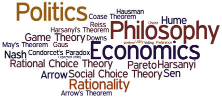
PHIL309P
Philosophy, Politics and Economics

Eric Pacuit
University of Maryland, College Park
pacuit.org

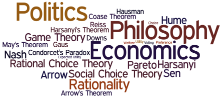
Announcements



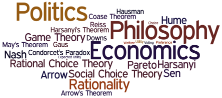
- Course website
  https://myelms.umd.edu/courses/1133211
- Reading
  - Gaus, Ch. 5
  - EP, Voting Methods (Stanford Encyclopedia of Philosophy)
  - C. List, Social Choice Theory (Stanford Encyclopedia of Philosophy)
  - M. Morreau, Arrow's Theorem (Stanford Encyclopedia of Philosophy)
- Quiz

Suppose that three experts *independently* formed opinions about three propositions. For example,

1. p: "Carbon dioxide emissions are above the threshold x"
2. $p \rightarrow q$: "If carbon dioxide emissions are above the threshold x, then there will be global warming"
3. q: "There will be global warming"

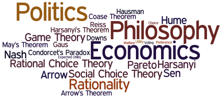


	p	$p \to q$	q
Expert 1			
Expert 2			
Expert 3			



	p	$p \to q$	q
Expert 1	True	True	
Expert 2			
Expert 3			

	p	$p \to q$	q
Expert 1	True	True	True
Expert 2			
Expert 3			



	p	$p \to q$	q
Expert 1	True	True	True
Expert 2	True		False
Expert 3			

	p	$p \to q$	q
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3			

	p	$p \to q$	q
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False

	p	$p \rightarrow q$	q
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False
Group			

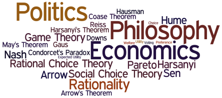
	p	$p \to q$	q
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False
Group	True		

	p	$p \rightarrow q$	q
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False
Group	True	True	

	p	$p \to q$	q
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False
Group	True	True	False

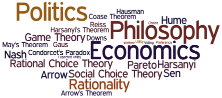
p: a valid contract was in place
q: there was a breach of contract
r: the court is required to find the defendant liable.

	p	q	$(p \wedge q) \leftrightarrow r$	r
1	yes	yes	yes	yes
2	yes	no	yes	no
3	no	yes	yes	no



Should we accept r?

	p	q	$(p \wedge q) \leftrightarrow r$	r
1	yes	yes	yes	yes
2	yes	no	yes	no
3	no	yes	yes	no



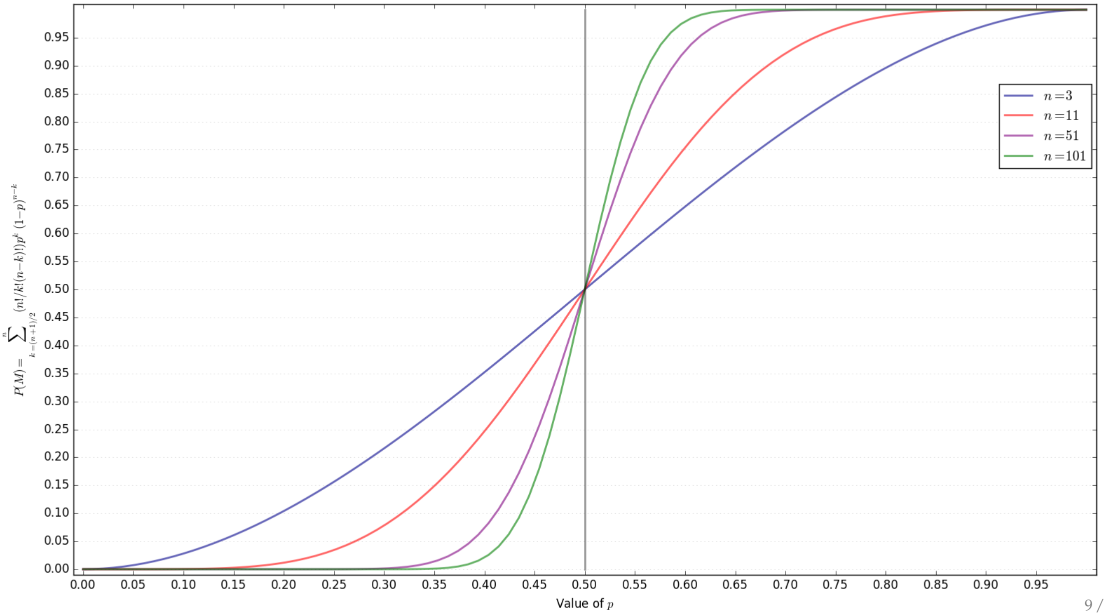
Should we accept r? No, a simple majority votes no.

	p	q	$(p \wedge q) \leftrightarrow r$	r
1	yes	yes	yes	yes
2	yes	no	yes	no
3	no	yes	yes	no

Should we accept r? Yes, a majority votes yes for p and q and $(p \wedge q) \leftrightarrow r$ is a legal doctrine.

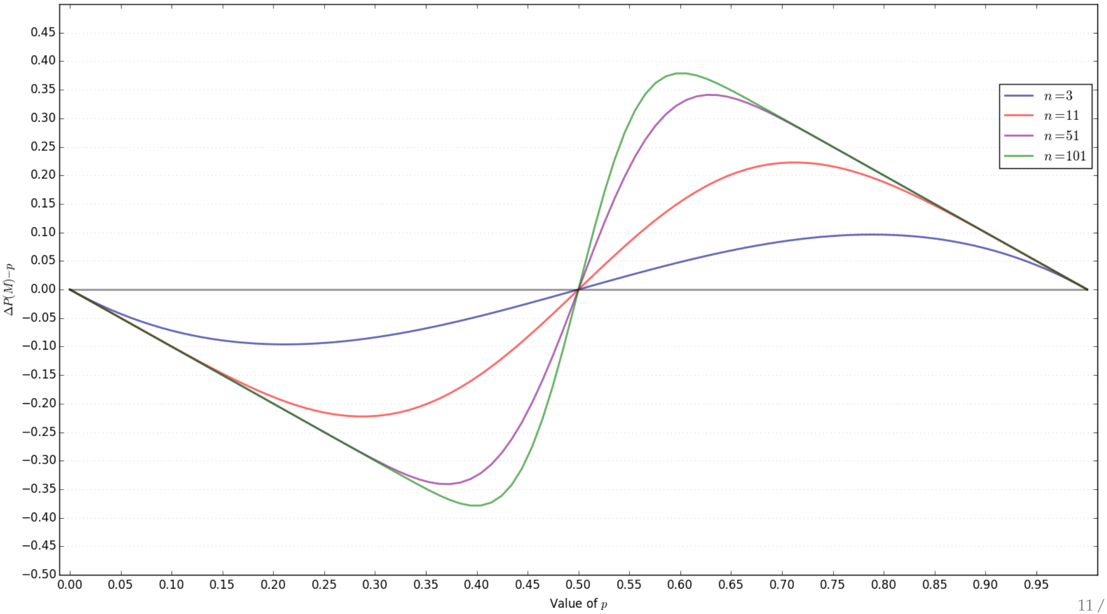
	p	q	$(p \wedge q) \leftrightarrow r$	r
1	yes	yes	yes	yes
2	yes	no	yes	no
3	no	yes	yes	no

$$P(M) = \sum_{k=(n+1)/2}^{n} \binom{n}{k} p^k (1-p)^{n-k}$$



$P(M) = \sum_{k=(n+1)/2}^{n} (n!/k!(n-k)!) p^k (1-p)^{n-k}$
Value of p
$n=3$
$n=11$
$n=51$
$n=101$

$$\Delta = P(M) - p$$


$n = 3$
$n = 11$
$n = 51$
$n = 101$
$\Delta P(M) - p$
Value of p
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
−0.35
−0.40
−0.45
−0.50
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95

	S	F	$D \leftrightarrow (F \wedge S)$
C1	T	T	T
C2	T	F	F
C3	F	T	F
C4	F	F	F

	S	F	$D \leftrightarrow (F \wedge S)$
$C1$	T	T	T
$C2$	T	F	F
$C3$	F	T	F
$C4$	F	F	F

$$P(C1) = q^2$$

$$P(C2) = P(C3) = q(1 - q)$$

$$P(C4) = (1 - q)^2$$

$$P(V \mid C1) = p^2$$

$$P(V \mid C2) = p^2 + p(1-p) + (1-p)^2$$

$$P(V \mid C4) = p^2 + 2p(1-p)$$

$$P(V) = \sum_{i=1}^{4} P(V \mid Ci)P(Ci)$$

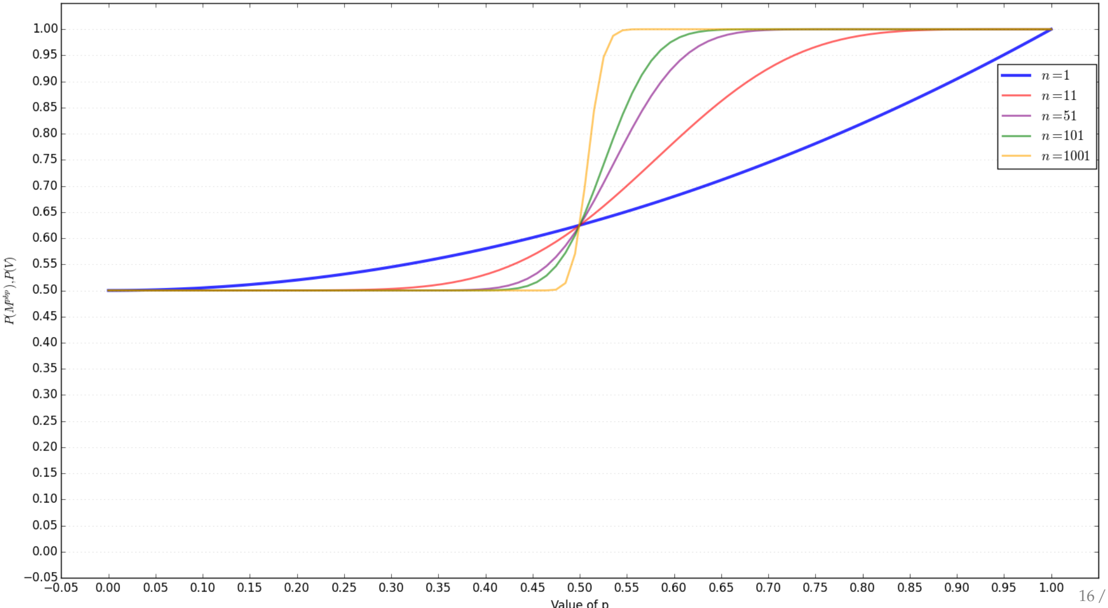
$$P(M^{pbp} \mid C1) = P(M)^2$$

$$P(M^{pbp} \mid C2) = P(M^{pbp} \mid C3) = P(M)^2 + P(M)(1 - P(M)) + (1 - P(M))^2$$

$$P(M^{pbp} \mid C4) = P(M)^2 + 2P(M)(1 - P(M))$$

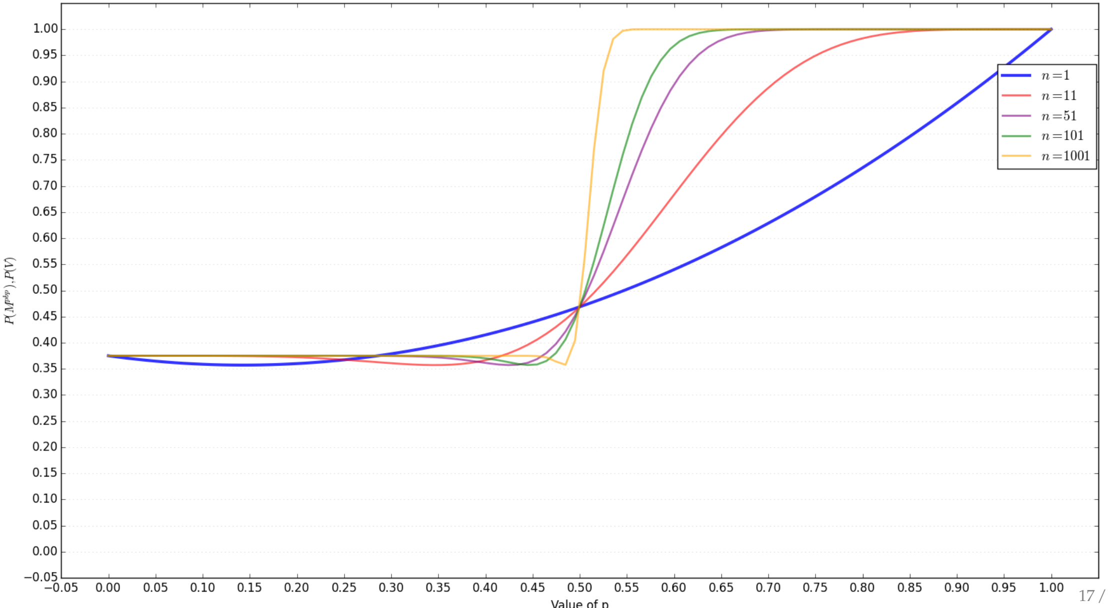
$$P(M^{pbp}) = \sum_{i=1}^{4} P(M^{pbp} \mid Ci)P(Ci)$$

$q = 0.5$



1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
$P(M^{pbp}), P(V)$
−0.05
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
Value of p
$n=1$
$n=11$
$n=51$
$n=101$
$n=1001$

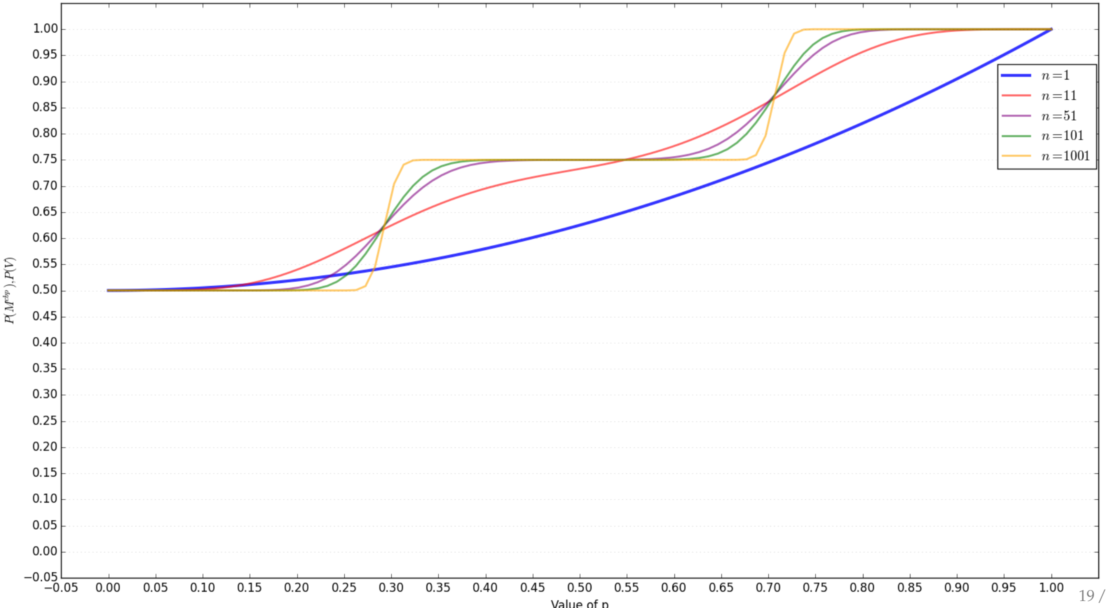
$q = 0.75$



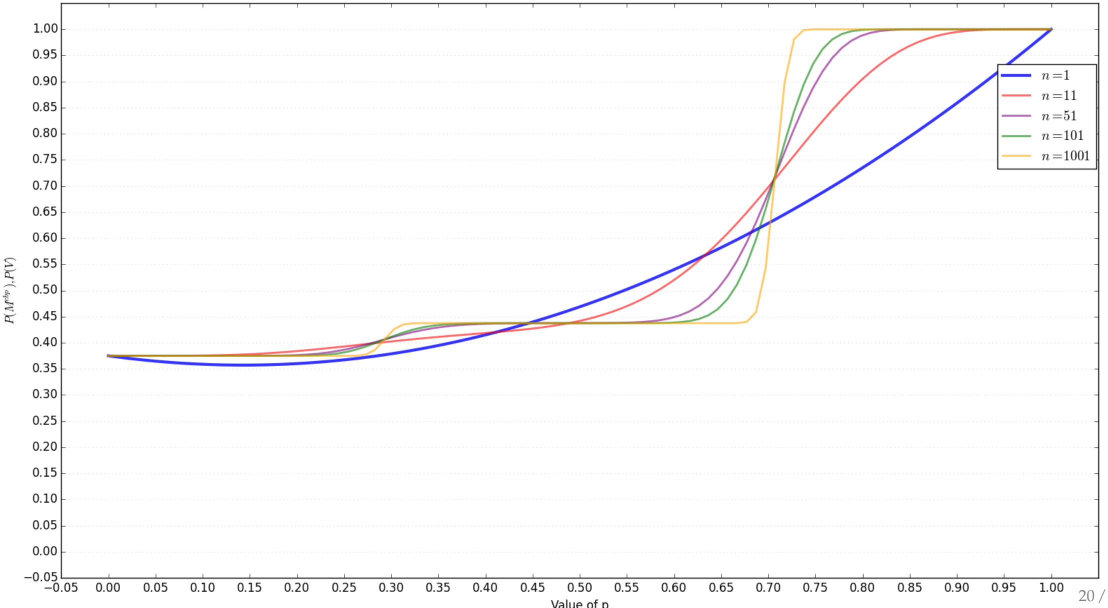
$$P(M^{cbp} \mid Ci) = \sum_{k=\frac{n+1}{2}}^{n} \binom{n}{k} P(V \mid Ci)^k (1 - P(V \mid Ci))^{n-k}$$

$$P(M^{cbp}) = \sum_{i=1}^{4} P(M^{cbp} \mid Ci) P(Ci)$$

$q = 0.5$



$q = 0.5$



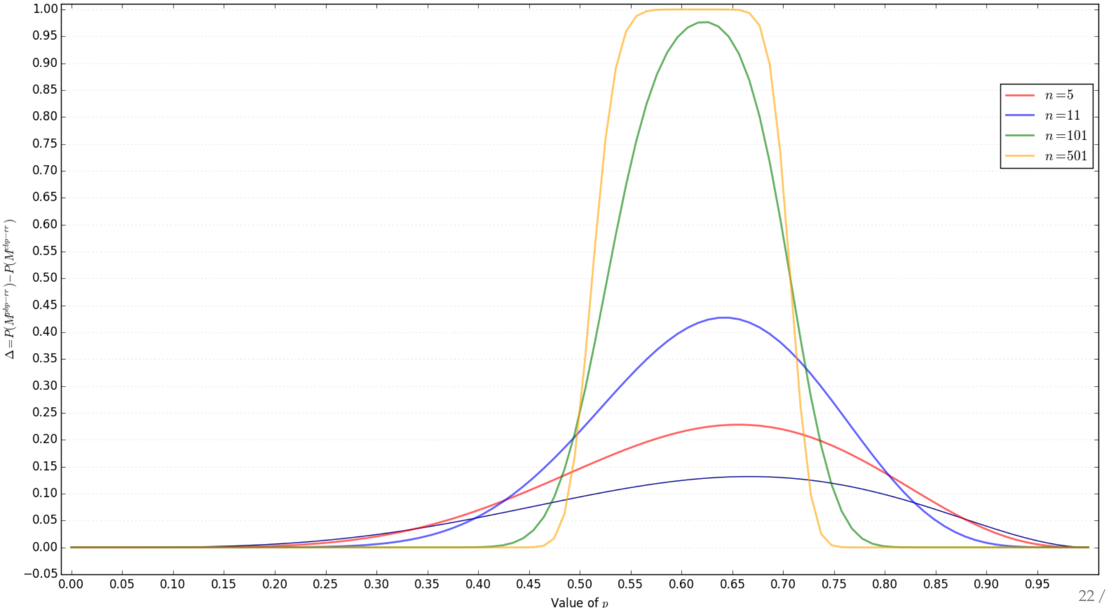
$$P(M^{pbp}) = \sum_{i=1}^{4} P(M^{pbp} \mid Ci)P(Ci)$$

$$P(M^{pbp-rr}) = P(M)^2$$

$$P(M^{cbp}) = \sum_{i=1}^{4} P(M^{cbp} \mid Ci)P(Ci)$$

$$P(M^{cbp-rr}) = \sum_{k=\frac{n+1}{2}}^{n} \binom{n}{k} p^2(1-p^2)^{n-k}$$

$q = 0.5$


$\Delta = P(M^{pbp-rr}) - P(M^{cbp-rr})$
Value of p
$n = 5$
$n = 11$
$n = 101$
$n = 501$
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95

An employee-owned bakery must decide whether to buy a pizza oven (P) or a fridge to freeze their outstanding Tiramisu (F). The pizza oven and the fridge cannot be in the same room. So they also need to decide whether to rent an extra room in the back (R). They all agree that they will rent the room if they decide to buy both the pizza oven and the fridge: $((P \wedge F) \rightarrow R)$, but they are contemplating renting the room regardless of the outcome of the vote on the appliances.

F. Cariani. *Judgement Aggregation*. Philosophy Compass, 6, 1, pgs. 22 - 32.

P, F are reasons for R

$\neg P, \neg F$ are not reasons for $\neg R$

$\neg R, P$ are reasons for $\neg F$

In 1991, the German parliament staged a debate on whether the parliament should move from Bonn to Berlin

Among the motions considered were A (the parliament should move to Berlin), and B (the seat of government should move to Berlin)

	A	B	$A \wedge B$
1	T	T	T
2	T	F	F
3	F	T	F

In 1991, the German parliament staged a debate on whether the parliament should move from Bonn to Berlin

Among the motions considered were A (the parliament should move to Berlin), and B (the seat of government should move to Berlin)

Should the parliament and the government should not be geographically separated?

	A	B	$A \wedge B$	$A \leftrightarrow B$
1	T	T	T	T
2	T	F	F	F
3	F	T	F	F

Another decision frame might have looked equally good. In the new frame, the basic motions they consider are whether to move the parliament (A'), and whether parliament and government should be in the same city (B').

	A	B	$A \wedge B$	$A \leftrightarrow B$
1	T	T	T	T
2	T	F	F	F
3	F	T	F	F

	A'	B'	$A' \wedge B'$
1	T	T	T
2	T	F	F
3	F	F	F

The Propositions

Propositions: Let $\mathcal{L}$ be a logical language (called **propositions** in the literature) with the usual boolean connectives.

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Consistency: The standard notion of logical consistency.

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Consistency: The standard notion of logical consistency.

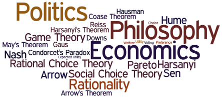
Aside: We actually need

1. $\{p, \neg p\}$ are inconsistent
2. all subsets of a consistent set are consistent
3. $\emptyset$ is consistent and each $S \subseteq \mathcal{L}$ has a consistent maximal extension (not needed in all cases)

The Agenda

Definition The **agenda** is a non-empty set $X \subseteq \mathcal{L}$, interpreted as the set of propositions on which judgments are made (note: X is a union of proposition-negation pairs $\{p, \neg p\}$).

The Agenda



Definition The **agenda** is a non-empty set $X \subseteq \mathcal{L}$, interpreted as the set of propositions on which judgments are made (note: X is a union of proposition-negation pairs $\{p, \neg p\}$).

Example: In the discursive dilemma: $X = \{p, \neg p, q, \neg q, p \to q, \neg(p \to q)\}$.

The Judgement Sets

Definition: Given an agenda X, each individual i's judgement set is a subset $A_i \subseteq X$.

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Definition: Given an agenda X, each individual i's judgement set is a subset $A_i \subseteq X$.

Rationality Assumptions:

1. A_i is **consistent**
2. A_i is **complete**, if for each $p \in X$, either $p \in A_i$ or $\neg p \in A_i$

Aggregation Rules

Let X be an agenda, $N = \{1, \ldots, n\}$ a set of voters, a **profile** is a tuple $(A_i, \ldots, A_n)$ where each A_i is a judgement set. An **aggregation function** is a map from profiles to judgment sets. I.e., $F(A_1, \ldots, A_n)$ is a judgement set.

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Examples:

- **Propositionwise majority voting**: for each $(A_1, \ldots, A_n)$,

$$F(A_1, \ldots, A_n) = \{p \in X \mid |\{i \mid p \in A_i\}| \geq |\{i \mid p \notin A_i\}|\}$$

- **Dictator of** i: $F(A_1, \ldots, A_n) = A_i$
- **Reverse Dictator of** i: $F(A_1, \ldots, A_n) = \{\neg p \mid p \in A_i\}$

Input

Universal Domain: The domain of F is the set of all possible profiles of consistent and complete judgement sets.

Output

Collective Rationality: F generates consistent and complete collective judgment sets.

Anonymity: For all profiles $(A_1, \ldots, A_n)$, $F(A_1, \ldots, A_n) = F(A_{\pi(1)}, \ldots, A_{\pi(n)})$ where π is a permutation of the voters.

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Unanimity: For all profiles $(A_1, \ldots, A_n)$ if $p \in A_i$ for each i then $p \in F(A_1, \ldots, A_n)$

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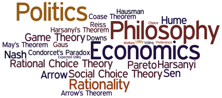
Monotonicity: For any $p \in X$ and all $(A_1, \ldots A_i, \ldots, A_n)$ and $(A_1, \ldots, A_i^*, \ldots, A_n)$ in the domain of F,

$$\text{if } [p \notin A_i, p \in A_i^* \text{ and } p \in F(A_1, \ldots, A_i, \ldots A_n)]$$
$$\text{then } [p \in F(A_1, \ldots, A_i^*, \ldots A_n)].$$

Systematicity: For any $p, q \in X$ and all $(A_1, \ldots, A_n)$ and $(A_1^*, \ldots, A_n^*)$ in the domain of F,

$$\text{if [for all } i \in N, p \in A_i \text{ iff } q \in A_i^*]$$
$$\text{then } [p \in F(A_1, \ldots, A_n) \text{ iff } q \in F(A_1^*, \ldots A_n^*) \text{]}.$$



Systematicity: For any $p, q \in X$ and all $(A_1, \ldots, A_n)$ and $(A_1^*, \ldots, A_n^*)$ in the domain of F,

$$\text{if [for all } i \in N, p \in A_i \text{ iff } q \in A_i^*]$$
$$\text{then } [p \in F(A_1, \ldots, A_n) \text{ iff } q \in F(A_1^*, \ldots A_n^*)\ ].$$

- independence
- neutrality

Independence: For any $p \in X$ and all $(A_1, \ldots, A_n)$ and $(A_1^*, \ldots, A_n^*)$ in the domain of F,

$$\text{if [for all } i \in N,\ p \in A_i \text{ iff } p \in A_i^*] \\ \text{then } [p \in F(A_1, \ldots, A_n) \text{ iff } p \in F(A_1^*, \ldots A_n^*)\ ].$$

Non-dictatorship: There exists no $i \in N$ such that, for any profile $(A_1, \ldots, A_n)$, $F(A_1, \ldots, A_n) = A_i$

Baseline Result

Theorem (List and Pettit, 2001) If $X \subseteq \{a, b, a \wedge b\}$, there exists no aggregation rule satisfying universal domain, collective rationality, systematicity and anonymity.

Sen’s Liberal Paradox

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Two members of a small society Lewd and Prude each have a personal copy of *Lady Chatterley's Lover*, consider

Sen's Liberal Paradox

Two members of a small society Lewd and Prude each have a personal copy of *Lady Chatterley's Lover*, consider

l: Lewd reads the book;
p: Prude reads the book;
$l \to p$: If Lewd reads the book, then so does Prude.

Sen’s Liberal Paradox

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Lewd desires to read the book, and if he reads it, then so does Prude (Lewd enjoys the thought of Prude's moral outlook being corrupted)

Sen’s Liberal Paradox

Lewd desires to read the book, and if he reads it, then so does Prude (Lewd enjoys the thought of Prude’s moral outlook being corrupted)

Prude desires to not read the book, and that Lewd not read it either, but in case Lewd does read the book, Prude wants to read the book to be informed about the dangerous material Lewd has read.

Sen’s Liberal Paradox

	l	p	$l \rightarrow p$

Sen’s Liberal Paradox

	l	p	$l \rightarrow p$
Lewd	True	True	True

Sen’s Liberal Paradox

	l	p	$l \to p$
Lewd	True	True	True
Prude	False	False	True

Sen’s Liberal Paradox

	l	p	$l \to p$
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Prude	False	False	True

1. Society assigns to each individual the liberal right to determine the collective desire on those propositions that concern only the individual’s private sphere
   l is Lewd’s case, p is Prude’s case

Sen's Liberal Paradox

	l	p	$l \rightarrow p$
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Prude	False	False	True

1. Society assigns to each individual the liberal right to determine the collective desire on those propositions that concern only the individual's private sphere
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2. Unanimous desires of all individuals must be respected.

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Lewd	True	True	True
Prude	False	False	True

1. Society assigns to each individual the liberal right to determine the collective desire on those propositions that concern only the individual's private sphere
   l is Lewd's case, p is Prude's case

2. Unanimous desires of all individuals must be respected.

So, society must be inconsistent!

$$F : L(X)^n \to (\wp(X) - \emptyset)$$

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Pareto: For all profiles $\mathbf{R} \in L(X)^n$ and alternatives A, B, if $A\ R_i\ B$ for all $i \in N$, then $B \notin F(\mathbf{R})$.

$F : L(X)^n \to (\wp(X) - \emptyset)$

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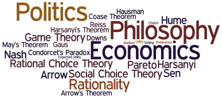
Liberalism: For all voters $i \in N$, there exists two alternatives A_i and B_i such that for all profiles $\mathbf{R} \in L(X)^n$, if $A_i \ R_i \ B_i$, then $B \notin F(\mathbf{R})$. That is, i is **decisive** over A_i and B_i.



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Minimal Liberalism: There are two distinct voters i and j such that there are alternatives A_i, B_i, A_j, and B_j such that i is decisive over A_i and B_i and j is decisive over A_j and B_j.

Sen's Impossibility Theorem. Suppose that X contains at least three elements. No social choice function $F : L(X)^n \to (\wp(X) - \emptyset)$ satisfies (universal domain) and both minimal liberalism and the Pareto condition.

A. Sen. *The Impossibility of a Paretian Liberal*. Journal of Political Economy, 78:1, pp. 152 - 157, 1970.

Suppose that X contains at least three elements and there are elements A, B, C and D such that

1. Voter 1 is decisive over A and B: for any profile $\mathbf{R} \in L(X)^n$, if $A\ R_1\ B$, then $B \notin F(\mathbf{R})$
2. Voter 2 is decisive over C and D: for any profile $\mathbf{R} \in L(X)^n$, if $C\ R_2\ D$, then $D \notin F(\mathbf{R})$

Two cases: 1. $B \neq C$ and 2. $B = C$.

Suppose that $X = \{A, B, C, D\}$ and

- Voter 1 is decisive over the pair A, B
- Voter 2 is decisive over the pair C, D

1	2
D	B
A	C
B	D
C	A

1	2
D	B
A	C
B	D
C	A

Voter 1 is decisive for A, B implies $B \notin F(\mathbf{R})$

1	2
D	B
A	C
B	D
C	A

Voter 1 is decisive for A, B implies $B \notin F(\mathbf{R})$

Voter 2 is decisive for C, D implies $D \notin F(\mathbf{R})$

1	2
D	B
A	C
B	D
C	A

Voter 1 is decisive for A, B implies $B \notin F(\mathbf{R})$

Voter 2 is decisive for C, D implies $D \notin F(\mathbf{R})$

Pareto implies $A \notin F(\mathbf{R})$

1	2
D	B
A	C
B	D
C	A

Voter 1 is decisive for A, B implies $B \notin F(\mathbf{R})$

Voter 2 is decisive for C, D implies $D \notin F(\mathbf{R})$

Pareto implies $A \notin F(\mathbf{R})$

Pareto implies $C \notin F(\mathbf{R})$

Suppose that $X = \{A, B, C\}$ and

- Voter 1 is decisive over the pair A, B
- Voter 2 is decisive over the pair B, C
- Voter 1's preference $R_1 \in L(X)$ is $C \ R_1 \ A \ R_1 \ B$
- Voter 2's preference $R_2 \in L(X)$ is $B \ R_2 \ C \ R_2 \ A$

1	2
C	B
A	C
B	A

1	2
C	B
A	C
B	A

Voter 1 is decisive for A, B implies $B \notin F(\mathbf{R})$

1	2
C	B
A	C
B	A

Voter 1 is decisive for A, B implies $B \notin F(\mathbf{R})$

Voter 2 is decisive for B, C implies $C \notin F(\mathbf{R})$

1	2
C	B
A	C
B	A

Voter 1 is decisive for A, B implies $B \notin F(\mathbf{R})$

Voter 2 is decisive for B, C implies $C \notin F(\mathbf{R})$

Pareto implies $A \notin F(\mathbf{R})$

“What is the moral?

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"What is the moral? It is that in a very basic sense liberal values conflict with the Pareto principle. If someone takes the Pareto principle seriously, as economists seem to do, then he has to face problems of consistency in cherishing liberal values, even very mild ones.... While the Pareto criterion has been thought to be an expression of individual liberty, it appears that in choices involving more than two alternatives it can have consequences that are, in fact, deeply illiberal." (pg. 157)

A. Sen. *The Impossibility of a Paretian Liberal*. Journal of Political Economy, 78:1, pp. 152 - 157, 1970.