CMSC427 Fall 2020
Hw 3 – Vectors SOLUTIONS

1. Warm up. Is the angle between v1=<2,3,2> and v2=<-3,1,2> obtuse or acute?

Solution: Since v1•v2 = 1 > 0, the angle is less than 90 degrees (acute)

2. For what values of α is **a** orthogonal to **b** − α**a**? What about the special case where ||**a**|| = 1? The case where ||**a**|| = 0?

So***lution:*** For **a** orthogonal to **b** – α**a**, we need **a•(b** – α**a) = 0**
which gives **a•b** – α**a•a = 0** and α = **a•b/a•a.**

If ||**a**|| = 1, then α = **a•b**
If ||**a**|| = 0, then α is not defined (if the length of **a** is zero, then **b** – α**a = b**, and if **b** is arbitrary, then **a** and **b** need not be orthogonal.

3. Given a helix curve in parametric vector form as P(t) = < r*cos(t), h*t, r*sin(t)>, what is the tangent vector T to the curve? What is the normal vector N (which is T')? And what is the binormal vector B (which is T x N)?

Solution: Given P(t) = < r*cos(t), h*t, r*sin(t) >

We have P'(t) = <-r*sin(t), h, r*cos(t) >

P''(t) = <-r*cos(t), 0, -r*sin(t) >

And now P'(t) x P''(t) =

$$\begin{vmatrix} i & j & k \\ -r*\sin(t) & h & r*\cos(t) \\ -r*\cos(t) & 0 & -r*\sin(t) \end{vmatrix}$$

= <-rh*sin(t), -r^2*(cos^2+sin^2), rh*cos(t) >

= < -rh*sin(t), r^2, rh*cos(t) >

4. For the previous problem the appropriate range of t isn't important – for actually drawing a helix, picking the range of t to appropriate scale h and the number of twists is important. Redo the helix equation so as t goes from 0 to 1, the helix makes N full turns and rises to a height h.

Solution: Given the height h, radius r, and the number of turns N, the equation would be:

P(t) = < R*cos(2πNt), ht, R*sin(2πNt) >

5. For the lecture example for the midpoint of a triangle, calculated by first blending the line segment between two points P0 and P1, and then blending that equation with the third point P2,

show that (a) if you hold s constant then varying t sweeps out a line, and (b) those lines of constant s are parallel to the line from P0 to P1.

Solution: The equation for the blending is developed as following:

Blending P0 and P1 gives the following, with v = P1-P0

$m(t) = (1-t)P0 + tP1 \quad = P0 - tP0 + tP1 = P0 + t(P1-P0) = P0+tv$

Blending m(t) and P2 gives

$$
\begin{aligned}
m0(s,t) &= (1-s)m(t) + sP2 \\
&= (1-s)(P0+tv) + sP2 \\
&= P0 + tv - s(P0 + tv) + sP2 \\
&= (P0 -sP0 + sP2) + (1-s)tv \\
&= P + t\,(cv) \qquad \text{with } P=(P0 -sP0 + sP2),\ c = (1-s)
\end{aligned}
$$

Assuming s is constant, then P is a fixed point, and (1-s) is constant so cv is scaled version of v.

Since m0(s,t) = P + t(cv) with s constant, then m0(s,t) is a line
Since each of the lines generated as we change s is in the direction (cv) or v,
the lines are all parallel (two parametric lines are parallel if they have the same vector).

The key to the solution is rewriting the blending equation in terms of the vector v.

6. If you have a • (b x c) = 0, what does it mean for the relationship of the three vectors?

Solution: It means they are co-planar and don't span 3D space. Since b x c is normal to both b and c, then if a is normal to b x c then it is in the same plane as b and c. A single plane is defined if two or three of a, b and c are not degenerate vectors (equal to the zero vector). Otherwise the three vectors define a single plane. a•(b x c) is called the triple scalar product.

7. Find the normal vector to the triplets below, if it exists:

a) P1=(1,1,1), P2=(1,2,1), P3=(3,0,4)

b) P1=(8,16,2), P2=(-8,-16,-2), P3=(4,8,1)

Solution:

a) $v1 = P2-P1= \langle 0,1,0\rangle \qquad v2 = P3-P1 = \langle 2,0,3\rangle$

and $v1 \times v2 = \langle 3, 0, -2\rangle$

b) $v1 = P2-P1 = \langle -16, -32, -4 \rangle \quad v2 = P3-P1 = \langle -4, -8, -1 \rangle$

and $v1 \times v2 = \langle 0, 0, 0 \rangle$