades are $\{6, 6, 7, 7, \mathbf{7}, 8, 9, 10, 10\}$: The median grade is 7.

Suppose B's grades are $\{6, 6, 6, \mathbf{6}, 9, 9, 9, 10\}$: The median grade is 6.

Majority Judgement

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The candidate with the largest median grade is declared the winner.

The *median* grade is the grade that is in the middle of the list when the grades are ordered (If there is an even number of judges, then the median grade is the lowest grade in the middle interval.)

Suppose that A's grades are $\{6, 6, 7, 7, \mathbf{7}, 8, 9, 10, 10\}$: The median grade is 7.

Suppose B's grades are $\{6, 6, 6, \mathbf{6}, 9, 9, 9, 10\}$: The median grade is 6.

Majority Judgement ranks B above A.

Majority Judgement: Tie-breaking rules

What happens when the median grades are the same?

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A's grades: $\{7, 9, \mathbf{9}, 11, 11\}$
B's grades: $\{8, 9, \mathbf{9}, 10, 11\}$

Majority Judgement: Tie-breaking rules

What happens when the median grades are the same?

A's grades: $\{7, 9, \mathbf{9}, 11, 11\}$
B's grades: $\{8, 9, \mathbf{9}, 10, 11\}$

The second median grade is found:
A's grades: $\{7, \mathbf{9}, 9, 11, 11\}$
B's grades: $\{8, \mathbf{9}, 9, 10, 11\}$

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A's grades: $\{7, 9, \mathbf{9}, 11, 11\}$
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The second median grade is found:
A's grades: $\{7, \mathbf{9}, 9, 11, 11\}$
B's grades: $\{8, \mathbf{9}, 9, 10, 11\}$

The third median grade is found:
A's grades: $\{7, 9, 9, \mathbf{11}, 11\}$
B's grades: $\{8, 9, 9, \mathbf{10}, 11\}$

Majority Judgement: Tie-breaking rules

What happens when the median grades are the same?

A's grades: $\{7, 9, \mathbf{9}, 11, 11\}$
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The second median grade is found:
A's grades: $\{7, \mathbf{9}, 9, 11, 11\}$
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The third median grade is found:
A's grades: $\{7, 9, 9, \mathbf{11}, 11\}$
B's grades: $\{8, 9, 9, \mathbf{10}, 11\}$

So, A is ranked above B.

Example

Suppose that there are five voters, $1, \ldots, 5$ and three candidates I, II, and III. The grades are A, B, C, D, or F (from best to worst).

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Suppose that there are five voters, $1, \ldots, 5$ and three candidates *I*, *II*, and *III*. The grades are A, B, C, D, or F (from best to worst).

	1	2	3	4	5
I	A	A	C	D	D
II	B	B	F	B	F
III	D	C	B	A	D

Example

Suppose that there are five voters, $1, \ldots, 5$ and three candidates *I*, *II*, and *III*. The grades are A, B, C, D, or F (from best to worst).

	1	2	3	4	5	Median
I	A	A	C	D	D	C
II	B	B	F	B	F	B
III	D	C	B	A	D	C

Candidate II is the majority judgement winner.

Example

Suppose that there are five voters, $1, \ldots, 5$ and three candidates *I*, *II*, and *III*. The grades are A, B, C, D, or F (from best to worst).

	1	2	3	4	5	Median
I	A	A	C	D	D	C
II	B	B	F	B	F	B
III	D	C	B	A	D	C

Candidate II is the majority judgement winner. *If asked about their preference, 4 voters would rank candidate I above candidate II*

Example

Suppose that there are five voters, $1, \ldots, 5$ and three candidates *I*, *II*, and *III*. The grades are $A = 4, B = 3, C = 2, D = 1$, or $F = 0$ (from best to worst).

	1	2	3	4	5	Average
I	4	4	2	1	1	2.4
II	3	3	0	3	0	1.8
III	1	2	3	4	1	2.2

Candidate II is the Majority Judgement winner. Candidate I is the Score Voting winner

More Information

M. Balinski and R. Laraki. *Majority Judgement: Measuring, Ranking and Electing*. The MIT Press, 2010.

W. D. Smith. *www.rangevoting.org*. .

S. Brams and R. Potthoff. *The Paradox of Grading Systems*. Manuscript, 2015.

A grading system is a voting system in which a voter can give any of g grades, $\{w_1, \ldots, w_g\}$, to each candidates.

AG winner: Candidate(s) that receives the largest average grade

SG winner: compare each candidate's grades with the grades of all other candidates. Candidate X beats candidate Y if the number of voters who grade X higher than Y exceed the number of voters that grade Y higher than X. The candidate(s) that beat every other candidate is(are) the SG winner(s).

Weak Paradox of Grading Systems

Grades: $\{0, 1, 2\}$

Candidates: $\{A, B, C\}$

9 Voters

# voters	2	3	4	Avg
A	2	0	1	
B	1	2	0	
C	0	1	2	

Grades: $\{0, 1, 2\}$

Candidates: $\{A, B, C\}$

9 Voters

# voters	2	3	4	Avg
A	2	0	1	8/9
B	1	2	0	8/9
C	0	1	2	11/9

Grades: $\{0, 1, 2\}$

Candidates: $\{A, B, C\}$

9 Voters

# voters	2	3	4	Avg
A	2	0	1	8/9
B	1	2	0	8/9
C	0	1	2	11/9

Average Grade Winner: C

Grades: $\{0, 1, 2\}$

Candidates: $\{A, B, C\}$

9 Voters

# voters	2	3	4	Avg
A	2	0	1	
B	1	2	0	
C	0	1	2	

Average Grade Winner: C

$$A \succ B$$

Grades: $\{0, 1, 2\}$

Candidates: $\{A, B, C\}$

9 Voters

# voters	2	3	4	Avg
A	2	0	1	
B	1	2	0	
C	0	1	2	

Average Grade Winner: C

$$A > B > C$$

Grades: $\{0, 1, 2\}$

Candidates: $\{A, B, C\}$

9 Voters

# voters	2	3	4	Avg
A	2	0	1	
B	1	2	0	
C	0	1	2	

Average Grade Winner: C

$$A \succ B \succ C \succ A$$

Grades: $\{0, 1, 2\}$

Candidates: $\{A, B, C\}$

9 Voters

# voters	2	3	4	Avg
A	2	0	1	
B	1	2	0	
C	0	1	2	

Average Grade Winner: C

Superior Grade Winners: A, B, C

Strong Paradox of Grading Systems

Grades: $\{0, 1, 2, 3\}$

Candidates: $\{A, B, C\}$

3 Voters

# voters	1	1	1	Avg
A	3	2	0	
B	0	3	1	
C	0	3	1	

Grades: $\{0, 1, 2, 3\}$

Candidates: $\{A, B, C\}$

3 Voters

# voters	1	1	1	Avg
A	3	2	0	5/3
B	0	3	1	4/3
C	0	3	1	4/3

Average Grade Winner: A

Grades: $\{0, 1, 2, 3\}$

Candidates: $\{A, B, C\}$

3 Voters

# voters	1	1	1	Avg
A	3	2	0	
B	0	3	1	
C	0	3	1	

Average Grade Winner: A

$$B \succ A$$

Grades: $\{0, 1, 2, 3\}$

Candidates: $\{A, B, C\}$

3 Voters

# voters	1	1	1	Avg
A	3	2	0	
B	0	3	1	
C	0	3	1	

Average Grade Winner: A

$$C \sim B \succ A$$

Grades: $\{0, 1, 2, 3\}$

Candidates: $\{A, B, C\}$

3 Voters

# voters	1	1	1	Avg
A	3	2	0	
B	0	3	1	
C	0	3	1	

Average Grade Winner: A

$$C \sim B \succ A$$

Grades: $\{0, 1, 2, 3\}$

Candidates: $\{A, B, C\}$

3 Voters

# voters	1	1	1	Avg
A	3	2	0	
B	0	3	1	
C	0	3	1	

Average Grade Winner: A

Superior Grade Winners: C, B

Grades: $\{0, 1, 2, 3, 4, 5\}$

Candidates: $\{A, B, C\}$

5 Voters

# voters	1	4	Avg
A	5	0	5/5
B	0	1	4/5
C	0	1	4/5

Average Grade Winner: A

Superior Grade Winner: B, C

To conclude, we have identified a paradox of grading systems, which is not just a mirror of the well-known differences that crop up in aggregating votes under ranking systems. Unlike these systems, for which there is no accepted way of reconciling which candidate to choose when, for example, the Hare, Borda and Condorcet winners differ, AV provides a solution when the aG and SG winners differ.

Theorem. When there are two grades, the AG and SG winners are identical.

Manipulating an election outcome

It has long been noted that a voter can achieve a preferred election outcome by misrepresenting his or her actual preferences.

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C.L. Dodgson refers to a voters tendency to

> *"adopt a principle of voting which makes it a game of skill than a real test of the wishes of the elector."*

and that in his opinion

> *"it would be better for elections to be decided according to the wishes of the majority than of those who happen to be more skilled at the game."*

Manipulating an election outcome

"If we assume society discourages the concentration of power, then at least two methods of manipulation are always available, no matter what method of voting is used: First, those in control of procedures can manipulate the agenda (by, for example, restricting alternatives or by arranging the order in which they are brought up). Second, those not in control can still manipulate by false revelation of values." (p. 137)

W. Riker. *Liberalism Against Populism*. Waveland Press, 1988.

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Literature

A. Taylor. *Social Choice and the Mathematics of Manipulation*. Cambridge University Press, 2005.

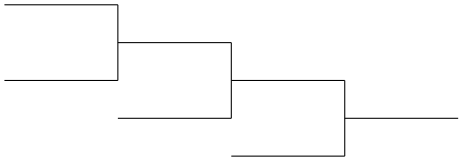
W. Poundstone. *Gaming the Vote*. Hill and Wang Publishers, 2008.

Manipulation: setting the agenda

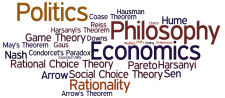
# voters	1	1	1
	B	A	C
	D	B	A
	C	D	B
	A	C	D

Manipulation: setting the agenda

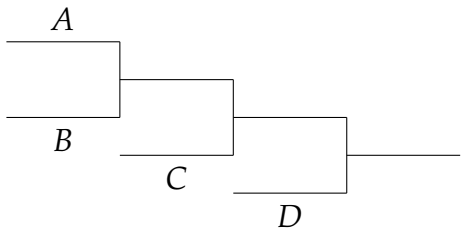
# voters	1	1	1
	B	A	C
	D	B	A
	C	D	B
	A	C	D



Manipulation: setting the agenda



# voters	1	1	1
	B	A	C
	D	B	A
	C	D	B
	A	C	D



Manipulation: setting the agenda

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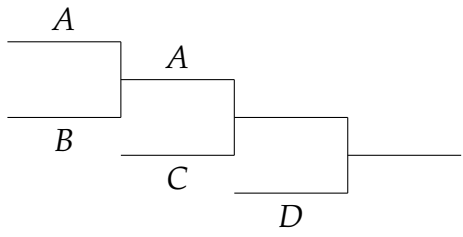
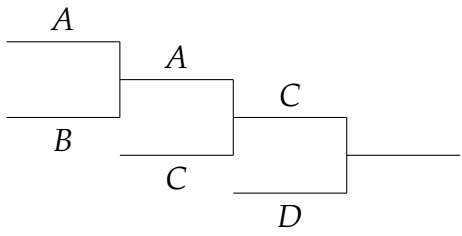
	B	A	C
	D	B	A
	C	D	B
	A	C	D



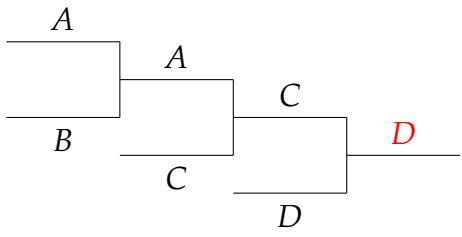
Manipulation: setting the agenda

# voters	1	1	1
	B	A	C
	D	B	A
	C	D	B
	A	C	D



Manipulation: setting the agenda

# voters	1	1	1
	B	A	C
	D	B	A
	C	D	B
	A	C	D



Manipulation: misrepresenting preferences

# voters	3	3	1
	A	B	C
	B	A	A
	C	C	B

Borda Winner: A

Manipulation: misrepresenting preferences

# voters	3	3	1
	A	B	C
	B	A	A
	C	C	B

Borda Winner: A

Manipulation: misrepresenting preferences

# voters	3	3	1
	A	B	C
	B	A	A
	C	C	B

Borda Winner: A

# voters	3	3	1
	A	B	C
	B	C	A
	C	A	B

Manipulation: misrepresenting preferences

# voters	3	3	1
	A	B	C
	B	A	A
	C	C	B

Borda Winner: A

# voters	3	3	1
	A	B	C
	B	C	A
	C	A	B

Borda Winner: B

Manipulation: misrepresenting preferences

# voters	3	3	1
	A	B	C
	B	A	A
	C	C	B

Borda Winner: A

# voters	3	3	1
	A	B	C
	B	C	A
	C	A	B

Borda Winner: B

Borda: “My procedure is only meant for honest men!”