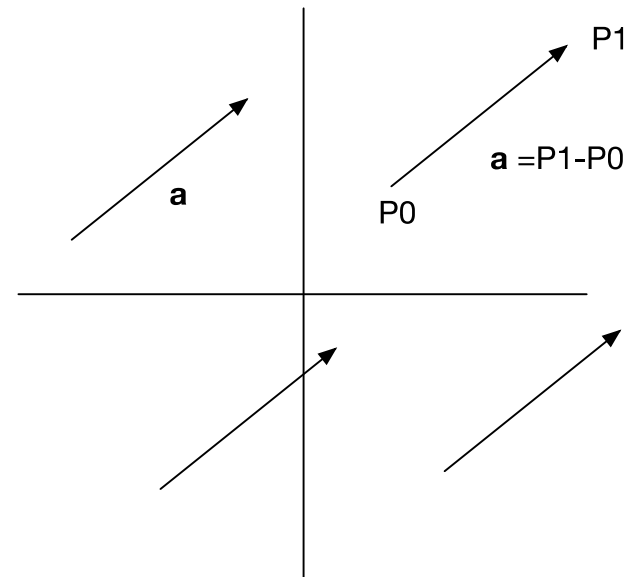
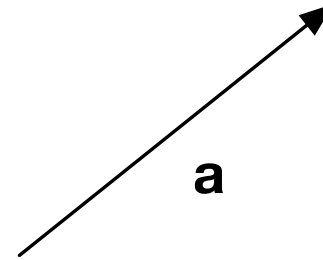


CMSC427
Geometry and
Vectors:
Homogenous
Coordinates

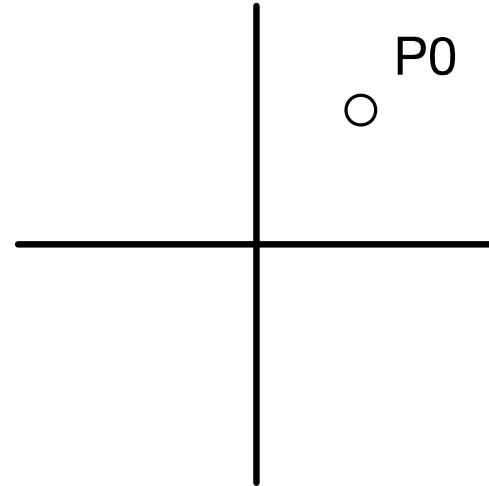
Vectors

- Direction and distance
- Describe
 - Difference between points
 - Speed, translation, axes
- Notation
 - In bold **a**
 - Angle brackets $\mathbf{a} = \langle x, y \rangle$
- Free vectors
 - No anchor point
 - Displacement, not location



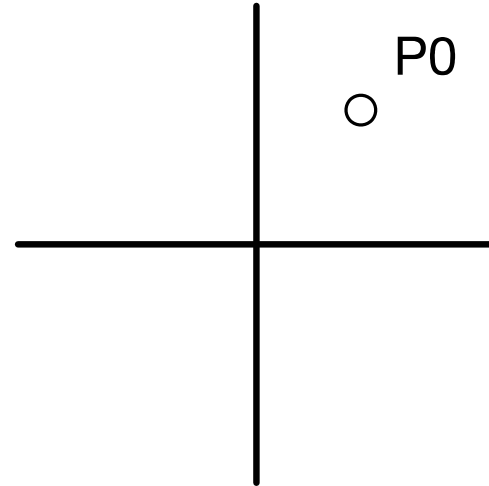
Points

- Location
- Describe
 - Position relative to origin
- Notation
 - In normal font
 - Parens $p = (x,y)$
- Not free
 - Locked in position
 - Not free to linearly combine



Points

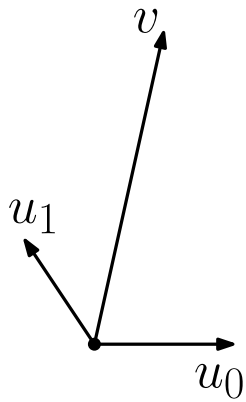
- Location
- Describe
 - Position relative to origin
- Notation
 - In normal font
 - Parens $p = (x,y)$
- Not free
 - Locked in position
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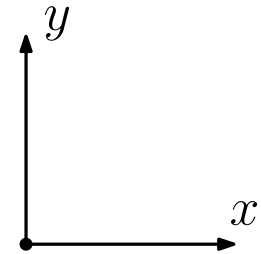
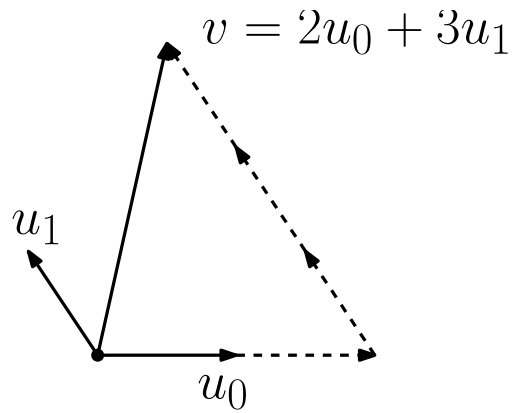
Linear combination of vectors

- Vectors (2D)

$$\vec{v} = \alpha_0 \vec{u}_0 + \alpha_1 \vec{u}_1$$



(a)



(b)

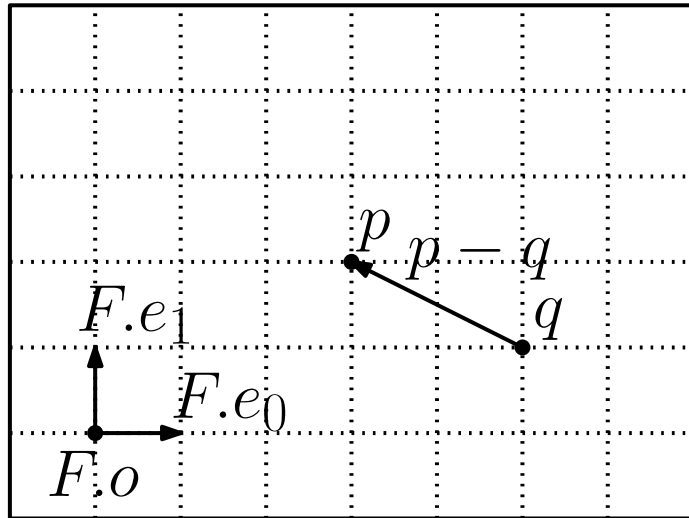
Coordinate system

- Add fixed point as origin (3D)

$$p = \alpha_0 \vec{u}_0 + \alpha_1 \vec{u}_1 + \alpha_2 \vec{u}_2 + o$$

Homogenous coordinates

- Add third scalar to coordinates



$$p_{[F]} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \quad q_{[F]} = \begin{pmatrix} 5 \\ 1 \\ 1 \end{pmatrix}$$

$$(p - q)_{[F]} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$$

Affine and convex combinations

Linear combinations of points vs. vectors

Point – point yields a

Vector – vector yields a ...

Point + vector yields a ...

Point + point yields a ...

Linear combinations of points vs. vectors

Point – point yields a ... vector

Vector – vector yields a ... vector

Point + vector yields a ... point

Point + point yields a ... ??? Not defined

Vectors are closed under addition and subtraction
Any linear combination valid

Points are not
Affine combination that sums to 0 yields vector
Affine combination that sums to 1 yields point
Convex combination yields point in convex hull

Moral: When programming w/ pts & vtrs, know the output type

What you should know

1. Notation for vectors $\langle x, y \rangle$ and pts (x, y)
2. Vectors and points are different data types
3. Coordinate system defined as vector plus origin
4. Use of fourth coordinate in homogeneous coordinates
5. How fourth coordinate value of 0 and 1 is preserved or created in convex and affine combinations