SOLUTIONS

CMSC427 Midterm practice questions

1. Give code plus the openGL vertex calls needed to draw the following parametric curve, given the appropriate variables t and R.   P(t) = (t cos 2πt, sin 2πt, 2t)

```
beginShape();
for (float t=0; t<1; t+=inc) {
    float x = t*cos(2πt)
    float y = sin 2πt
    float z = 2t
    vertex(x,y,z);
endShape();
```

2. Compute the Frenet frame for the parametric curve $p(t) = < t^3, t, \sin t >$
Don't normalize for this exercise.

$$p(t) = \langle t^3, t, \sin t \rangle$$

$$T = p'(t) = \langle 3t^2, 1, \cos t \rangle$$

$$N = p''(t) = \langle 6t, 0, -\sin t \rangle$$

$$B = p'(t) \times p''(t) = \begin{vmatrix} i & j & k \\ 3t^2 & 1 & \cos t \\ 6t & 0 & -\sin t \end{vmatrix}$$

$$= \langle -\sin t, 3t^3 \sin t + 6t \cos t, 6t \rangle$$

3. Which of the following pairs of transformations do *not* commute?

A. Non-uniform scale and rotation in 2 or 3D.

$$\begin{bmatrix} s & 0 \\ 0 & t \end{bmatrix}\begin{bmatrix} \cos & -\sin \\ \sin & \cos \end{bmatrix} = \begin{bmatrix} s\cos & -s\sin \\ t\sin & t\cos \end{bmatrix}$$

Do not.

$$\begin{bmatrix} \cos & -\sin \\ \sin & \cos \end{bmatrix}\begin{bmatrix} s & 0 \\ 0 & t \end{bmatrix} = \begin{bmatrix} s\cos & -t\sin \\ s\sin & t\cos \end{bmatrix}$$

off diagonal terms not equal.

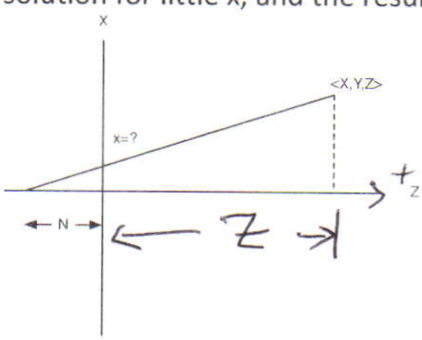
B. Two rotations around the origin in 2D.

These commute

C. Two general rotations in 3D.

These commute

4. From this diagram set up the perspective equation for *x*. Here N is the offset of the eye point back from the origin – useful if you'd like to take the eye point to negative infinity to see what happens. The image plane is the X axis. Give the similar triangle equation, the perspective solution for little x, and the resulting matrix. (This is similar to previous homework).



$$\frac{x}{N} = \frac{X}{N+Z}$$

$$x = \frac{NX}{N+Z}$$

$$M_p = \begin{bmatrix} N & 0 & 0 & 0 \\ 0 & N & 0 & 0 \\ 0 & 0 & N & 0 \\ 0 & 0 & 1 & N \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/N & 1 \end{bmatrix}$$

BOTH WORK

5. Given the input data below, compute the camera coordinate system xc, yc and zc, and the camera matrix.

at = (1,0,0) lookAt = (0,0,0) up = < 0, 1, 0>

$$Z_c = at - lookAt = (1,0,0) - (0,0,0)$$
$$= \langle 1,0,0 \rangle$$

$$X_c = UP \times Z_c = \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = \cancel{\langle 0,0,0 \rangle} \; \langle 0,0,-1 \rangle$$

$$Y_c = Z_c \times X_c = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{vmatrix} = \langle 0,1,0 \rangle$$

$$d = \langle x_c \cdot at, y_c \cdot at, z_c \cdot at \rangle$$
$$= \langle \langle 0,0,-1 \rangle \cdot \langle 1,0,0 \rangle, \langle 0,1,0 \rangle \cdot \langle 1,0,0 \rangle,$$
$$\langle 1,0,0 \rangle \cdot \langle 1,0,0 \rangle \rangle$$
$$= \langle 1, 0, 0 \rangle$$

$$M_c = C^{-1} = \begin{bmatrix} x_c & -dx \\ y_c & -dy \\ z_c & -dz \\ 0\,0\,0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6. If there is no rotation in the position of the camera, how does the camera matrix relate to the input values of at, lookAt and up?

The camera matrix is a rotation plus a translation

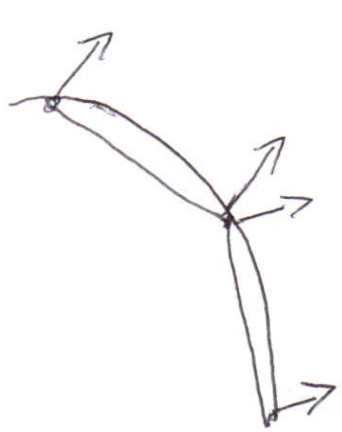
$$\begin{bmatrix} R & & & T \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -d_x \\ 0 & 1 & 0 & -d_y \\ 0 & 0 & 1 & -d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -at.x \\ 0 & 1 & 0 & -at.y \\ 0 & 0 & 1 & -at.z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

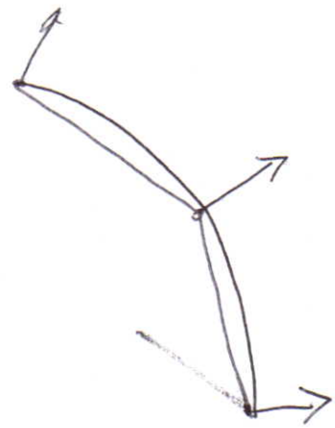
Since $d = R \cdot at$, and $R = I$,

we have $d = at$

7. What is the difference between facet and true normals, and why we assign normals to each vertex rather than each face?



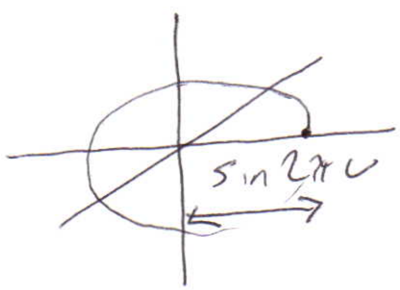
facet normals are normal to the flat face



true normals are normal to the local approximated surface

8. Given the following 2D curve in the x-y plane, give the parametric surface of revolution around the y-axis.

$$P(u) = \left\langle \sin 2\pi u \quad u^2+1 \quad 0 \right\rangle$$



$$P(u,v) = \langle (\sin 2\pi u)\cos 2\pi v,\ u^2+1,\ (\sin 2\pi u)(\sin 2\pi v) \rangle$$

take $P(u)_x$ as the radius of a circle

$$(P(u)_x \cos 2\pi v,\ P(u)_x \sin 2\pi v)$$

9. *Shading models*. What is the effect of increasing p (or f) in the shading equation (the exponent for the specular component)?

we have

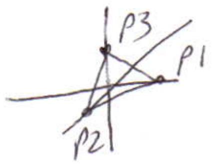
$$c_s = k_s (R \cdot e)^p c_l$$

increasing p sharpens the curve and tightens the specular highlight

10. Given the following data (original version of this problem had variables from a different textbook):

p1 = (1,0,0)	light at = (2,0,2)	cl_d = 100	ka = 0.2
p2 = (0,0,1)	eye at = (2,2,0)	cl_s = 80	kd = 0.5
p3 = (0,1,0)	f = p= 2	cl_a = 40	ks = 0.5

Compute the geometric elements (L,R,e,n) for vertex p1, for Phong specular model.
Compute the ambient, diffuse and specular components
Compute the final value of I = cd+cs+ca (diffuse plus specular plus ambient)



The normal to the facet is $\langle 1,1,1 \rangle$

$\bar{n} = \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$

$L = \text{light at} - p1 = (2,0,2) - (1,0,0) = \langle 1,0,2 \rangle$

$e = \text{eye at} - p1 = (2,2,0) - (1,0,0) = \langle 1,2,0 \rangle$

$\bar{L} = \langle \frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}} \rangle \quad \bar{e} = \langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0 \rangle$

$R = 2(\bar{L} \cdot \bar{n})\bar{n} - \bar{L} = 2(\langle \frac{1}{\sqrt{3}}, 0, \frac{2}{\sqrt{5}} \rangle \cdot \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle)$

$\quad \times \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle - \langle \frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}} \rangle$

$\quad = \langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0 \rangle$

$\text{diffuse} = kd * (n \cdot L) * cl\_d$

$\quad = kd * \langle \frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}} \rangle \cdot \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle * cl\_d$

$\quad = 0.5 * (\frac{1}{\sqrt{15}} + \frac{2}{\sqrt{15}}) * 100 = 50 * \frac{\sqrt{3}}{\sqrt{5}}$

$\text{specular} = ks * cl\_s * (\bar{R} \cdot \bar{e})^p$

$\quad = 0.5 * 80 * (\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0 \rangle \cdot \langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0 \rangle)$

$\quad = 0.5 * 80 * 1 = 40$

$\text{ambient} = ka * cla = 0.2 * 40 = 8$