

CMSC427
Parametric surfaces:
computing normals



Computing normal vectors for mesh

- Approach 1: Cross product of numeric data
 - Find v1 and v2 from vertices (which?)
 - $N = v1 \times v2$
- Approach 2: Partial derivatives of parametric curve
 - Given vector $P(u,v) = < x(u,v), y(u,v), z(u,v) >$
 - Derive vectors $dU = dP(u,v)/du$ and $dV = dP(u,v)/dv$
 - $N = dU \times dV$

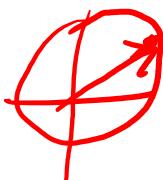
- Other approaches:

- Newell's method

- Gradient vector of implicit surface

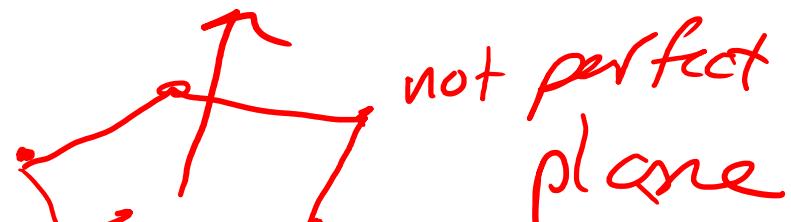
- Given implicit function $f(x,y,z)$

- Derive gradient $< df/dx, df/dy, df/dz >$



$$< x, y, z >$$

$$< 2x, 2y, 2z >$$

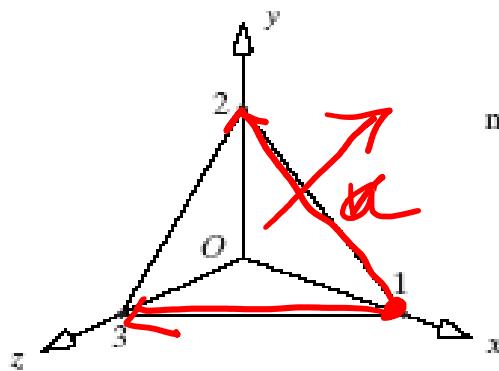


$$\underline{x^2 + y^2 + z^2 - R^2 = 0}$$



Example: Tetrahedron

a)



b)

| | | | | | |
|----------|---|-----|----|----|----|
| numVerts | 4 | 0 | 1 | 0 | 0 |
| pt | | 0 | 0 | 1 | 0 |
| numNorms | 4 | 577 | 0 | -1 | 0 |
| norm | | 577 | 0 | 0 | -1 |
| numFaces | 4 | 577 | -1 | 0 | 0 |
| face | | 3 | 3 | 3 | 3 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 | 0 | 2 | 2 | 3 | 3 |
| 2 | 0 | 2 | 1 | 3 | 2 | 3 | 2 | 3 | 3 |
| 3 | 0 | 1 | 1 | 2 | 2 | 0 | 3 | 0 | 3 |

✓

vector
vertices - P

$$\mathbf{a} = \mathbf{P}_2 - \mathbf{P}_1 \quad \mathbf{a} \times \mathbf{b}$$
$$\mathbf{b} = \mathbf{P}_3 - \mathbf{P}_1$$



Example: Cylinder

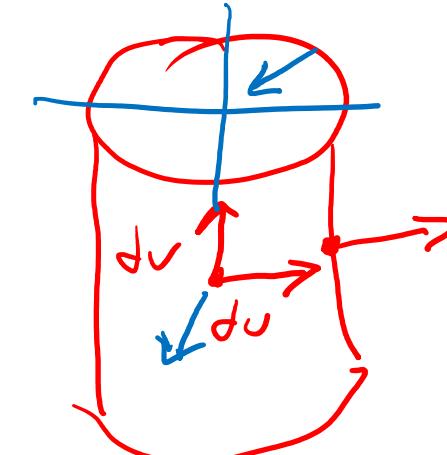
- Parametric cylinder (full derivation on Elms handout)

$$p(u, v) = \langle R \cos u, h*v, R \sin u \rangle$$



$$\frac{\partial p(u, v)}{\partial u} = \langle -R \sin u, 0, R \cos u \rangle$$

$$\frac{\partial p(u, v)}{\partial v} = \langle 0, h, 0 \rangle$$



$$n = \frac{\partial p(u, v)}{\partial u} \times \frac{\partial p(u, v)}{\partial v} = \langle -R \sin u, 0, R \cos u \rangle \times \langle 0, h, 0 \rangle$$

$$= \langle -h R \cos u, 0, -h R \sin u \rangle$$

$$\bar{n} = \frac{n}{\|n\|} = \langle \cos u, 0, \sin u \rangle$$

