CMSC427 Fall 2020
Exercises for Tuesday and Thursday, Nov. 10^{th} and 12^{th}
Modeling – parametric curves and surfaces

This week we will cover piecewise parametric curves such as Hermite, Catmull Rom and Bezier, and different construction methods for deriving them – plus the generalization to surface patches.

You'll be expected to know Hermite, Catmull Rom and Bezier – we'll look at B-splines, but only quickly and mainly to explain global vs local control for control points. For surfaces, you should know bilinear and Bezier. We may look at subdivision approaches to surface generation as time allows this week. We'll focus on interpolating curves, leaving approximating curves aside.

Objectives. You should be able to:
1. Explain why piecewise parametric curves are an alternative to basic curves like circles, ellipses, and similar.
2. Explain why cubic curves are generally good enough for modeling given shape and psychological reasons.
3. Use the Hermite curve solution approach to derive parametric coefficients given constraints similar, but not identical, to the four point case we gave.
4. Work with multiple versions of the Hermite curve: cubic coefficients (version 1 in the handout), matrix form (version 2), and blending functions (version 3), and explain the usefulness of each.
5. Show the Hermite blending functions, or another set, represent an affine combination.
6. Given blending functions, explain how they weight the control points and tangents, and how the curve varies given a particular set of blending functions.
7. Derive the data matrix (the matrix H in the handout) for a given set of constraints.
8. Explain the convex hull property for Bezier curves.
9. Convert from matrix, to coefficient, to blending function representation of curves.
10. Sketch and explain the de Casteljau algorithm for Bezier curves.
11. Derive the Bezier coefficients by de Casteljau.
12.Derive a bilinear patch (we have done this before, will review).
13. Derive a patch by interpolation from two curves, that may not be linear – generalize the bilinear patch.
14. Sketch out the derivation of a Bezier patch by interpolating Bezier curves.

Elms modules:
See Week 10, lectures for Tuesday and Thursday Nov. 10^{th} and 12^{th}.

Readings:
The handout CMSC427PiecewiseParametricNotes.pdf explains the basic terminology and derivations for curves.

https://www.scratchapixel.com/lessons/advanced-rendering/bezier-curve-rendering-utah-teapot

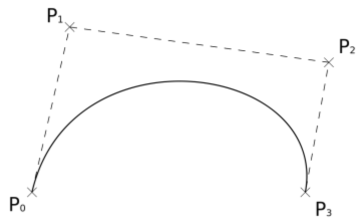
http://ibiblio.org/e-notes/Splines/bezier.html

https://webglfundamentals.org/webgl/lessons/webgl-3d-geometry-lathe.html

http://math.hws.edu/eck/cs424/graphicsbook2018/demos/c2/cubic-bezier.html

Exercises:

1. The Bezier curve can be defined with the four constraints below – the tangent vectors are defined by P0 to P1, and P2 to P3. Using these constraints derive a basis matrix for the curve with



P0 = P0
P1 = P3
T0 = 3(P1-P0)
T1 = 3(P3-P2)

2. Show the Hermite blending functions h00, h01, h10, h11 are an affine combination (and why they are not a convex combination). (BTW, Blending functions for some curves are.)

3. Give three points P0, P1 and P2, give the derivation of a quadratic Bezier curve. Interpolate P0 and P2, and use P1 to define the tangents through P0 and P2. Use

4. Based on (3), give the blending functions and determine if they are an affine and/or convex combination.