CMSC427 Transformations: Concatenating

Credit: slides 9+ from Prof. Zwicker

Today

Transformations & matrices

- Introduction
- Matrices
- Homogeneous coordinates
- Affine transformations
- Concatenating transformations
- Change of coordinates
- Common coordinate systems

Concatenating transformations

- Build "chains" of transformations

$$\mathbf{M}_3, \mathbf{M}_2, \mathbf{M}_1 \in \mathbf{R}^{4\times 4}$$

- Apply $\mathbf{M}_1$ followed by $\mathbf{M}_2$ followed by $\mathbf{M}_3$

$$\mathbf{M} = \mathbf{M}_3\mathbf{M}_2\mathbf{M}_1$$

- Overall transformation is an affine transformation

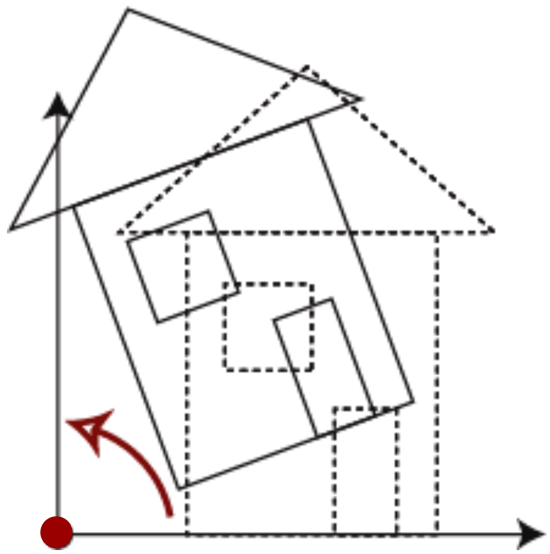
$$\mathbf{p}' = \mathbf{M}_3\mathbf{M}_2\mathbf{M}_1\mathbf{p} = \mathbf{M}\mathbf{p}$$

- Multiplication on the left

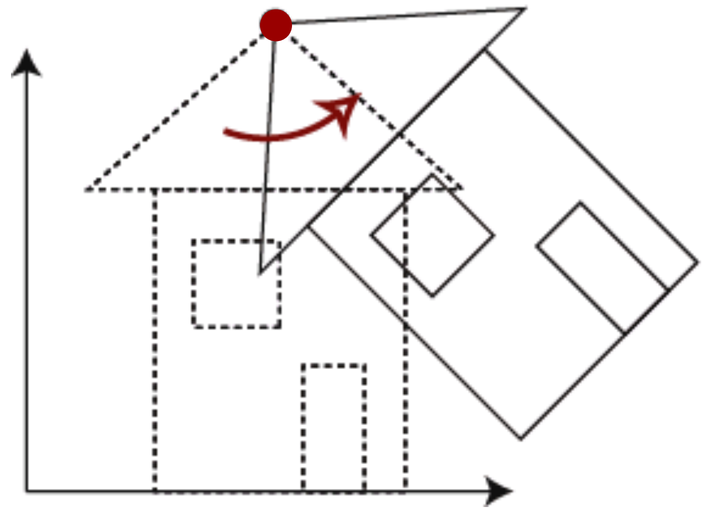
Concatenating transformations

- Result depends on order because matrix multiplication not commutative
- Thought experiment
  - Translation followed by rotation vs. rotation followed by translation

Rotating with pivot

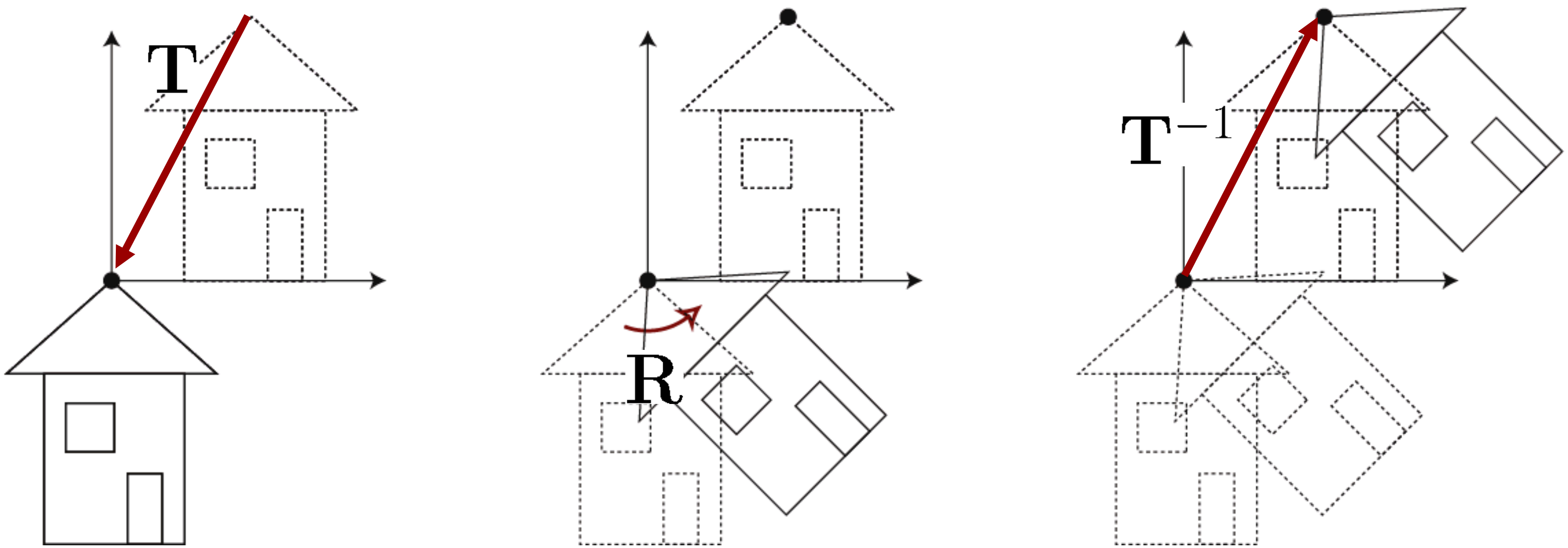


Rotation around origin



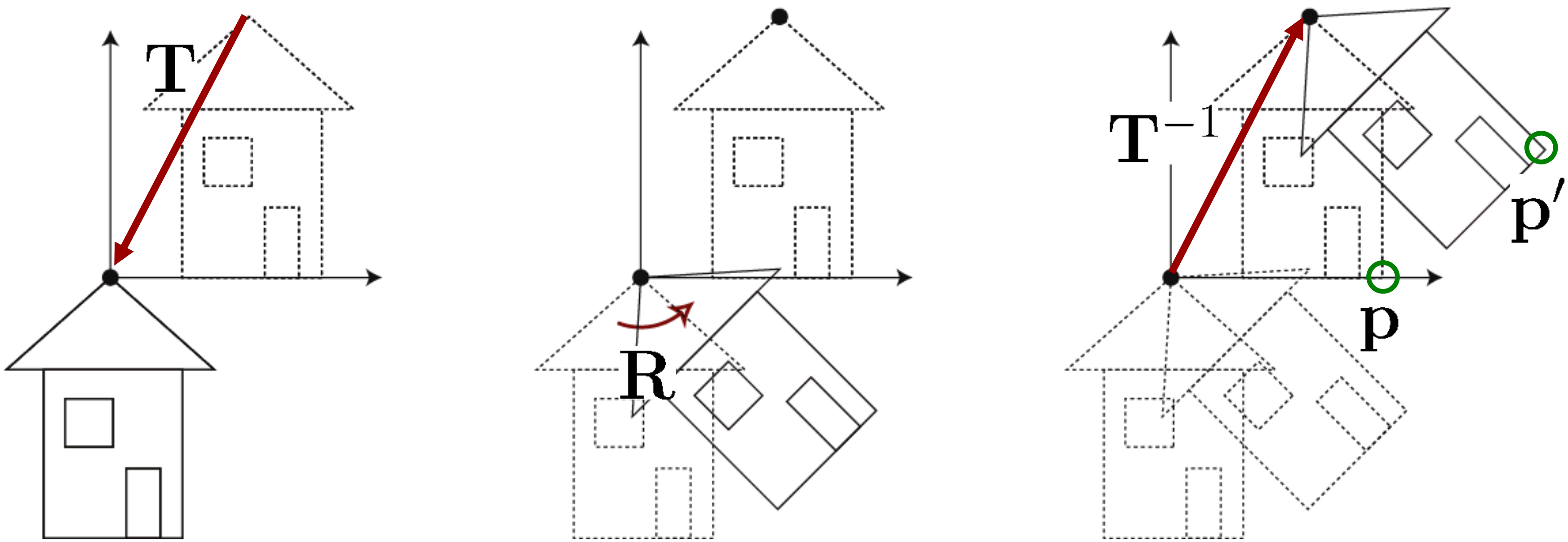
Rotation with pivot

Rotating with pivot



1. Translation $\mathbf{T}$ 2. Rotation $\mathbf{R}$ 3. Translation $\mathbf{T}^{-1}$

Rotating with pivot



1. Translation $\mathbf{T}$ 2. Rotation $\mathbf{R}$ 3. Translation $\mathbf{T}^{-1}$

$$\mathbf{p}' = \mathbf{T}^{-1}\mathbf{R}\mathbf{T}\mathbf{p}$$

Concatenating transformations

- Arbitrary sequence of transformations

$$\mathbf{p}' = \mathbf{M}_3\mathbf{M}_2\mathbf{M}_1\mathbf{p}$$

$$\mathbf{M}_{total} = \mathbf{M}_3\mathbf{M}_2\mathbf{M}_1$$

$$\mathbf{p}' = \mathbf{M}_{total}\mathbf{p}$$

- Note: associativity

$$\mathbf{M}_{total} = (\mathbf{M}_3\mathbf{M}_2)\mathbf{M}_1 = \mathbf{M}_3(\mathbf{M}_2\mathbf{M}_1)$$

So either is valid

T=M3.multiply(M2); Mtotal=T.multiply(M1)

or

T=M2.multiply(M1); Mtotal=M3.multiply(T)