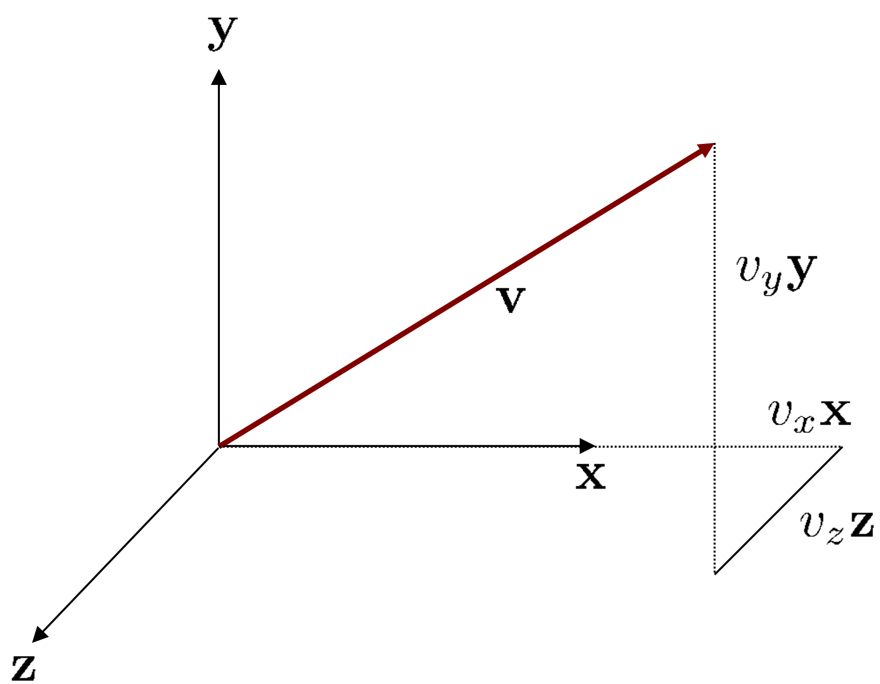
CMSC427 Transformations: Homogeneous coordinates (again)

Credit: slides 9+ from Prof. Zwicker

Vectors & coordinate systems

- Vectors defined by orientation, length
- Describe using three basis vectors

$$\mathbf{x}, \mathbf{y}, \mathbf{z}$$



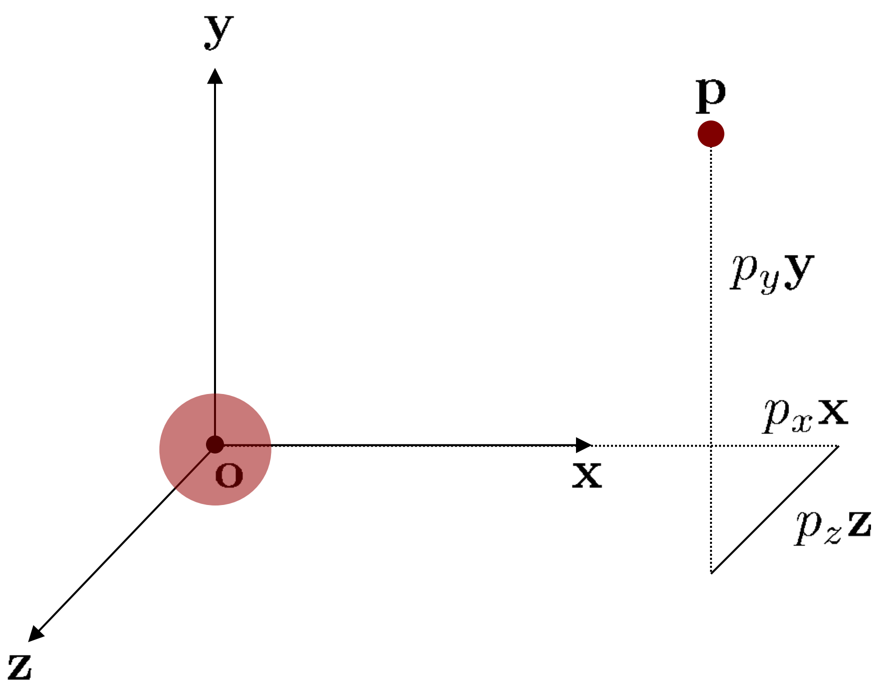
$$\mathbf{v} = v_x\mathbf{x} + v_y\mathbf{y} + v_z\mathbf{z}$$

Points in 3D

- How do we represent 3D points?
- Are three basis vectors enough to define the location of a point?

Points in 3D

- Describe using three basis vectors and reference point, origin



$$\mathbf{p} = p_x\mathbf{x} + p_y\mathbf{y} + p_z\mathbf{z} + \mathbf{o}$$

Vectors vs. points

- Vectors

$$\mathbf{v} = v_x\mathbf{x} + v_y\mathbf{y} + v_z\mathbf{z} + \boxed{0 \cdot \mathbf{o}} \qquad \begin{bmatrix} v_x \\ v_y \\ v_z \\ \boxed{0} \end{bmatrix}$$

- Points

$$\mathbf{p} = p_x\mathbf{x} + p_y\mathbf{y} + p_z\mathbf{z} + \boxed{1 \cdot \mathbf{o}} \qquad \begin{bmatrix} p_x \\ p_y \\ p_z \\ \boxed{1} \end{bmatrix}$$

- Representation of vectors and points using 4th coordinate is called homogeneous coordinates

Homogeneous coordinates

- Represent an affine space
  http://en.wikipedia.org/wiki/Affine_space
- Intuitive definition
  - Affine spaces consist of a vector space and a set of points
  - There is a subtraction operation that takes two points and returns a vector
  - Axiom I: for any point **a** and vector **v**, there exists point **b**, such that **(b-a) = v**
  - Axiom II: for any points **a, b, c** we have **(b-a)+(c-b) = c-a**

Affine space

Vector space,

http://en.wikipedia.org/wiki/Vector_space

- [xyz] coordinates
- represents vectors

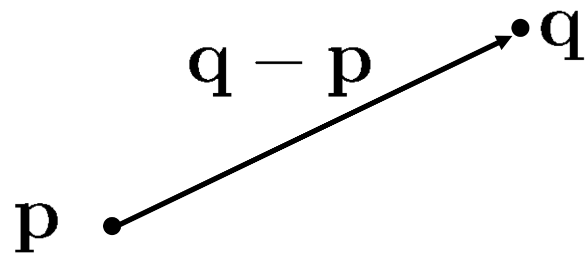
Affine space

http://en.wikipedia.org/wiki/Affine_space

- [xyz1], [xyz0] homogeneous coordinates
- distinguishes points and vectors

Homogeneous coordinates

- Subtraction of two points yields a vector



- Using homogeneous coordinates

$$\mathbf{p} = \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} \qquad \mathbf{q} = \begin{bmatrix} q_x \\ q_y \\ q_z \\ 1 \end{bmatrix} \qquad \mathbf{q} - \mathbf{p} = \begin{bmatrix} q_x - p_x \\ q_y - p_y \\ q_z - p_z \\ 0 \end{bmatrix}$$