SOLUTIONS

CMCS427 Midterm prep and sample questions

Questions on the quiz will be from Hw2 and Hw3, with some from basic transforms.

1. Given the two vectors <3,2,1> and <3,-1,4>, is the angle between them equal to, greater than or less than 90 degrees? What is the fastest way to answer this?

$a = \langle 3,2,1\rangle$ $\quad a \cdot b > 0 \Rightarrow$ angle < 90

$b = \langle 3,-1,4\rangle$ $\quad a \cdot b = 0 \Rightarrow$ angle $= 0$

$a \cdot b < 0 \Rightarrow$ angle > 90

$a \cdot b = 3^2 - 2 + 4 = 11 > 0 \Rightarrow$ angle less than 90°

2. Considering the two vectors in the previous question, exactly how would you compute the angle theta between them? Set up the formula but don't compute any inverse trig functions.

$$a \cdot b = |a||b|\cos\theta$$

so $$\cos\theta = \frac{a \cdot b}{|a||b|} \Rightarrow \theta = \arccos\left(\frac{\langle 3,2,1\rangle \cdot \langle 3,-1,4\rangle}{|\langle 3,2,1\rangle||\langle 3,-1,4\rangle|}\right)$$

3. Give matrices to rotate, scale and translate a shape around its center of mass. Assume the center of mass is at (x,y,z), the scale is uniform and value s, and the rotation is around the Z axis with angle t. Give the matrices, and then give the composition of matrices you'd need to perform the operations in the right order. Do not calculate out the final matrix. Given that matrix multiplications can be non-commutative, are there more than one composition that would work?

$$M_{R\theta}^{z} = \begin{bmatrix} \cos t & -\sin t & 0 & 0 \\ \sin t & \cos t & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad M_S = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\underline{\theta = t}$

$$M_T = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

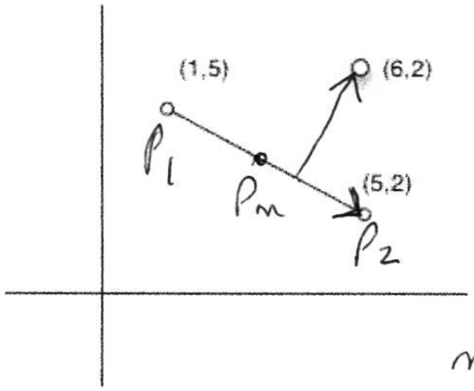
CAN YOU CHANGE THE ORDER? YES - THE SCALE AND ROTATE MATRICES COMMUTE

Composition of matrices:

$$M * P = M_T * M_{R\theta}^{z} * M_S * M_T^{-1} * P$$

First translate center of mass to origin, then scale, then rotate, finally translate back.

4. Given the two points (1,5) and (5,2), in the diagram below, give the following:



$V = P_2 - P_1$

$= (5,2) - (1,5) = \langle 4,-3\rangle$

$V^\perp = \langle 3,4\rangle$ perp vector

midpt $P_M = \frac{P_2 + P_1}{2} = \frac{(6,7)}{2} = (3,3.5)$

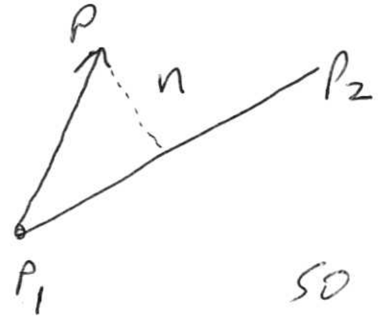
a) A parametric equation for the perpendicular bisector line.

$P(t) = P_m + t\,V^\perp$ midpoint + t×perp-vector

$= (3,3.5) + t\langle 3,4\rangle$

b) The point-normal form of the same line.

The normal $n = v^\perp$, or normalized, $\bar{n} = \frac{\langle 3,4\rangle}{\sqrt{9+16}} = \frac{\langle 3,4\rangle}{5}$



A point P is ~~no~~ on the line if $(P - P_1)\cdot n = 0$

so $((x,y) - (1,5))\cdot\langle 3,4\rangle = 0$

e. What is the distance from (6,-5) to the line?

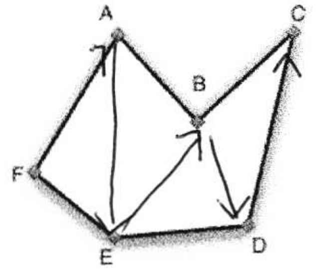
If you have the perp vector, normalized,

Then $d = |(P - P_1)\cdot\bar{n}| = \left|((6,2) - (1,5))\cdot\frac{\langle 3,4\rangle}{5}\right|$

$= \left|\langle 5,-3\rangle\cdot\frac{\langle 3,4\rangle}{5}\right|$

$= \left|\frac{15-12}{5}\right| = \frac{3}{5} = 0.6$

5. Give the sequence of vertices needed to draw this polygon using GL_TRIANGLE_STRIP.



F A E B D C

6. Give a fragment of Processing needed to draw a parametric ellipse, given the appropriate variables t, A and B. We have the equations x = A cos(t), y = B sin(t).

```
for (float t=0; t≤2π; t+=inc) {
  float x= A*cos(t);
  float y= B*sin(t);
  point(x,y);
}
```

OR

```
beginShape();
```

rest remains the same

```
vertex(x,y);
endShape();
```

7. Determine whether or not the parametric curve in the previous question satisfies the implicit equation

$$\frac{x^2}{A}+\frac{y^2}{B}=1$$

Substitute $x=A\cos(t)$, $y=B\sin(t)$ into equation:

$$\frac{(A\cos(t))^2}{A}+\frac{(B\sin(t))^2}{B}=1$$

$$\frac{A^2}{A}\cos^2 t+\frac{B^2}{B}\sin^2 t=1$$

$$A\cos^2 t + B\sin^2 t = 1$$

The equation <u>does not</u> satisfy the implicit equation unless $A=B=1$.