CMSC427
Points, polylines and polygons

## Issue: discretization of continuous curve

- In theory, smooth curve:

- In reality, piecewise discrete approximation:



## Modeling with discrete approximations

## Increase fidelity with more points


(a) 25,000 vertices.
(b) 5,000 vertices.
(c) 500 vertices.


## Points, polylines and polygons



Points
Also called vertices


Polyline
Continuous sequence of line segments


Polygon
Closed sequence of line segments

## Polygon properties I

- Simple
no self-intersections no duplicate points
- Non-simple self-intersections duplicate points



## Polygon properties II

- Convex polygon

Any two points in polygon can be connected by inside line

- Concave polygon

Not true of all point pairs


## Point in polygon problem

Is P inside or outside the polygon?

Case 1


Case 2


Case 3


## Point in polygon problem

Is P inside or outside the polygon?


## Point in polygon problem

Is P inside or outside the polygon?


## Other polygon problems

- Polygon collision
- Return yes/no

- Polygon intersection
- Return polygon of intersection (P)
- Polygon rasterization
- Return pixels that intersect

- Polygon winding direction
- Return clockwise (CW) or counterclockwise (CCW)


Moral: easier with simple, convex, low count polygons


## Triangular mesh



## Why triangles ...

Why triangles?

1. Easiest polygon to rasterize
2. Polygons with $n>3$ can be non-planar


Polygon triangulation


## Theorem: Every simple polygon has a triangulation

- Proof by induction

Base case: $\mathrm{n}=3$

Inductive case
A) Pick a convex corner $p$. Let $q$ and $r$ be pred and succ vertices.
B) If qr a diagonal, add it. By induction, the smaller polygon has a triangulation.
C) If qr not a diagonal, let $z$ be the
 reflex vertex farthest to qr inside $\Delta p q r$.
D) Add diagonal pz; subpolygons on both sides have triangulations.

Example: cutting off ears


## Parametric curves vs. polylines

- Parametric curves

1. Model objects by equation
2. Complex shapes from few values
3. Modeling arbitrary shape can be hard

- Polylines

1. Model objects by data points
2. Complex shapes need additional data
3. Can model any shape approximately

- Looking forward
- Use polylines to control general parametric curves
- B-splines, NURBS



## What you should know after today

1. Definitions of polyline and polygons
2. Definitions of properties of polygons (simple, convex/concave
3. Point in polygon problem and first solution
4. That polygons support interesting problems (Don't have to know them now)
5. Why we like triangles over more complex polygons
6. That simple polygons can be triangulated
