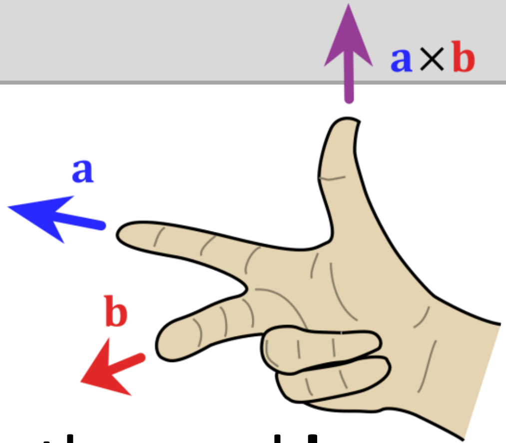
CMSC427
Geometry and Vectors: Cross Product

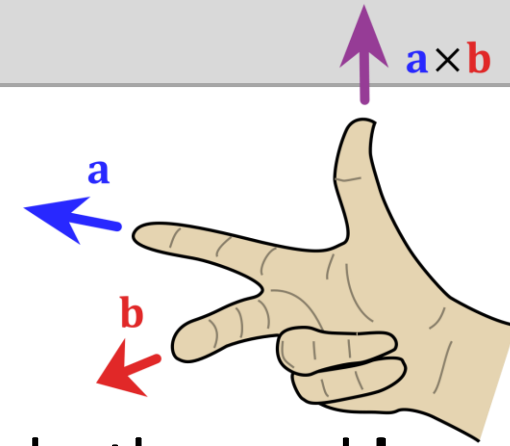
Cross product



$\mathbf{a} \times \mathbf{b}$

- Read as "a cross b"
- **a x b** is a vector perpendicular to both **a** and **b**, in the direction defined by the right hand rule

Cross product



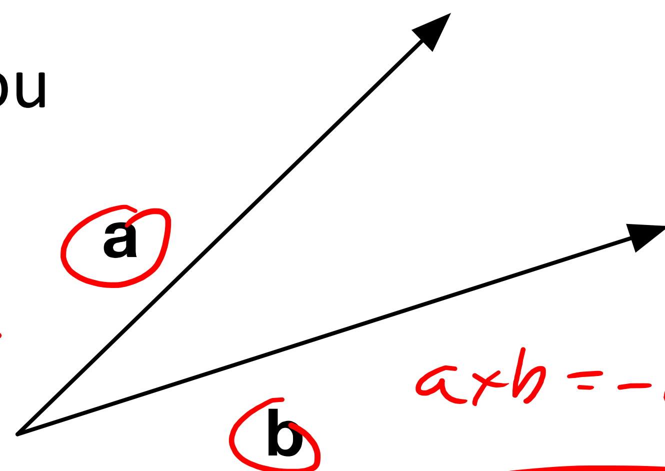
$\mathbf{a} \times \mathbf{b}$

- Read as “a cross b”
- **a x b** is a vector perpendicular to both **a** and **b**, in the direction defined by the right hand rule
- Vector a and b lie in the plane of the projection screen.
- Does **a x b** point towards you or away from you?

into the screen, away

What about **b x a**?

out of the screen, towards you

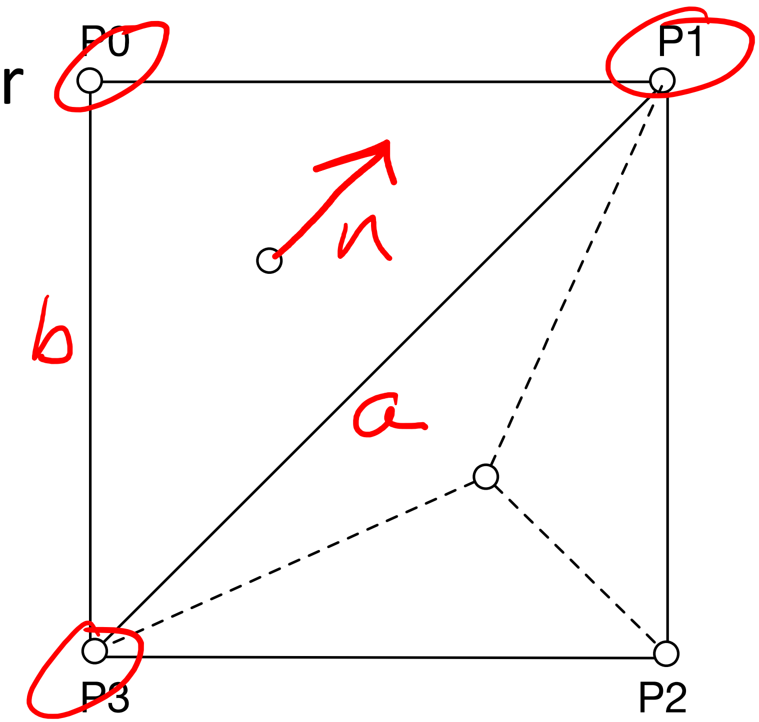


$a \times b = -b \times a$

Cross product: normal to plane

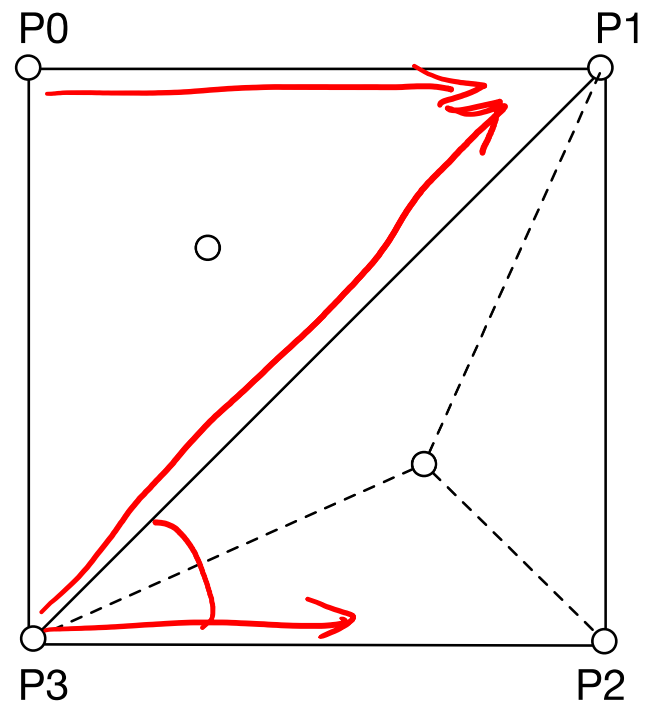
- How compute normal vector to triangle P0, P1, P3?
- Assume up is out of page

$$n = a \times b$$
$$= (P1 - P3) \times (P0 - P3)$$



Cross product: normal to plane

- How compute normal vector to triangle P0, P1, P3?
- Assume up is out of page
- One answer:
  n = (P1-P3) x (P0-P1)



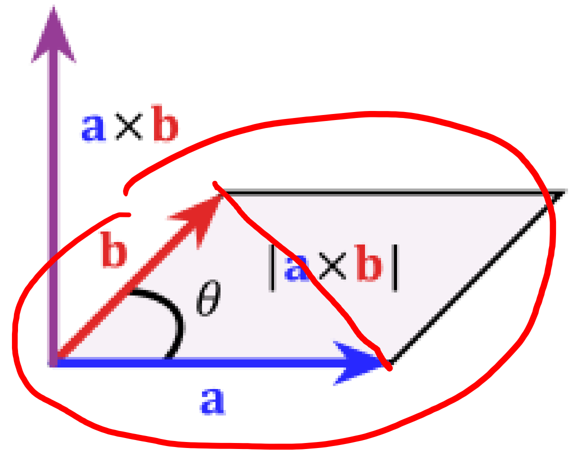
Cross product: length of a x b

- Parallelogram rule

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin(\theta)$$

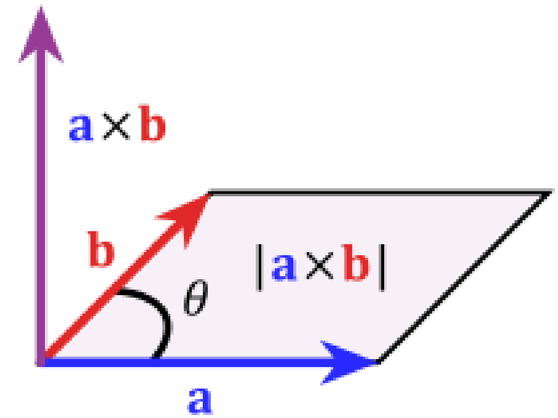
- Area of ~~mesh~~ triangle?
- When would cross product be zero?

$$|\mathbf{a} \times \mathbf{b}| = 0$$



$$\frac{|a \times b|}{2}$$

Cross product: length of a x b



- Parallelogram rule

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin(\theta)$$

- Area of mesh triangle?
- When would cross product be zero?

$$|\mathbf{a} \times \mathbf{b}| = 0$$

- Either a,b parallel, or either vector degenerate

a b

a,b $\theta = 0$ $\Rightarrow$ $a \times b = 0$

Cross product: computing, vector approach

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{bmatrix}$$

Cross product: computing, matrix approach

- Determinant of

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{bmatrix}$$

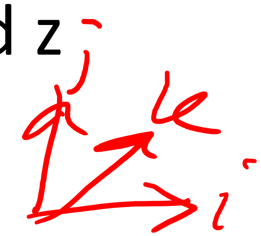
- Computed with 3 lower minors

$$\mathbf{a} \times \mathbf{b} = i \begin{bmatrix} a_y & a_z \\ b_y & b_z \end{bmatrix} - j \begin{bmatrix} a_x & a_z \\ b_x & b_z \end{bmatrix} + k \begin{bmatrix} a_x & a_y \\ b_x & b_y \end{bmatrix}$$

- with

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - cb$$

- i, j, k are unit vectors in directions of x, y and z axes
- i=<1,0,0>   j=<0,1,0>   k=<0,0,1>



Cross product: computing, matrix approach

- Determinant of

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{bmatrix}$$

$a = \langle 1, 1, 1 \rangle$

$b = \langle 2, 1, 1 \rangle$

$$a \times b = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

$= i * 0 - j(-1) + k \cdot -1$

$= \langle 0, 1, -1 \rangle$

$a = \langle 0, 1, 0 \rangle$ j

$b = \langle 0, 0, 1 \rangle$ k

$$a \times b = \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$= i * 1 - j * 0 + k\, 0$

$= \langle 1, 0, 0 \rangle$ i

What you should know after today

1. Cross product, right hand rule and sine rule
2. Computing cross product with determinant