PHIL309P
Philosophy, Politics and Economics

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- **Final Exam: Thu, May 12 8:00AM - 10:00AM**, EGR 2116 (see Testudo)
  - In-class exam
  - Consult problem sets (Problem sets 2 & 3 will be graded by Thursday or Friday), quizzes
  - Review sheet will be provided on Thursday or Friday
  - Multiple choice, short answers, short essay (questions will be provided).
- Final class: Tuesday, May 10
- Final comment: General reflections about the course, topics you found most interesting, topics you wish we spent more time on, etc.
- A couple quizzes coming.
- I'll be in my office on Wednesday of finals week in case you have questions about the final (you can schedule an appointment to be sure that I'm there).

Harsanyi’s Theorem

Assume that there is a finite number of citizens ($N = \{1, \ldots, n\}$), and a finite set of social states X.

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Assume that there is a *Planner*.

- The planner's utility function matches the social utility function
- If the Planner is a citizen, he is required to have two (but not necessarily different) preference orderings — his personal ordering and his moral ordering.

Individual and Social Rationality Each citizen and the Planner have a ranking $\succeq_1, \succeq_2, \ldots, \succeq_n, \succeq$ over $\mathcal{L}(X)$ (the set of lotteries over the social states X) satisfying the Von Neumann-Morgenstern axioms.

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- Each citizen's preference is represented by a linear utility function u_i
- The Planner's preference is represented by a linear utility function u
- Assume that all the citizens use 0 to 1 utility scales.
- Assume that 0 is the lowest utility scale for the Planner.

Strong Pareto

(P1) For each L, L' if $L \sim_i L'$ for all $i \in N$, then $L \sim L'$

(P2) For each L, L' if $L \succeq_i L'$ for all $i \in N$ and $L \succ_j L'$ for some $j \in N$, then $L \succ L'$

Each lottery L is associated with a vector of real numbers, $(u_i(L), \ldots, u_n(L)) \in \Re^n$. That is, the sequence of utility values of L for each agent.

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Defined the following two sets:

$$\mathcal{R}^n = \{(r_1, \ldots, r_n) \in \Re^n \mid \text{ there is a } L \in \mathcal{L} \text{ such that for all } i = 1, \ldots, n, u_i(L) = r_i\}$$

and

$$\mathcal{R} = \{r \in \Re \mid \text{there is a } L \in \mathcal{L} \text{ such that } u(L) = r\}$$

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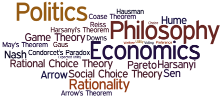
Define a function $f : \mathcal{R}^n \to \mathcal{R}$ as follows: for all $(r_1, \dots, r_n)$, let $f(r_1, \dots, r_n) = r$ where $r = u(L)$ with L a lottery such that $(u_1(L), \dots, u_n(L)) = (r_1, \dots, r_n)$.

Equity



(E) All agents should be treated equally by the Planner. Formally, this means that $f(r_1, \ldots, r_n) = f(r'_1, \ldots, r'_n)$ when there is a permutation $\pi : N \to N$ such that for each $i = 1, \ldots, n$, $r'_i = r_{\pi(i)}$.

Harsanyi's Theorem For all $(r_1, \ldots, r_n) \in \mathcal{R}^n$, $f(r_1, \ldots, r_n) = r_1 + \cdots + r_n$.

Observation. The function f is well-defined.

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Proof. Suppose that $L, L' \in \mathcal{L}$ such that $(u_1(L), \ldots, u_n(L)) = (u_1(L'), \ldots, u_n(L'))$. Then, for all $i \in N$, i is indifferent between L and L' (i.e., $L \sim_i L'$). Then, by axiom $P1$, we have $L \sim L'$. Thus, $u(L) = u(L')$; and so, f is well-defined.

For each $i \in N$ and $L \in \mathcal{L}$, we have $0 \leq u_i(L) \leq 1$.

For each $i \in N$, let $e_i = (0, 0, \ldots, 1, \ldots, 0)$ (where there is a 1 in the ith position and 0 everywhere else).

This corresponds to a situation in which a single agent gets her most preferred outcome while all the other agents get their least-preferred outcome.

Lemma. For each $i, j \in N$, $f(e_i) = f(e_j)$

Lemma. For all $a \in \mathfrak{R}$, $af(r_1, \ldots, r_n) = f(ar_1, \ldots, ar_n)$.

Let L be the lottery such that for each $i \in N$, $u_i(L) = r_i$. Consider the lottery $L' = [L : a, \mathbf{0} : (1 - a)]$, where $\mathbf{0}$ is the lottery in which everyone gets their lowest-ranked outcome.

Then, for each $i \in N$, $u_i(\mathbf{0}) = 0$. Furthermore, by the Pareto principle $P1$, we must have $u(\mathbf{0}) = 0$.

Then, for all $i \in N$, we have

1. $u_i(L') = au_i(L) + (1-a)u_i(\mathbf{0}) = au_i(L) = ar_i$; and
2. $u(L') = au(L) + (1-a)u(\mathbf{0}) = au(L)$

$$af(r_1, \ldots, r_n) \quad = \quad au(L) \qquad\qquad \text{(definition of } f\text{)}$$

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 & = & u(L') & \text{(item 2.)}
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 & = & f(u_1(L'),\ldots,u_n(L')) & \text{(definition of } f\text{)} \\
 & = & f(ar_1,\ldots ar_n) & \text{(item 1.)}
\end{array}
$$

Theorem. For all $(r_1, \ldots, r_n) \in \mathcal{R}^n, f(r_1, \ldots, r_n) = r_1 + \cdots + r_n$.

Consider a lottery L such that for all $i \in N$, $u_i(L) = r_i$. Consider lotteries L_i such that $u_i(L_i) = r_i$ and for all $j \neq i$, $u_j(L_i) = 0$. Consider the lottery $L' = [L_1 : 1/n, \ldots, L_n : 1/n]$.

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- $u_i(L') = \sum_{k=1}^{n} \frac{1}{n} u_i(L_k) = \frac{1}{n} u_i(L_i) = \frac{1}{n} r_i$.
- $f(0, \ldots, r_k, \ldots, 0) = r_k f(0, \ldots, 1, \ldots, 0) = r_k$

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	1	2	P
L_1	r_1	0	$f(r_1, 0) = r_1 f(1, 0)$
L_2	0	r_2	$f(0, r_2) = r_2 f(0, 1)$
L'	$\frac{1}{2}u(L_1) + \frac{1}{2}u(L_2) = \frac{1}{2}r_1$	$\frac{1}{2}u(L_1) + \frac{1}{2}u(L_2) = \frac{1}{2}r_1$	$f(\frac{1}{2}r_1, \frac{1}{2}r_2)$

$$\frac{1}{2} f(r_1, r_2) = f(\frac{1}{2}\, r_1, \frac{1}{2}\, r_2) = u(L') = \frac{1}{2}\, u(L_1) + \frac{1}{2}\, u(L_2) = \frac{1}{2}\, r_1 f(1, 0) + \frac{1}{2}\, r_2 f(0, 1)$$

$$u(L') = \sum_{k=1}^{n} \frac{1}{n}\, u(L_k)$$

$$
\begin{aligned}
u(L') &= \textstyle\sum_{k=1}^{n} \frac{1}{n}\, u(L_k) \\
&= \textstyle\sum_{k=1}^{n} \frac{1}{n} f(u_1(L_k), \ldots, u_k(L_k), \ldots, u_n(L_k))
\end{aligned}
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&= \tfrac{1}{n} f(r_1, \ldots, r_n)
\end{aligned}
$$

Thus,

$$\frac{1}{n} f(r_1, \ldots, r_k) = u(L') = \sum_{k=1}^{n} \frac{1}{n}\, r_k = \frac{1}{n} \sum_{k=1}^{n} r_k$$

Hence, $f(r_1, \ldots, r_n) = r_1 + \cdots + r_n$, as desired.

For 2 citizens, Harsanyi's Theorem require the existence of the following vectors of utilities:

$$(0,0) \quad (0,1) \quad (1,0) \quad (u_1,0) \quad (0,u_2) \quad (u_1,u_2)$$

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Problem. None of Harsanyi's conditions guarantee the existence of this social outcomes.

Suppose the problem is to give a scholarship to *exactly* one of the citizens.

- $(1, 0)$: give the scholarship to citizen 1
- $(0, 1)$: give the scholarship to citizen 2

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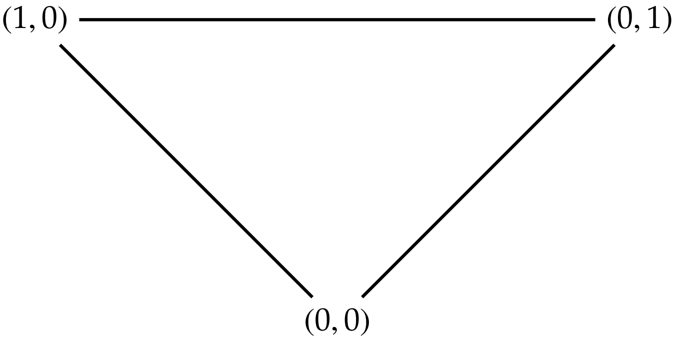
- $(1, 0)$: give the scholarship to citizen 1
- $(0, 1)$: give the scholarship to citizen 2
- What is the outcome $(0, 0)$?

Distributable Goods Assumption

For every vector of numbers $(u_1, \ldots, u_n)$ with $0 \leq u_i \leq 1$, there is at least one social option for which the distribution of citizens' utilities equals the vector in question.

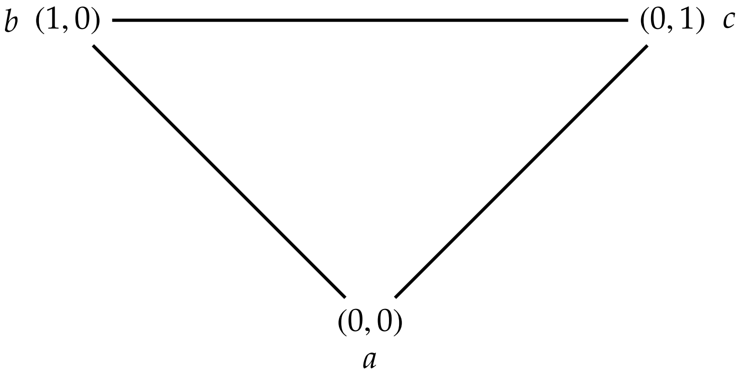
A distributable good is one, such as food, health, education, talent, friendship, for which all distributions throughout society are at least logically possible.

Problem: Philosophers also look to social choice theory for help in resolving problems in which interests conflict-situations, for example, in which citizens gain only at the expense of others, or ones in which the citizens envy each other, or prefer to sacrifice for each other. These are situations in which we cannot count on the distributable goods assumption to hold.



- $(1, 0)$ is the best for citizen 1 and worst for citizen 2
- $(0, 1)$ is the best for citizen 2 and worst for citizen 1
- $(0, 0)$ is the worst for both citizens

Special Prospects Assumption. There are three social options a, b and c such that (1) the first citizen prefers b to a and is indifferent between a and c, (2) the second citizen prefers c to a and is indifferent between a and$b$.



- 1 prefers b to a and is indifferent between a and c
- 2 prefers c to a and is indifferent between a and b.
- a, b, and c can be very similar or "close" to each other

- A defense of the theorem must argue either that a “true” representation of the citizens’ preferences will give rise to the appropriate vectors or that there is a set of “background” options sufficiently rich to support the same vectors, or that certain profiles, such as those in which considerations of envy or altruism are operative, should not be considered.

Mary $\quad$ seashore $\succ_M$ museums $\succ_M$ camping

Sam $\quad$ camping $\succ_S$ museums $\succ_S$ seashore

- The seashore is the only alternative that Mary finds bearable, although she feels more negative about going to the mountains than to the museums.
- Each choice is fine with Sam, although he would much prefer going to the mountains.

	Mary	Sam	Total
Seashore	20		
Museums	10		
Mountains	9		

	Mary	Sam	Total
Seashore	20	86	
Museums	10	93	
Mountains	9	100	

	Mary	Sam	Total
Seashore	20	86	106
Museums	10	93	103
Mountains	9	100	109

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Seashore	20	86	106
Museums	10	93	103
Mountains	9	100	109

For Mary, the difference between the seashore and the mountains crosses the threshold between the bearable and the intolerable. She feels that her "right to an emotionally recuperative vacation will be violated by following a utilitarian scheme.

	Mary	Sam	Total
Seashore	200	86	286
Museums	100	93	190
Mountains	90	100	190

Mary: My preferences are so intense in comparison with yours that my scale should range between 0 and 1,000, if yours range between 0 and 100.

	Mary	Sam	Total
Seashore	20	86	106
Museums	10	93	103
Mountains	9	100	109

Sam: You think that my preferences are rather weak, but the fact is I feel things quite deeply. I have been brought up in a culture very different from yours and have been trained to avoid emotional outbursts...But I have strong feelings all the same.

	Mary	Sam	Total
Seashore	20	86	106
Museums	10	93	103
Mountains	9	100	109

Sam: I do not think that extra weight should be given in a utilitarian calculation to those who are capable of more intense preferences. , the difference between the seashore and the mountains crosses the threshold between the bearable and the intolerable. She feels that her “right to an emotionally recuperative vacation will be violated by following a utilitarian scheme.

- Is Mary's preference for the seashore *really* stronger than Sam's for the mountains? Or, is Mary just a more vocal person?
- If some people's preferences are in fact stronger than others', how could we *know* this?
- Does it make any more sense to compare Sam's preferences with Mary's than it does to compare a dog's preference for steak bones with a horse's preference for oats?
- Even if we answer all these questions affirmatively, is it morally proper to respond to such information in making social choices?

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Is there more to emotions that our display of them?

1. An employer must choose between two equally qualified employees to promote. Assume that everything about their contributions to the firm, their length of service, personal financial needs, and so forth, is the same. The employer summons both employees to her office for separate conversations. The first is an impassive type who allows that he would be pleased to be promoted. The second, on the other hand, effusively tells the employer how long he has hoped for the promotion, etc.

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2. A politician must decide whether to demolish a block of old houses to make room for a new library. The residents of the houses are old and feeble, and the sponsors of the library are young and quite vocal. Both send delegates to speak to the politician. The politician finds it politically expedient to favor the young.

Those that believe in interpersonal comparison of utilities will grant that the two cases have been correctly described: The employer weighed the utilities of her two employees and the politician simply responded to political pressure.

Those who are skeptical about interpersonal comparisons of utility, will argue that in both cases the decision maker is simply behaving in accordance with cultural conditioning to respond in certain ways to the actions of others...the second employee’s effusiveness is just as much a form of pressure as the political activists’.

- interpersonal comparison of utility *levels*
- interpersonal comparison of utility *increments*

Harsanyi's social welfare function deals with incremental utilities and ignores utility levels.

- The ranking of x and y in terms of sums is preserved if adding (the same or different) numbers to both x and y. Adding these numbers is tantamount to changing the zero points of the citizens' utilities.
- Harsanyi's social welfare function does respond to changes in the units used to measure utility increments.

Some social choice methods respond only to changes in the utility origins, these presuppose the interpersonal comparison of *utility origins*.

Some social choice methods respond only to change in utility units and presuppose interpersonal comparison of utility units.

Some social choice methods respond to changes in both utility origins and units and presupposes interpersonal comparison of both.

Fair Division

Suppose that there is a set G of objects that must be divided among a group of individuals.

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Questions:

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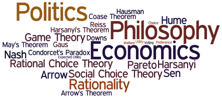
  - Several divisible objects
  - A single heterogeneous object

Fair Division



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  - A set of indivisible objects
  - Several divisible objects
  - A single heterogeneous object
- Are side-payments allowed?
- Dividing "goods" or "bads"? or both?

- Individual utilities of the goods: Ordinal? Cardinal?

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- Maximize social welfare:
  - Utilitarian: maximize $\sum_i u_i$
  - Egalitarian: maximize $\min_i\{u_i\}$
  - Nash: maximize $\Pi_i u_i$

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- Maximize social welfare:
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  - Egalitarian: maximize $\min_i\{u_i\}$
  - Nash: maximize $\Pi_i u_i$
- Preferences over bundles, or allocations: Separable? Additive? Lifted from an ordering over the objects?

Literature

H. Moulin. *Fair Division and Collective Welfare*. The MIT Press, 2003.

S. Brams and A. Taylor. *Fair Division: From cake-cutting to dispute resolution*. Cambridge University Press, 1998.

Suppose that G is a set of goods to be distributed among n individuals.

An **allocation** is a function $A : N \to \wp(G)$ assigning goods to individuals (note that, in general, it need not be the case that $\bigcup_{i \in N} A(i) = G$).

For each $i \in N$, u_i is i's utility function on *bundles* of goods. Then, the utility of an allocation is $u_i(A) = u_i(A(i))$.

A **profile** of utilities for an allocation A is a tuple $(u_1(A(1)), \ldots, u_n(A(n)))$, where $N = \{1, \ldots, n\}$ is the set of individuals.

Pareto Efficiency

Suppose that A and A' are allocations.

A **Pareto dominates** A' provided for all $i \in N$, $u_i(A(i)) \geq u_i(A'(i))$ and there is a $j \in N$ such that $u_j(A(j)) > u_j(A'(j))$.

A is **Pareto efficient** if it is not Pareto dominated. (That is, there is no A' such that A' Pareto dominates A)

Envy-Freeness

An allocation A is **envy-free** provided there is no individual i such that

$$u_i(A(j)) > u_i(A(i))$$

for some j.

Proportionality

Suppose that G is the set of all the objects and there are n individuals. An allocation A is **proportional** provided for all i:

$$u_i(A) \geq \frac{1}{n} u_i(G)$$

Note that this only makes sense when the utilities are monotonic: for all sets of goods $C \subseteq D \subseteq G$, $u_i(C) \leq u_i(D)$.

Equitability

An allocation A is **equitable** provided for all i, j:

$$u_i(A(i)) = u_j(A(j)))$$

Paradoxes of Fair Division: Indivisible Goods

S. Brams, P. Edelman and P. Fishburn. *Paradoxes of Fair Division*. Journal of Philosophy, 98:6, pgs. 300-314, 2001.

No Envy-Free Division

$$
\begin{array}{lccccc}
\text{Ann:} & 1 & \succ & 2 & \succ & 3 \\
\text{Bob:} & 1 & \succ & 3 & \succ & 2 \\
\text{Cath:} & 2 & \succ & 1 & \succ & 3
\end{array}
$$

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\end{array}
$$

There are no envy-free divisions.

Envy-Freeness and Efficiency

$$\begin{array}{rccccccccccc}
\text{Ann:} & 1 & \succ & 2 & \succ & 3 & \succ & 4 & \succ & 5 & \succ & 6 \\
\text{Bob:} & 4 & \succ & 3 & \succ & 2 & \succ & 1 & \succ & 5 & \succ & 6 \\
\text{Cath:} & 5 & \succ & 1 & \succ & 2 & \succ & 6 & \succ & 3 & \succ & 4
\end{array}$$

Envy-Freeness and Efficiency

Ann:	1	$\succ$	2	$\succ$	3	$\succ$	4	$\succ$	5	$\succ$	6
Bob:	4	$\succ$	3	$\succ$	2	$\succ$	1	$\succ$	5	$\succ$	6
Cath:	5	$\succ$	1	$\succ$	2	$\succ$	6	$\succ$	3	$\succ$	4

Ann: $\{1, 3\}$
Bob: $\{2, 4\}$
Cath: $\{5, 6\}$

Envy-Freeness and Efficiency

$$\text{Ann: } 1 \succ 2 \succ 3 \succ 4 \succ 5 \succ 6$$

$$\text{Bob: } 4 \succ 3 \succ 2 \succ 1 \succ 5 \succ 6$$

$$\text{Cath: } 5 \succ 1 \succ 2 \succ 6 \succ 3 \succ 4$$

Ann: $\{1, 3\}$	Ann: $\{1, 2\}$
Bob: $\{2, 4\}$	Bob: $\{3, 4\}$
Cath: $\{5, 6\}$	Cath: $\{5, 6\}$

Envy-Freeness and Efficiency

$$
\begin{array}{lccccccccccc}
\text{Ann:} & 1 & \succ & 2 & \succ & 3 & \succ & 4 & \succ & 5 & \succ & 6 \\
\text{Bob:} & 4 & \succ & 3 & \succ & 2 & \succ & 1 & \succ & 5 & \succ & 6 \\
\text{Cath:} & 5 & \succ & 1 & \succ & 2 & \succ & 6 & \succ & 3 & \succ & 4
\end{array}
$$

$$
\begin{array}{rl}
\text{Ann:} & \{1,3\} \\
\text{Bob:} & \{2,4\} \\
\text{Cath:} & \{5,6\}
\end{array}
\qquad\qquad
\begin{array}{rl}
\text{Ann:} & \{1,2\} \\
\text{Bob:} & \{3,4\} \\
\text{Cath:} & \{5,6\}
\end{array}
$$

Envy-Freeness and Efficiency

$$\text{Ann: } \boxed{\color{red}1} \succ \boxed{\color{red}2} \succ 3 \succ 4 \succ 5 \succ 6$$

$$\text{Bob: } 4 \succ 3 \succ 2 \succ 1 \succ 5 \succ 6$$

$$\text{Cath: } \boxed{\color{blue}5} \succ \boxed{\color{red}1} \succ \boxed{\color{red}2} \succ \boxed{\color{blue}6} \succ 3 \succ 4$$

Ann: $\{1,3\}$	Ann: $\color{red}\{1,2\}$
Bob: $\{2,4\}$	Bob: $\{3,4\}$
Cath: $\{5,6\}$	Cath: $\color{blue}\{5,6\}$

There is no other division that guarantees *envy freeness*