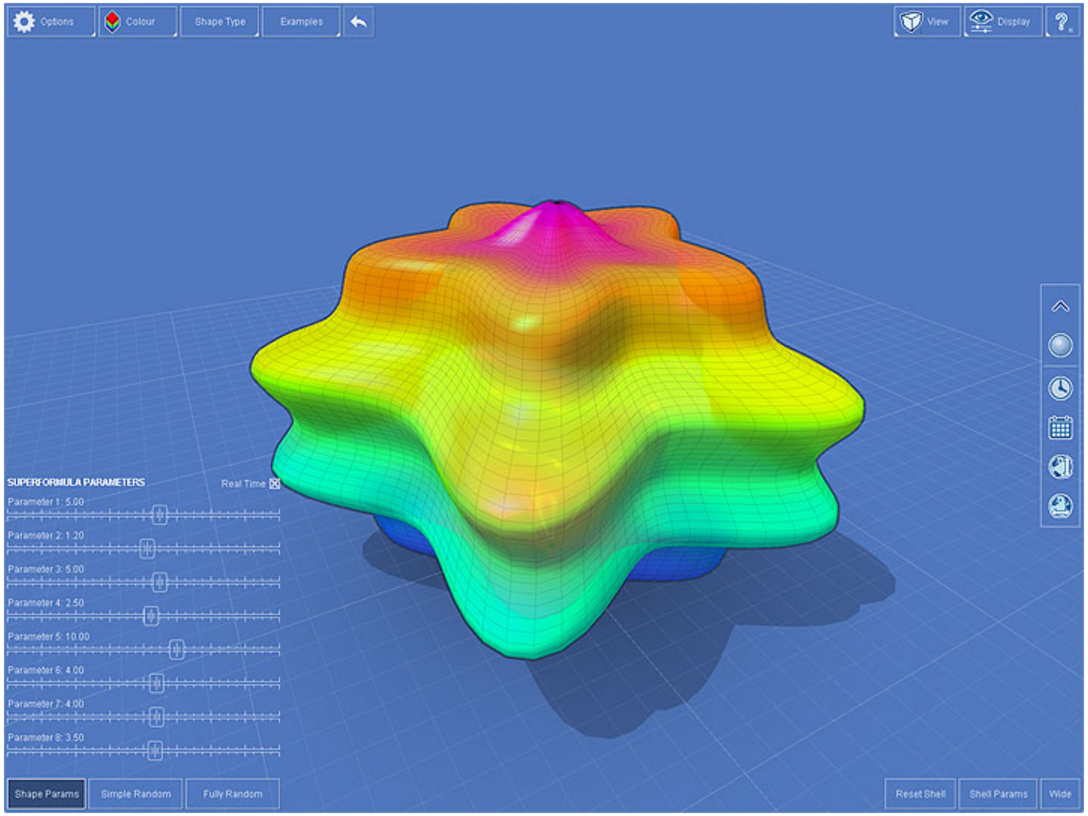
CMSC427
Fractals

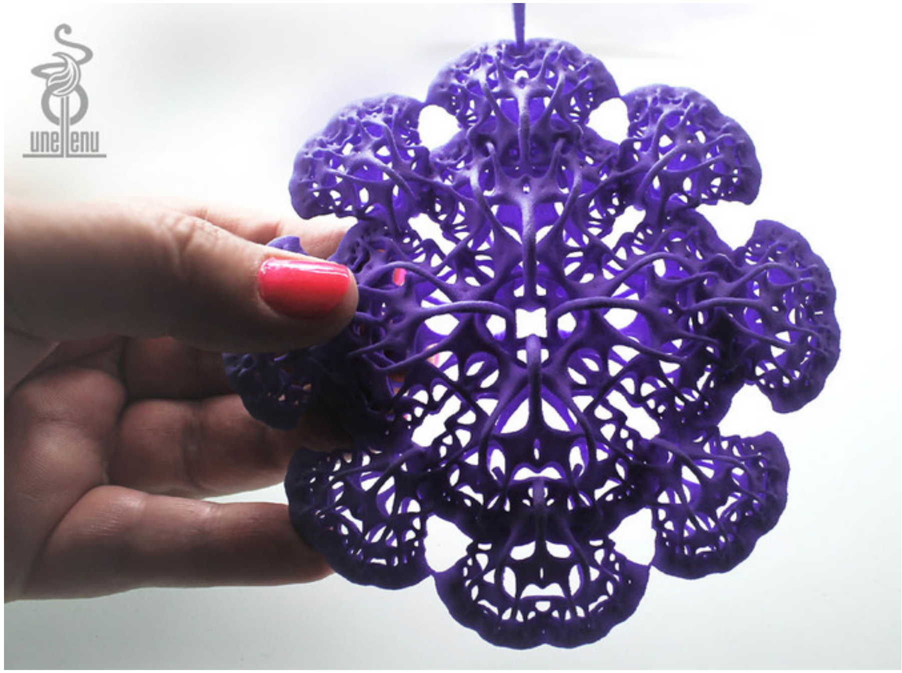
Parametric surfaces

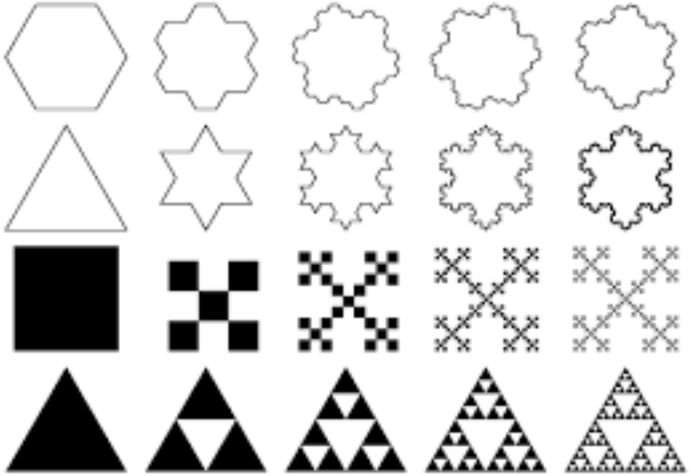
- Typically
  - Smooth
  - Compact

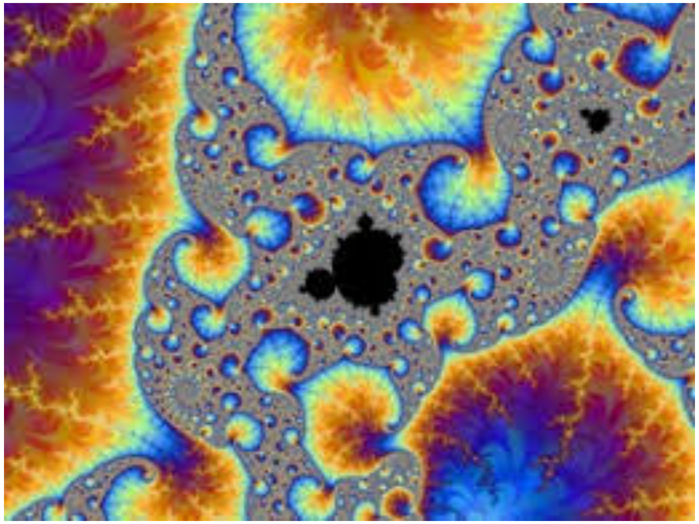


  - Andrew Marsh
  - From 1st day

More complex patterns and shapes?

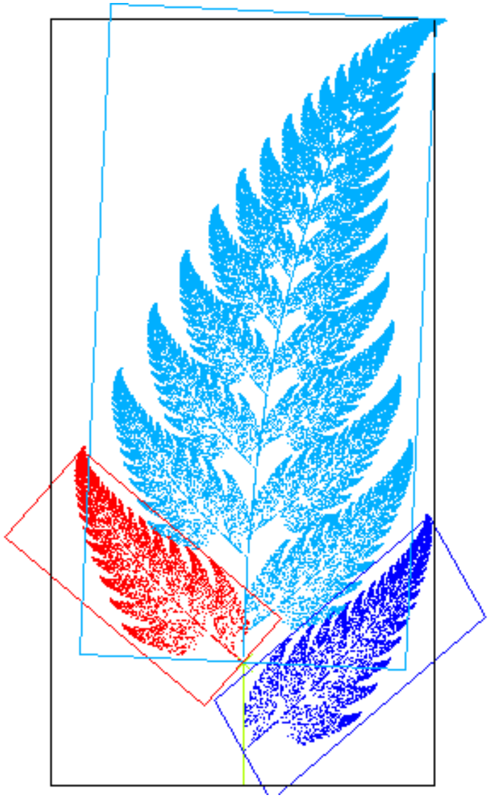




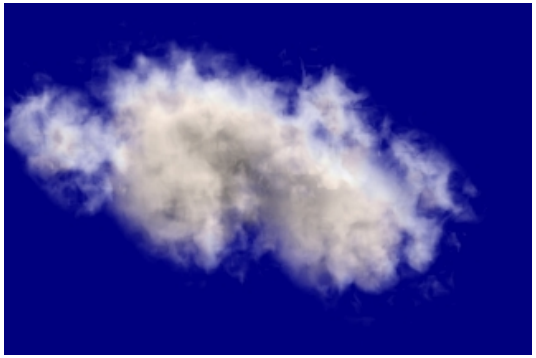


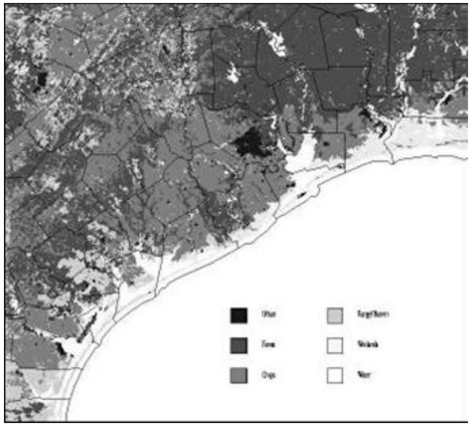
Fractals

- Class of shapes characterized by recursive structure
- Self-similarity
  - Parts are similar to each other and the whole

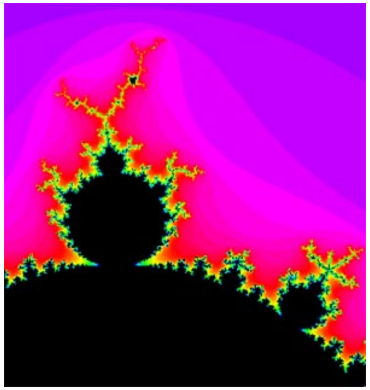


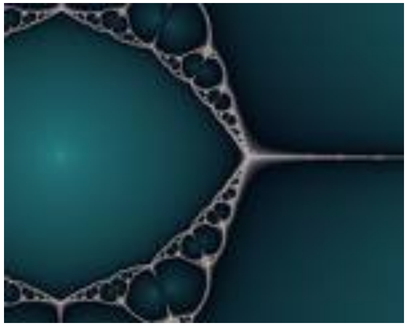
Self-similarity in nature - again





Artificial fractals

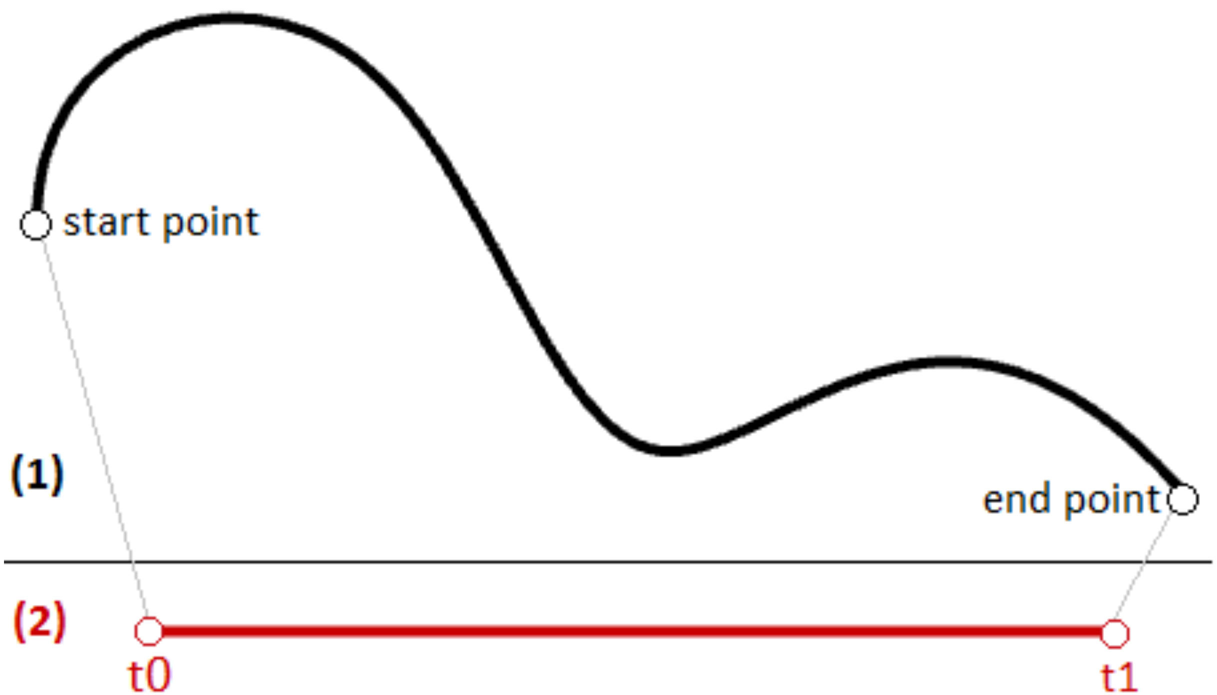


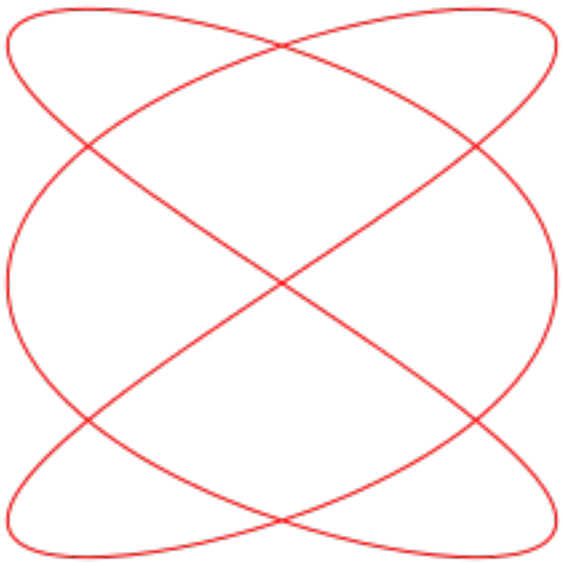


- fractal cow???

Dimensionality of curves and surfaces

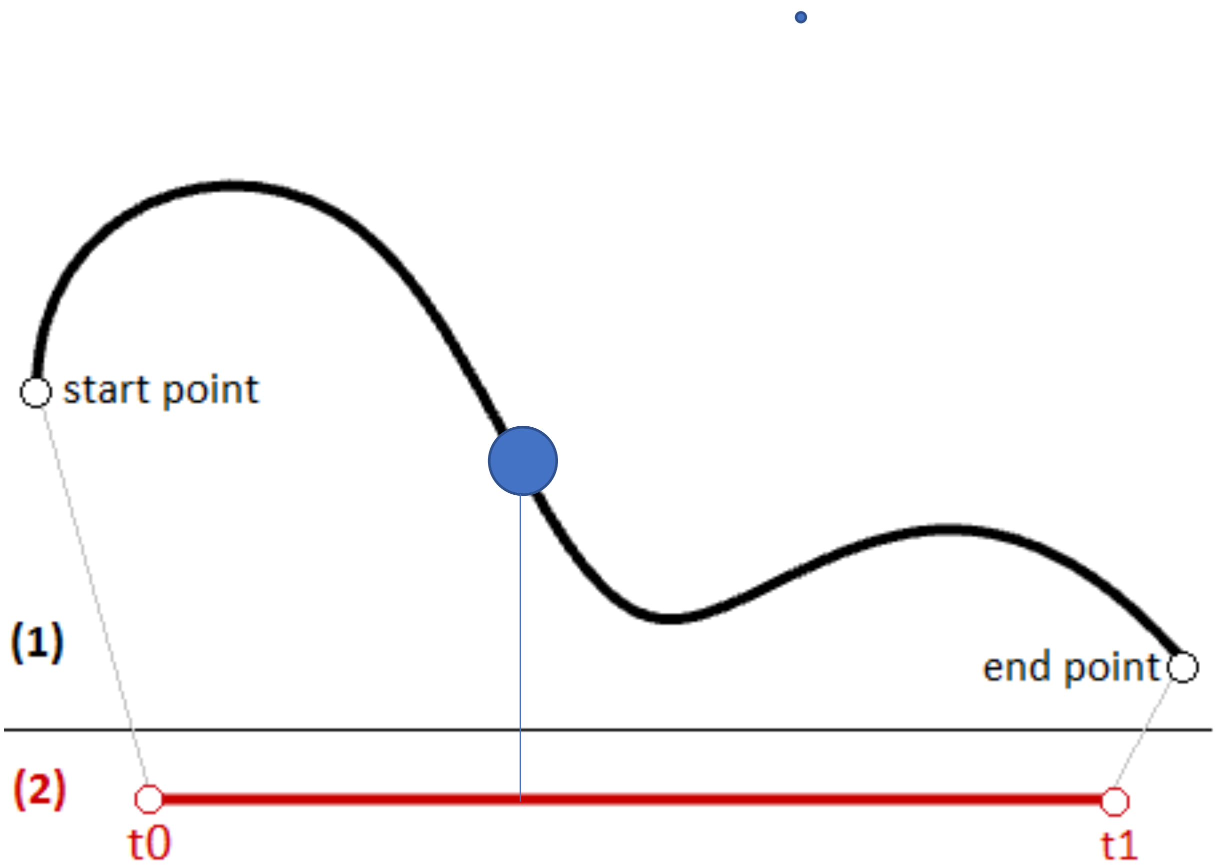
- How many dimensions is a curve?

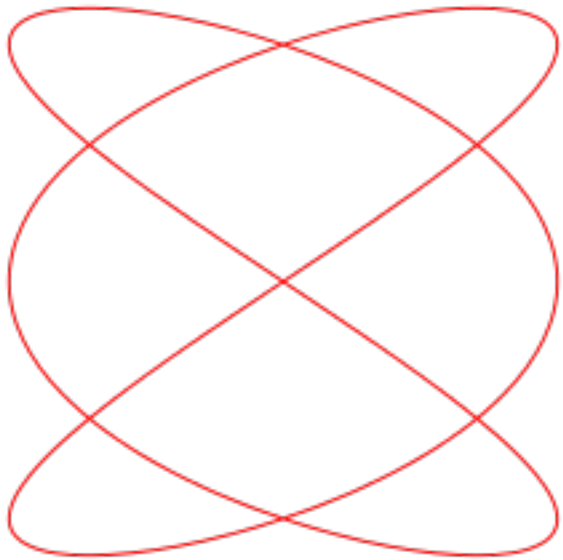




Dimensionality of curves and surfaces

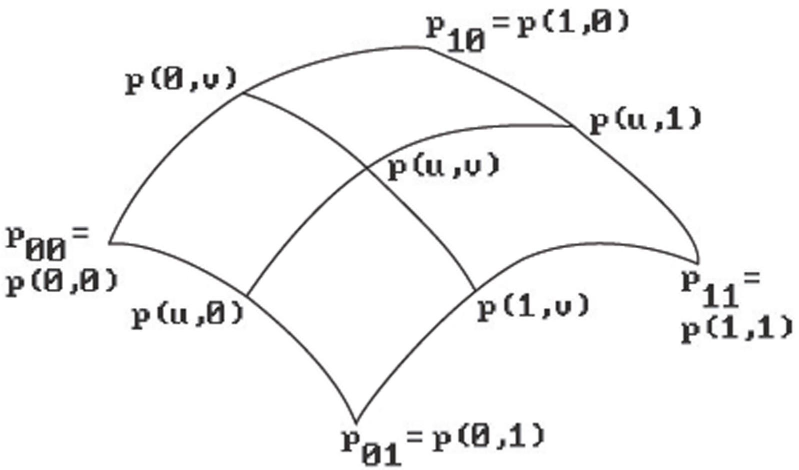
- How many dimensions is a curve?
- 1
- One variable describes where you are





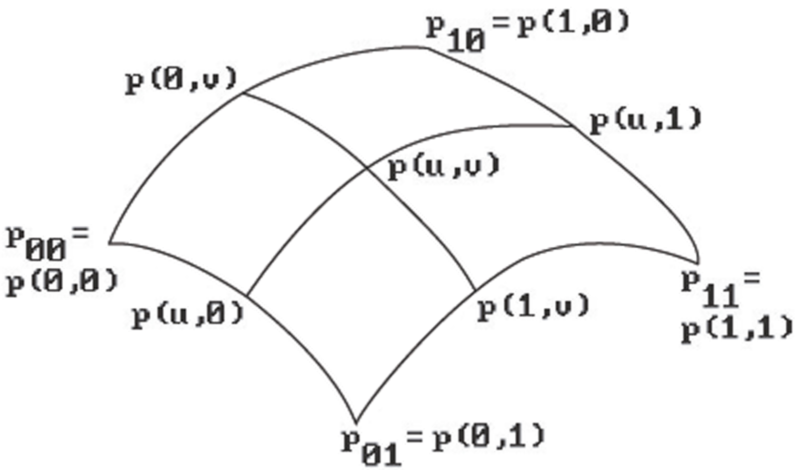
Surface?

- Number of dimensions?



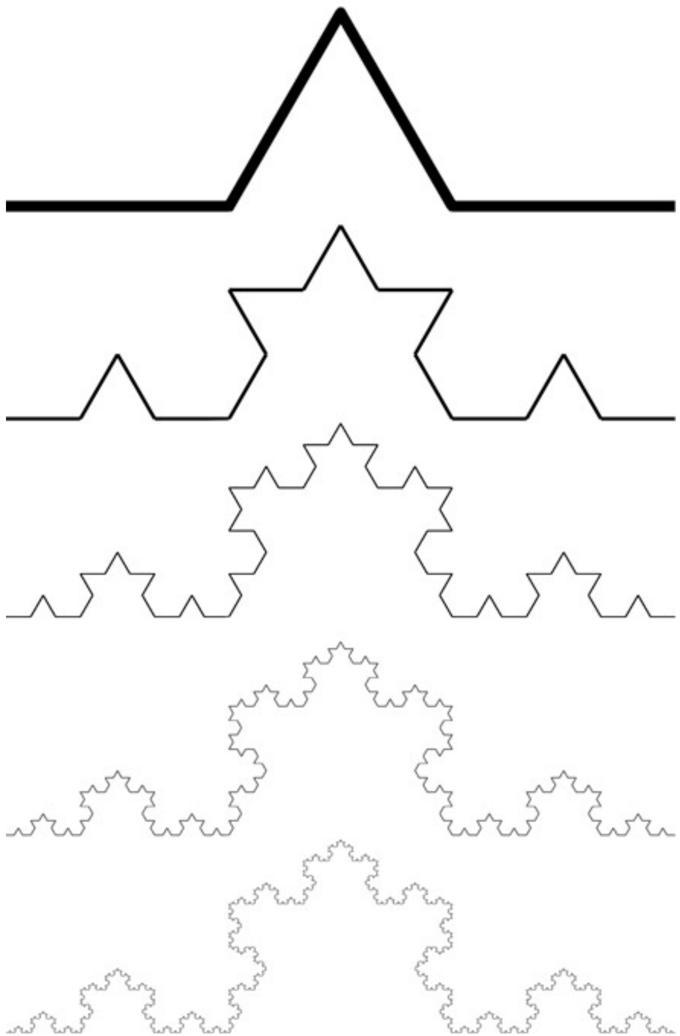
Surface?

- Number of dimensions?
- 2D
- Embedded in 3D space, but still 2D in (u,v)
- Terminology: manifold



Recursive rewrite process

- Koch curve



Recursive rewrite process

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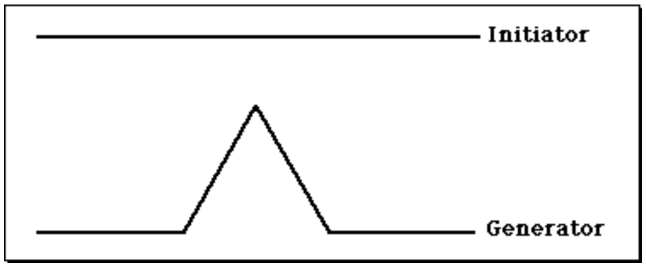
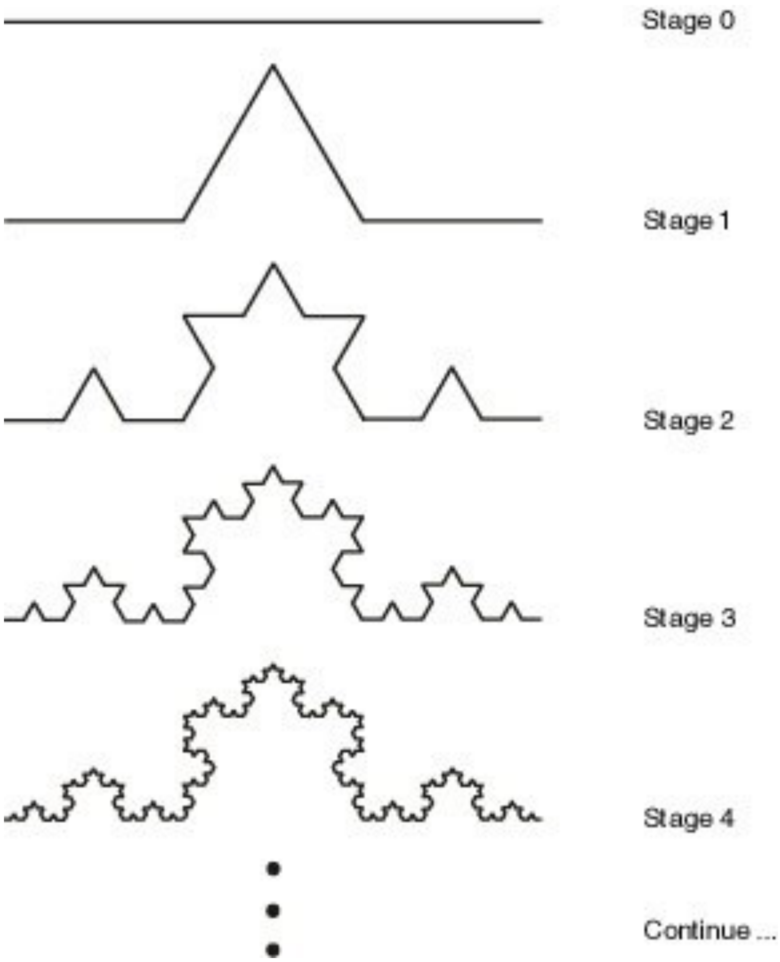


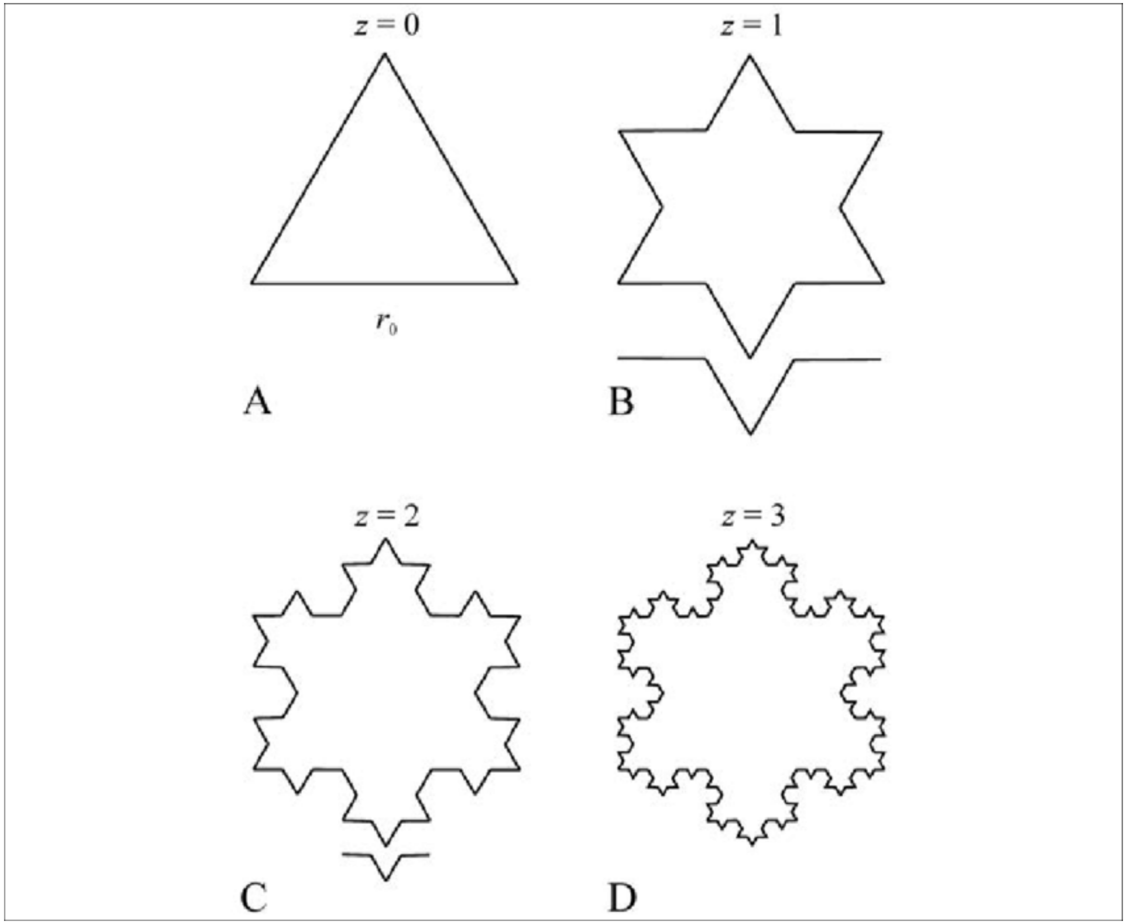


- Recursive replace lines by generator

- Koch curve is limit

Change initiator: Koch snowflake

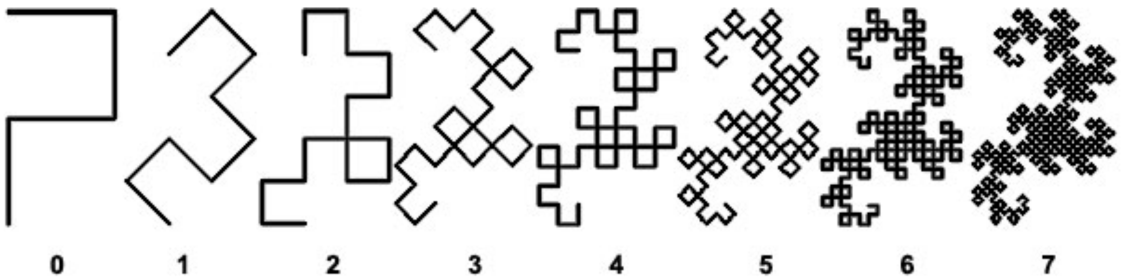
- Koch curve

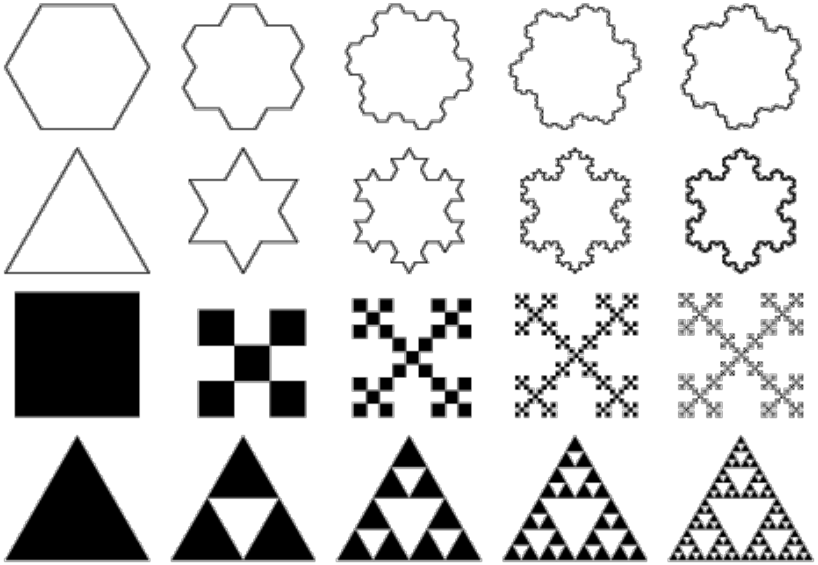


http://ecademy.agnesscott.edu/~lriddle/ifs/kcurve/kcurve.htm

Change generator: other curves

- Dragon curve

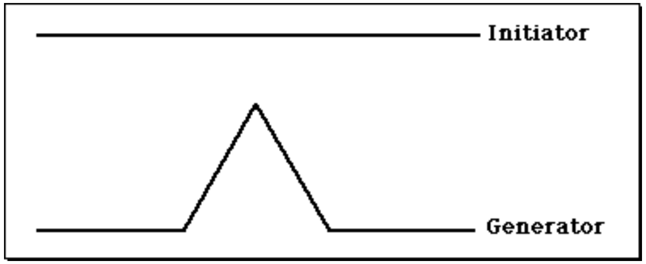


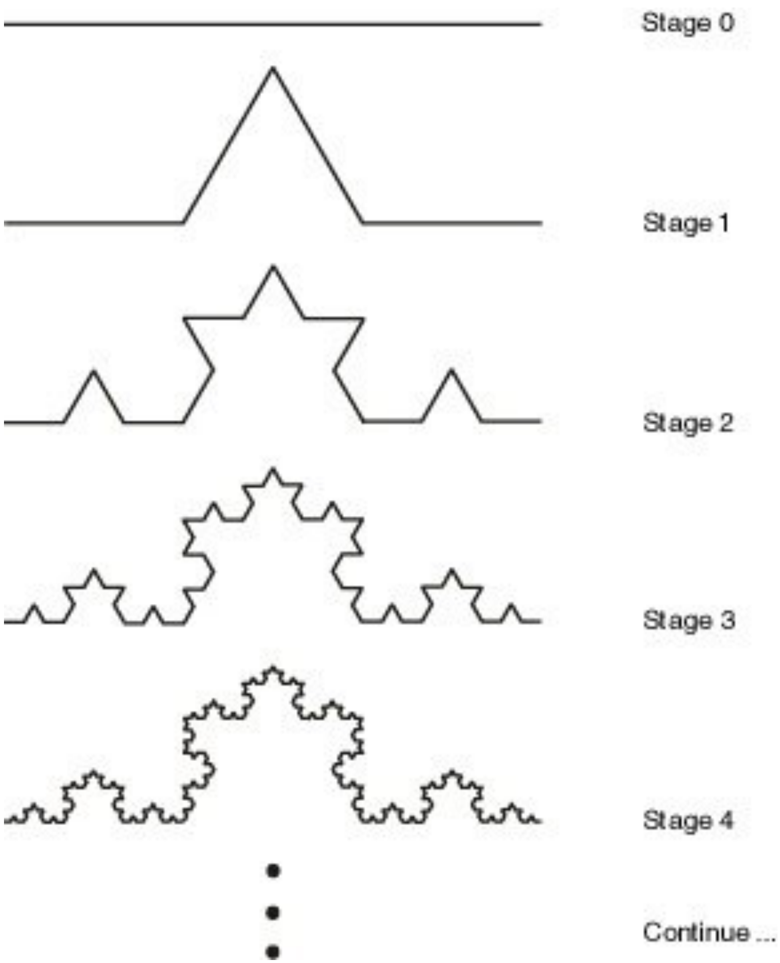


http://www.shodor.org/master/fractal/software/Snowflake.html

Length of Koch curve?

- Initiator – length 1
- Generator?





Length of Koch curve?

- Initiator – length 1
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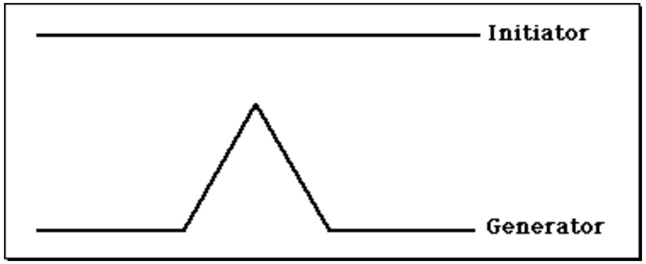
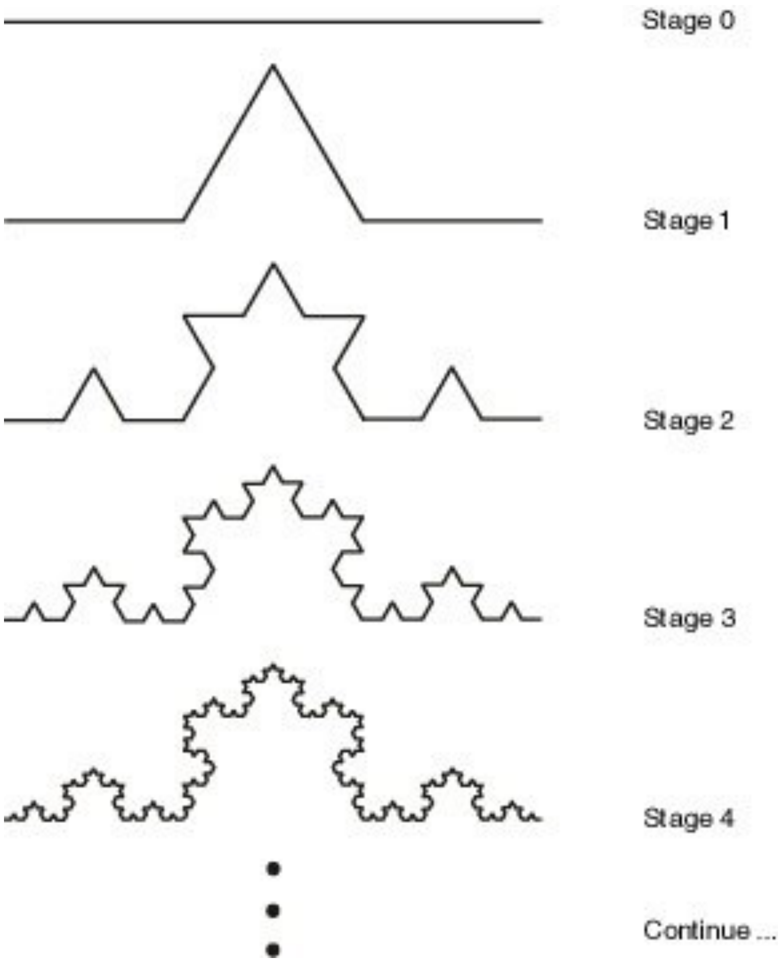
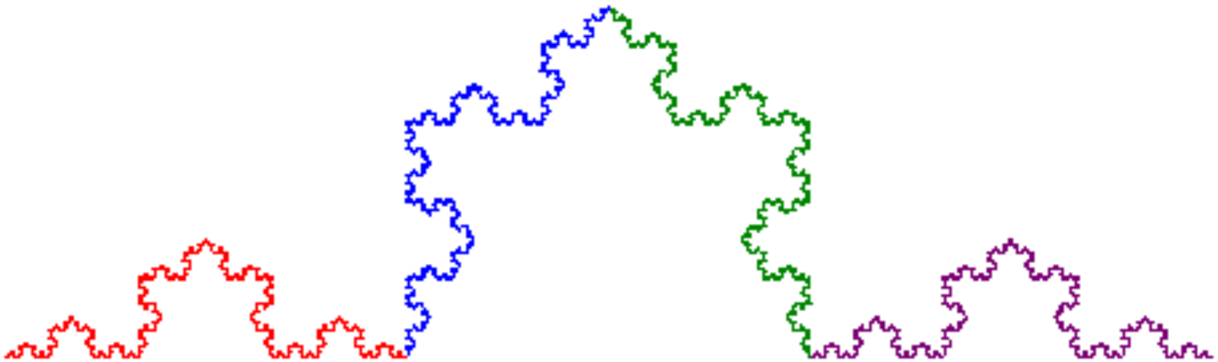




- G = 4/3
- Stage n: Length = $\left(\frac{4}{3}\right)^n$
- $\lim_{n\to\infty} \left(\frac{4}{3}\right)^n = \infty$

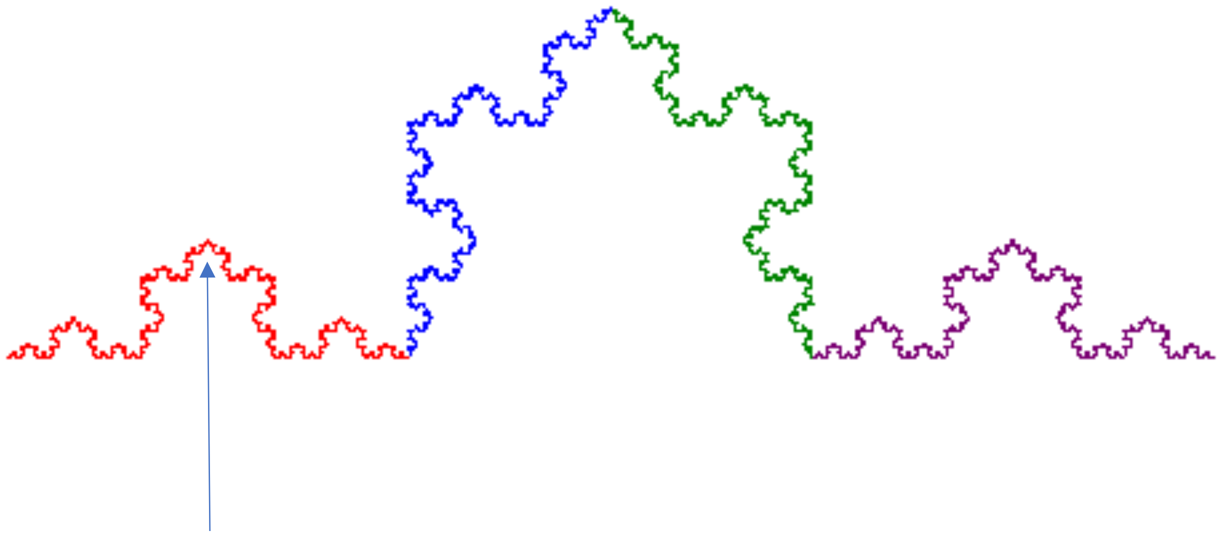
Infinite length curve in finite space

- Is one parameter t enough to describe where you are?



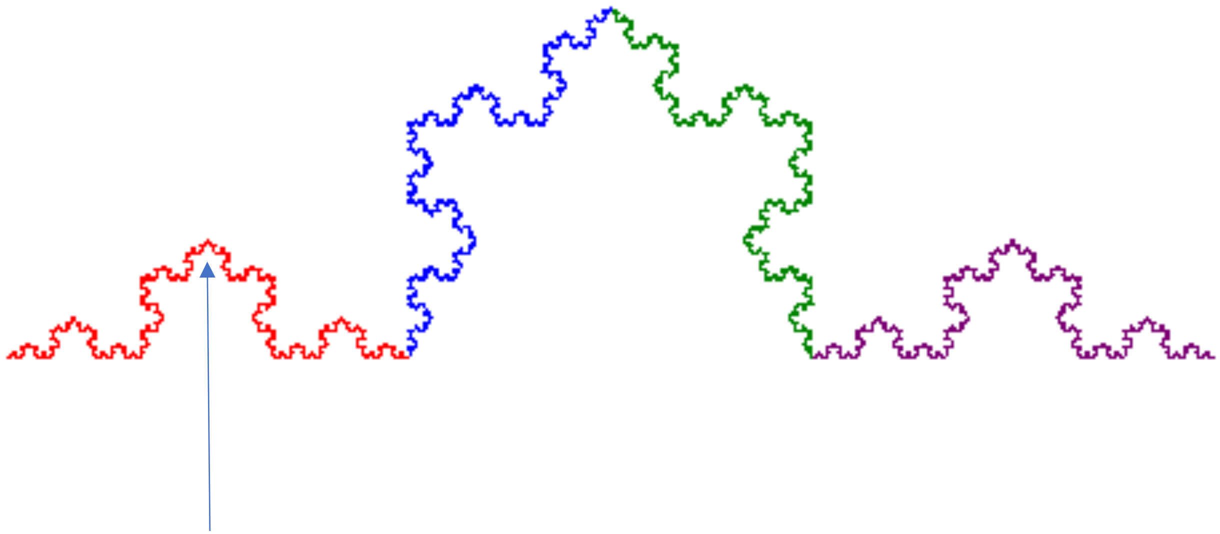
Infinite length curve in finite space

- Is one parameter t enough to describe where you are?
- No – takes infinite length to get to any position



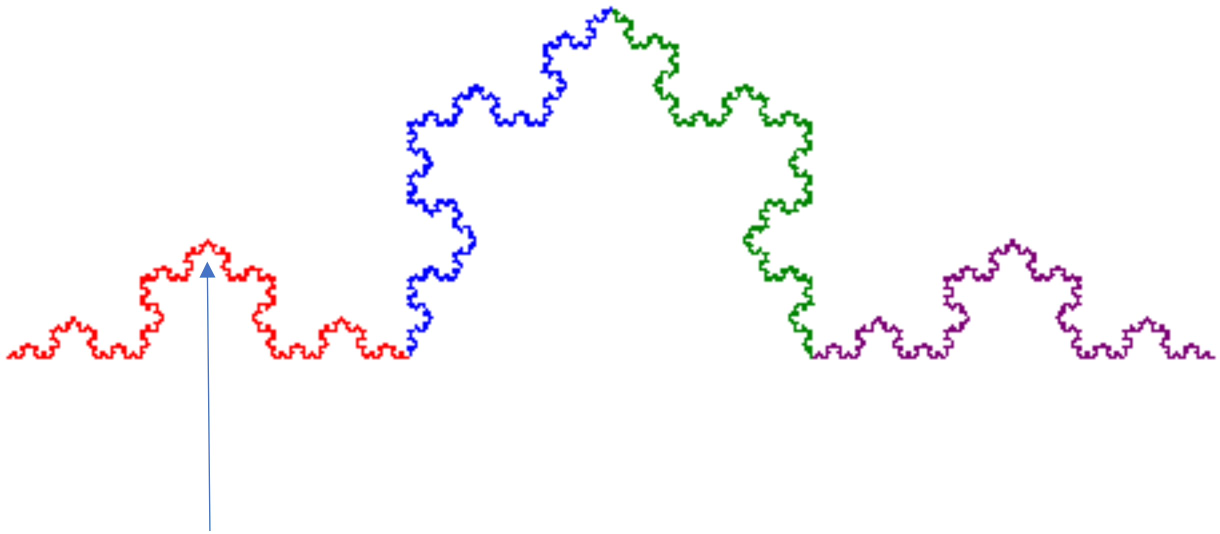
Infinite length curve in finite space

- Is one parameter t enough to describe where you are?
- No – takes infinite length to get to any position
- Does it take 2 parameters (u,v)?



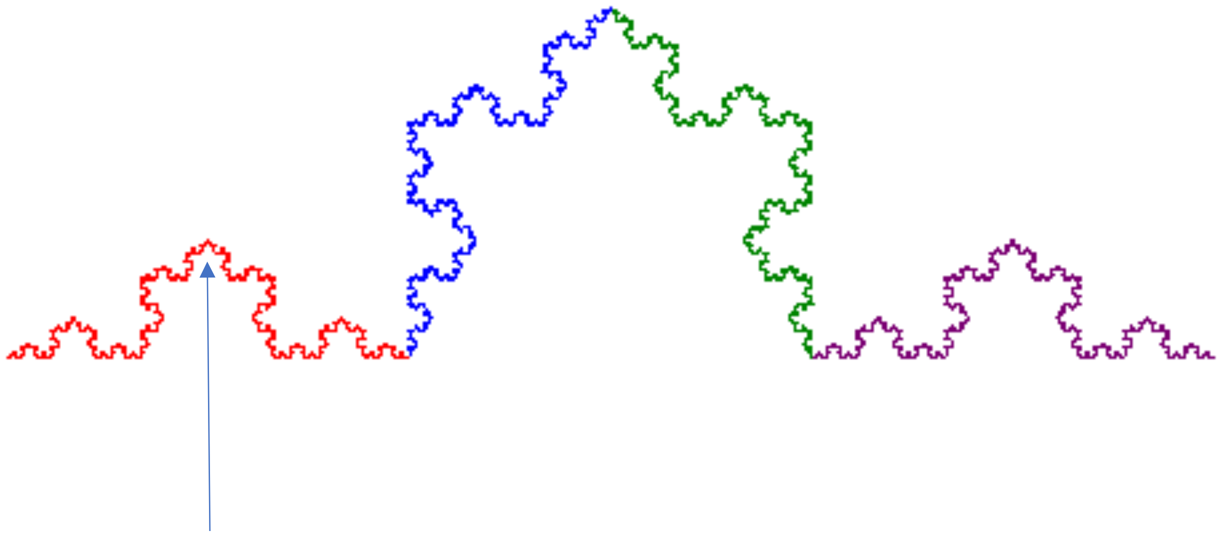
Infinite length curve in finite space

- Is one parameter t enough to describe where you are?
- No – takes infinite length to get to any position
- Does it take 2 parameters (u,v)?
- No – we can position anywhere in plane



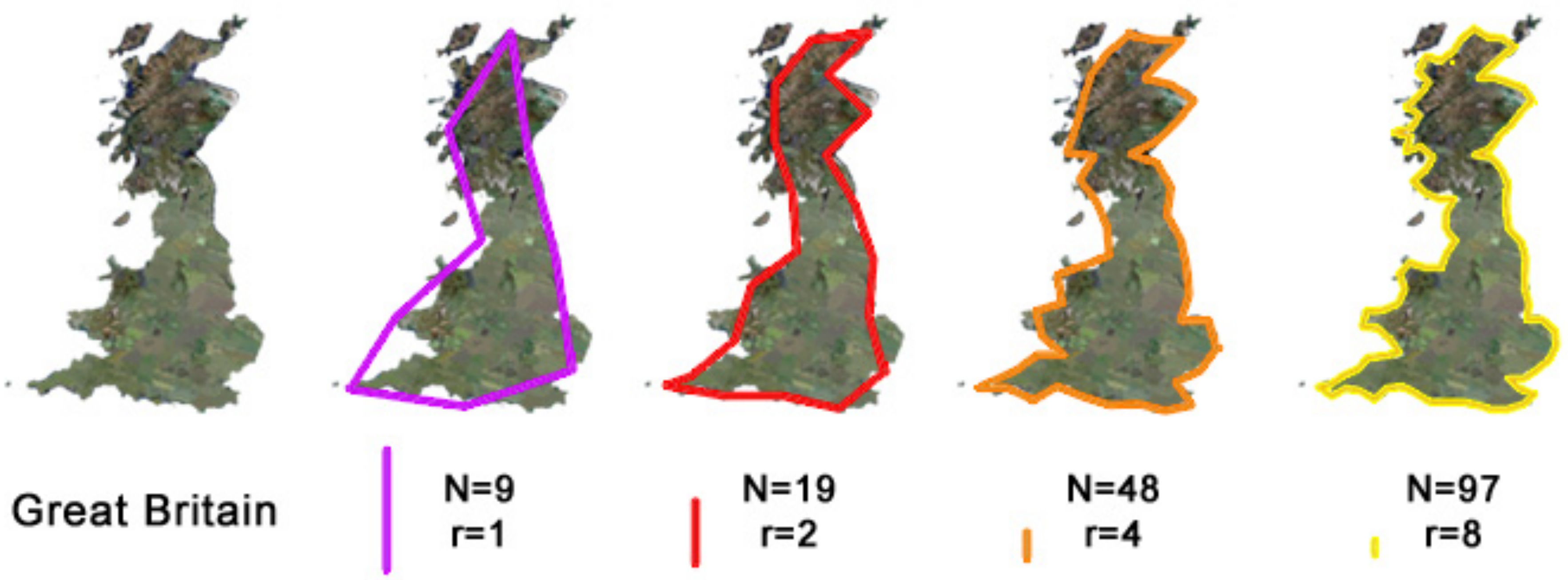
Fractal dimension

- Dimension of Koch curve is 1.26186
- Between 1 and 2 dimensions



Measuring fractal dimension

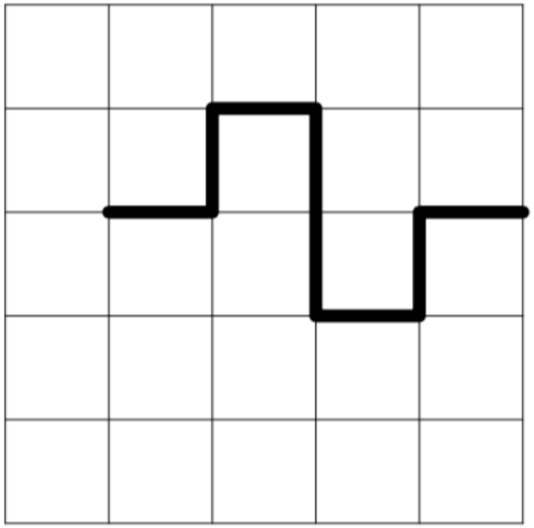
- Log ratio of how length increases as measuring rod decreases
- Measure coast with progressively shorter rods



Measuring fractal dimension

- Measuring generator dimension

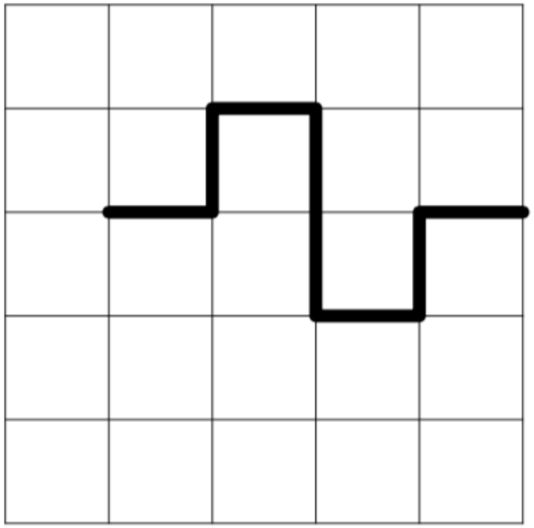
- Formula:
- $D = \frac{\log N}{\log \frac{1}{s}}$
- N – number of parts
- S – scale factor for one part



- N = ?
- S = ?

Measuring fractal dimension

- Measuring generator dimension



- Formula:
- $D = \frac{\log N}{\log \frac{1}{s}}$
- N – number of parts
- S – scale factor for one part

- N = 8
- S = 1/4

$$D = \frac{\log 8}{\log \frac{1}{1/4}} = \frac{\log 8}{\log 4} = \frac{3}{2} = 1.5$$

Measuring fractal dimension

- Measuring generator dimension

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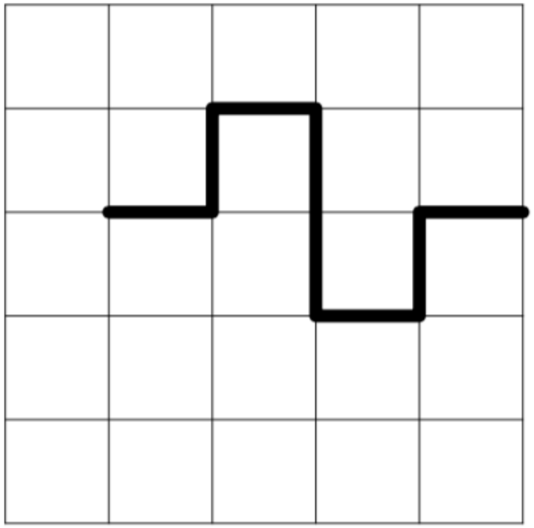

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Measuring fractal dimension

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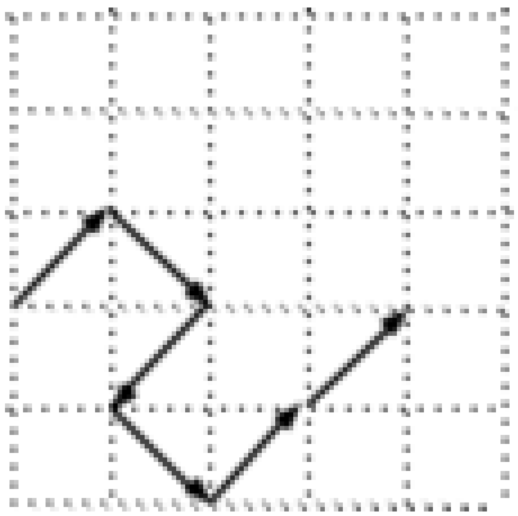
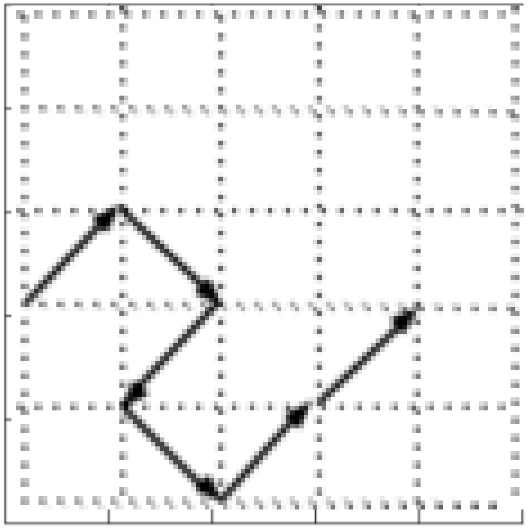


- N = ?
- S = ?

Measuring fractal dimension

- Measuring generator dimension



- Formula:
- $D = \frac{\log N}{\log \frac{1}{s}}$
- N – number of parts
- S – scale factor for one part

- N = 6
- S = $\frac{\sqrt{2}}{4}$

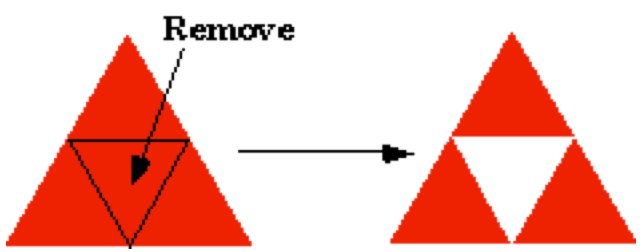
$$D = \frac{\log 6}{\log \frac{1}{\sqrt{2}/4}} = \frac{\log 8}{\log 4/\sqrt{2}} = \frac{3}{2} = 1.723$$

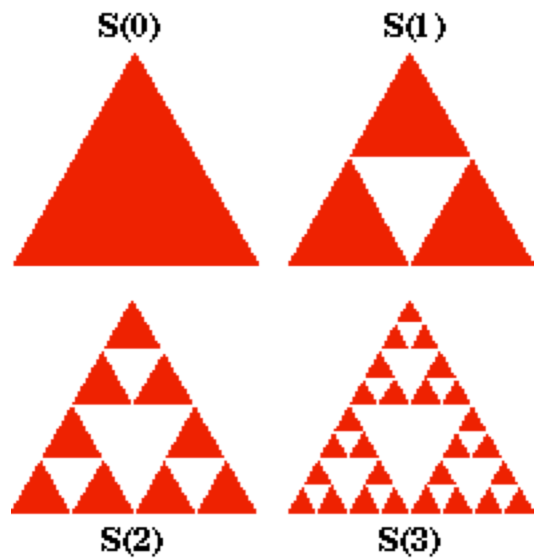
Creating fractals

- Recursive generators
- L-systems (Lindermeyer)
- Iterated function systems (IFS)
- Particle systems
- Midpoint displacement

Iterated Function Systems (IFS)

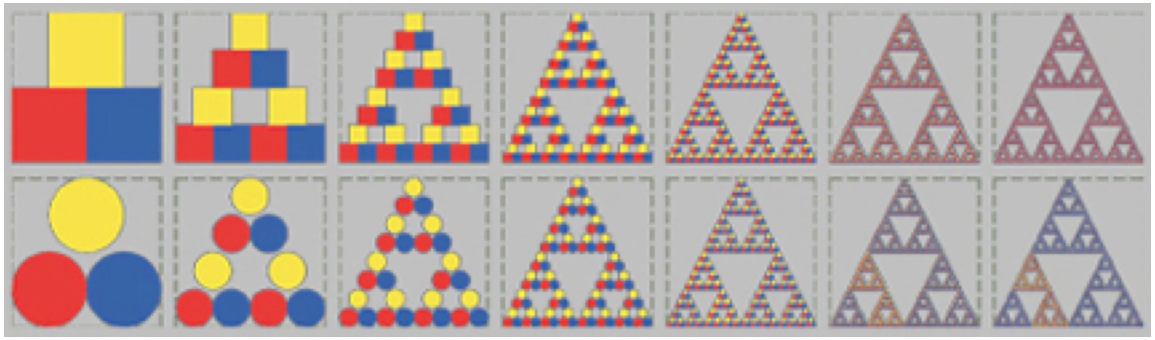
- Serpinski gasket





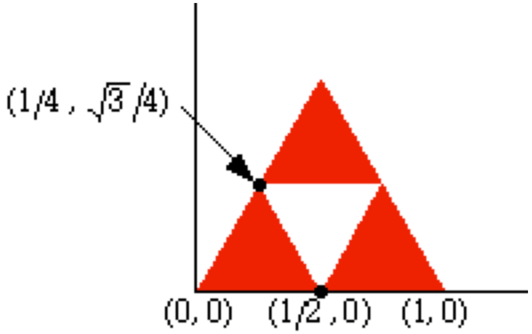
Copy machine version

- Reduce and duplicate



Copy machine version

- Triangle with 3 scaled and translated versions



$$f_1(\mathbf{x}) = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \mathbf{x}$$

scale by r

$$f_2(\mathbf{x}) = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$$

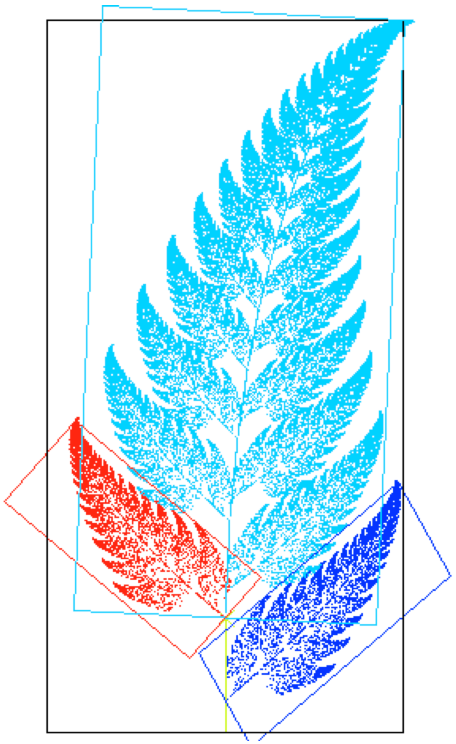
scale by r

$$f_3(\mathbf{x}) = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0.250 \\ 0.433 \end{bmatrix}$$

scale by r

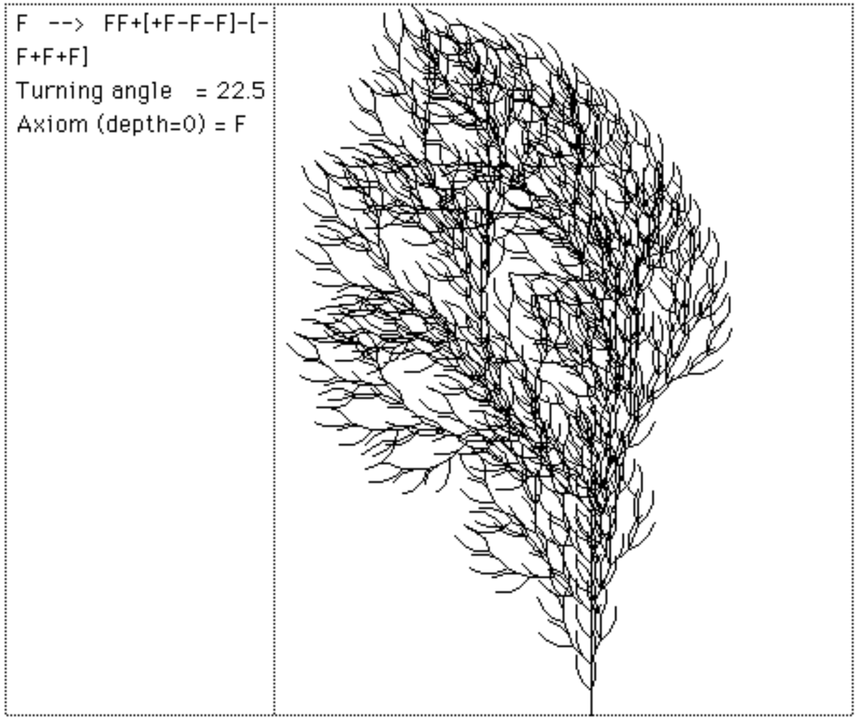
Barnsley Fern IFS

- http://www.zeuscat.com/andrew/chaos/spleenwort.fern.html



L-systems

- Grammar based technique
  - Represent shape as string of symbol
  - Each symbol has meaning in drawing shape
- Two parts
  - Grammar for generating strings
  - Rendering algorithm for interpreting strings as shapes

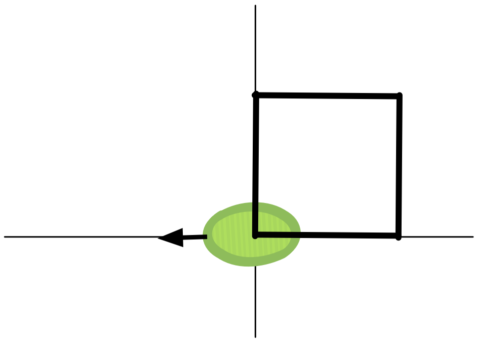


L-system turtle for rendering strings

- Turtle graphic commands
  - Turtle has state <angle, x, y>
  - Knows where it is and which direction it is pointed
- F - move forward a distance d, draw
- f - move forward a distance d, no draw
- + - turn left by angle delta
- - - turn right by angle delta
- [,] - push and pop turtle stack to remember state

Example: drawing F+F+F+F with angle=90 degrees

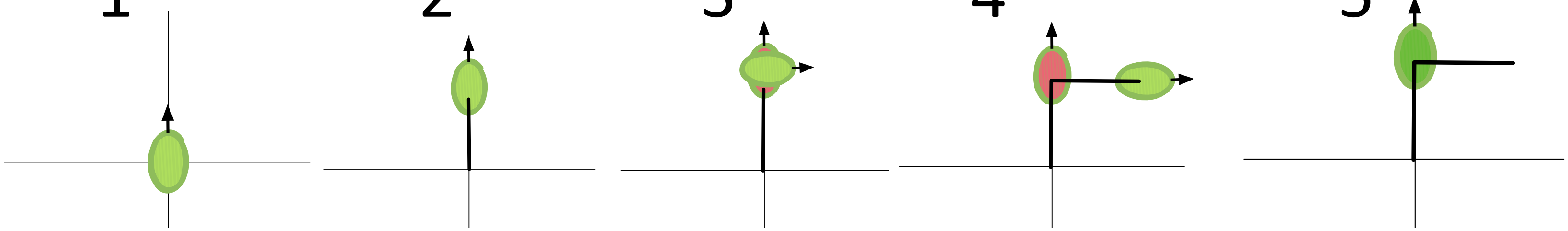
- Initial state <90,0,0> (default)
- 1) F – forward one unit
- 2) + - turn right 90 degrees
- 3) F – forward one unit
- 4) + - right 90
- And so on …
- Draws box
- Steps:



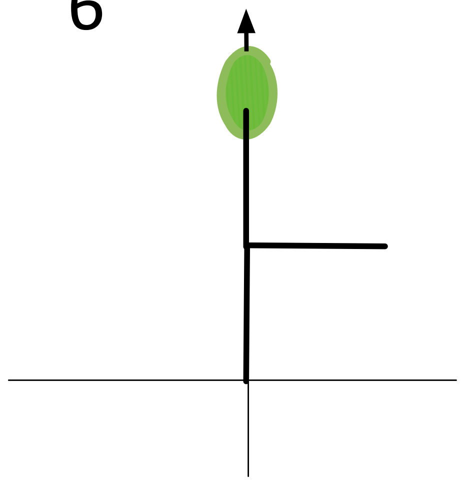
- 1 2 3 4

Example: drawing F[+F]F with angle=90 degrees

- Initial state <90,0,0>
- 1) F – forward one unit
- 2) [- push state (red)
- 3) + - turn right 90 degrees
- 4) F – forward one unit
- 5)] – pop state
- 6) F– forward one unit
- Steps:
- 1 2 3 4 5



6



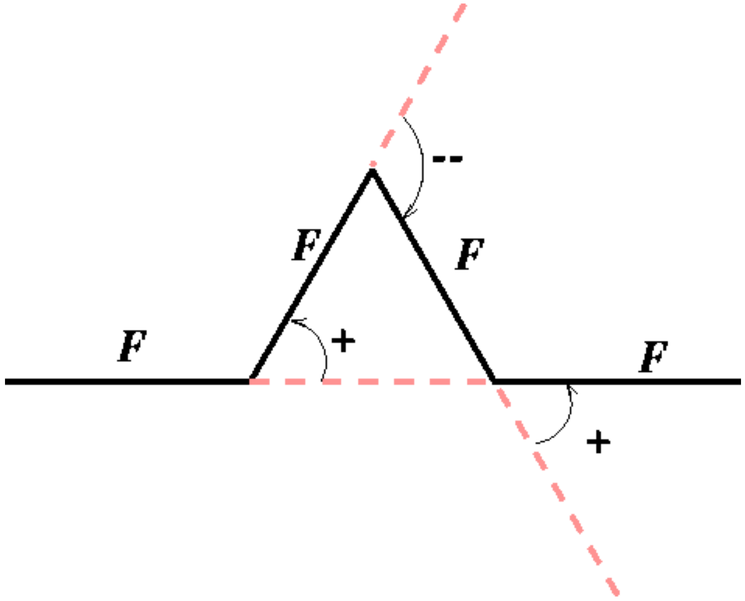
L-system for Koch curve

- Initiator

  F

- Replacement rule (no [])
  - F -> F+F--F+F

- Angle 60 degrees
- Distance 1 unit

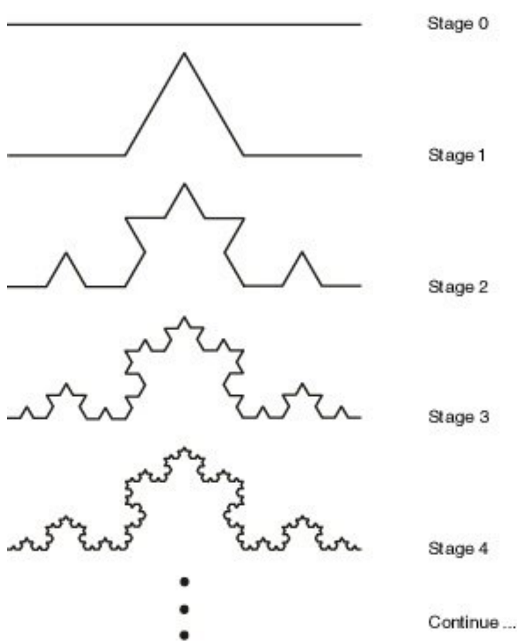


L-system for Koch curve: generating the string

- Stage 0
  - F
- Stage 1
  - F+F--F+F
- Stage 2
  - F+F--F+F+F+F--F+F--F+F--F+F+F+F--F+F

Replace F’s by rule F -> F+F--F+F

Don’t replace +, -, [,]



L-system for trees/shrubs

a
n=5, δ=25.7°
F
F → F[+F]F[-F]F

b
n=5, δ=20°
F
F → F[+F]F[-F][F]

c
n=4, δ=22.5°
F
F → FF-[-F+F+F]+[+F-F-F]

d
n=7, δ=20°
X
X → F[+X]F[-X]+X
F → FF

e
n=7, δ=25.7°
X
X → F[+X][-X]FX
F → FF

f
n=5, δ=22.5°
X
X → F-[[X]+X]+F[+FX]-X
F → FF

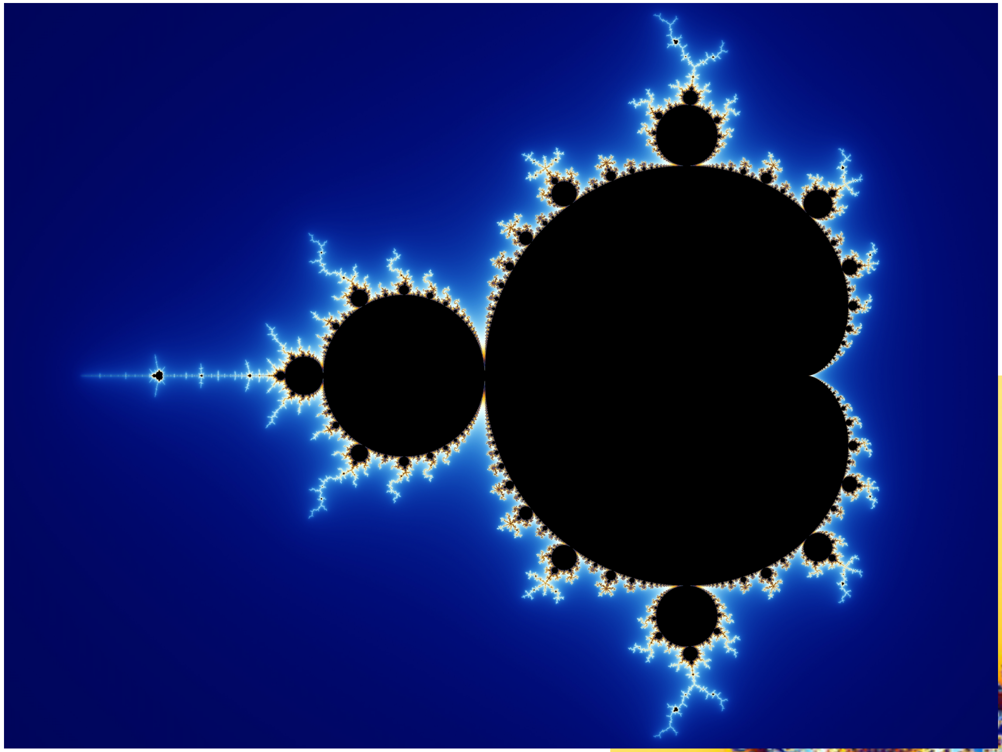
Stochastic L-system

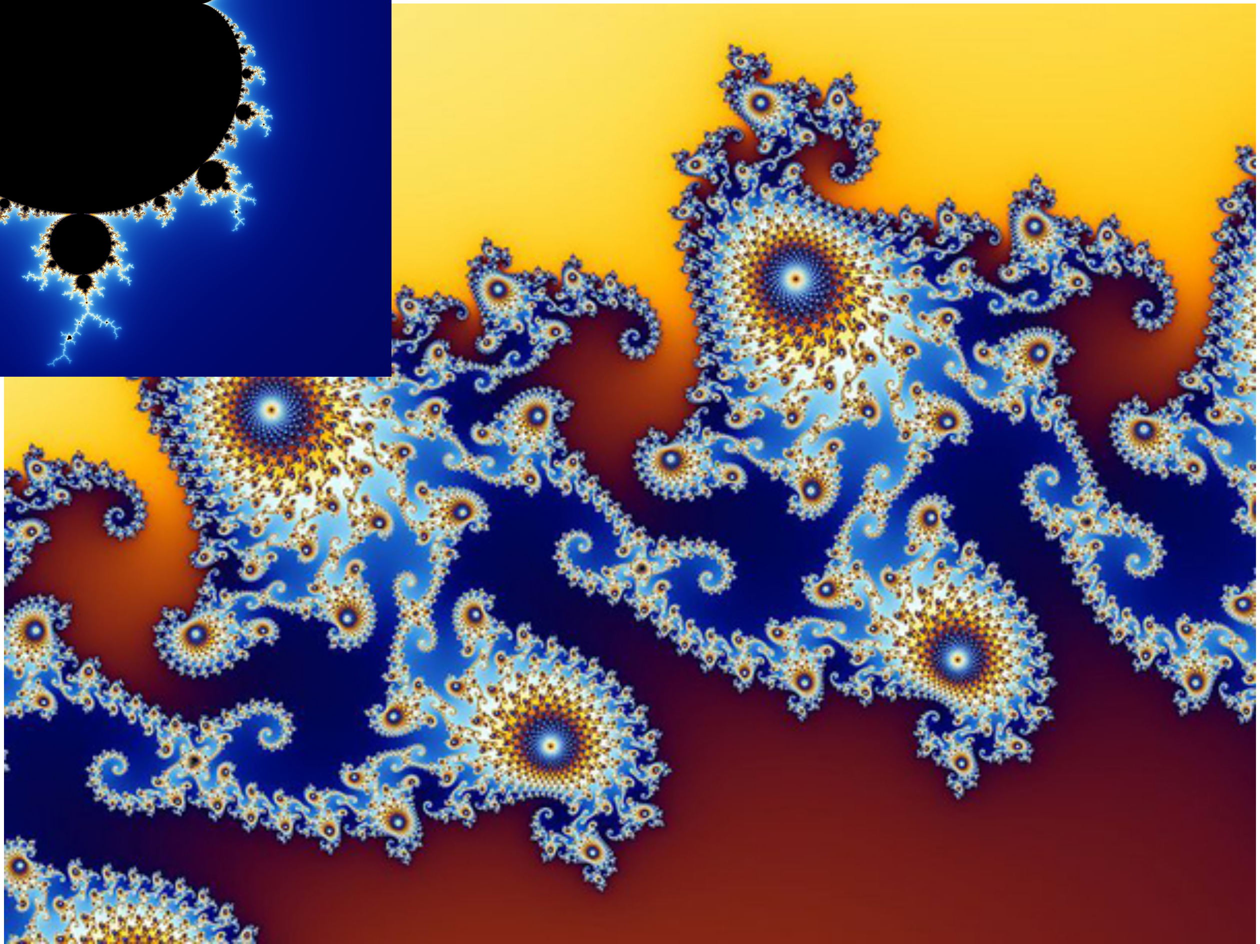
- Probability augmented replacement rules
- Choose each rule with given probabilty
- Generates more natural shapes (trees, shrubs)

```
V        =    { F, +, -, [, ] }
w        =    F
```

1. $F \xrightarrow{0.33} F\,[+F]\,F\,[-F]\,F$
2. $F \xrightarrow{0.33} F\,[+F]\,F$
3. $F \xrightarrow{0.34} F\,[-F]$

Mandelbrot and Julia sets

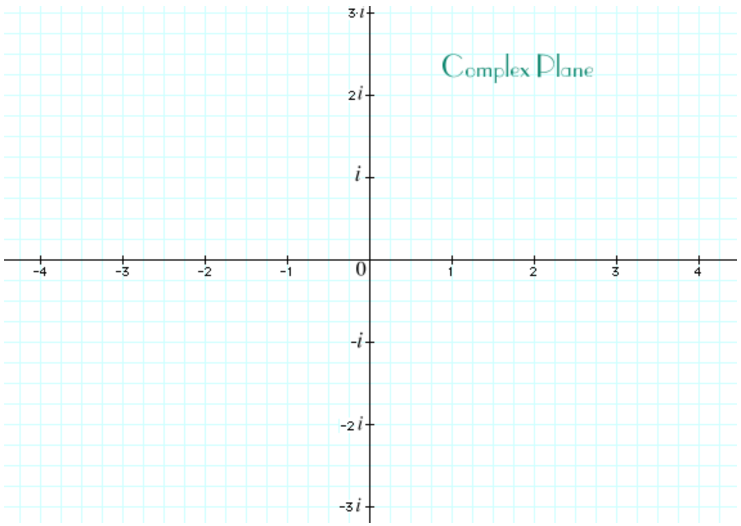




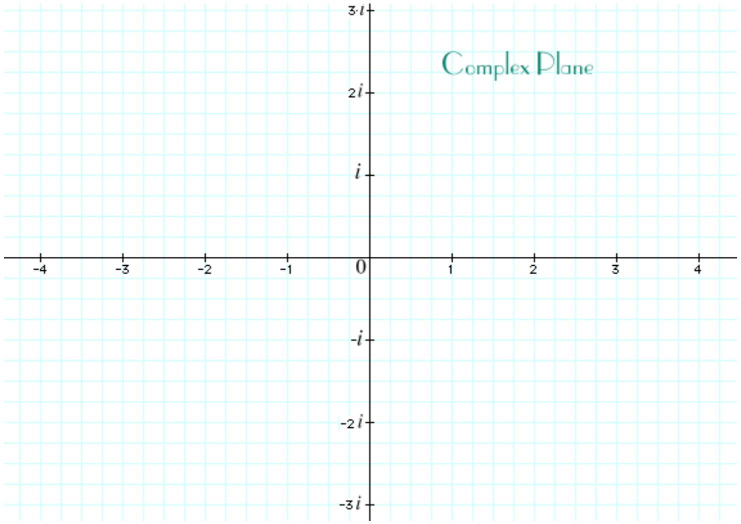
Mandelbrot equation

- Consider complex plane
- $C = x + yi$

- Iterate the function
- $Z = Z^2 + C$
- With Z0 = 0
- If the sequence Z0, Z1, Z2, remains bounded, Z is in the Mandelbrot set
- If it diverges, not in set – when |Z| > 2
- Color by number of iterations to divergence



L-Systems



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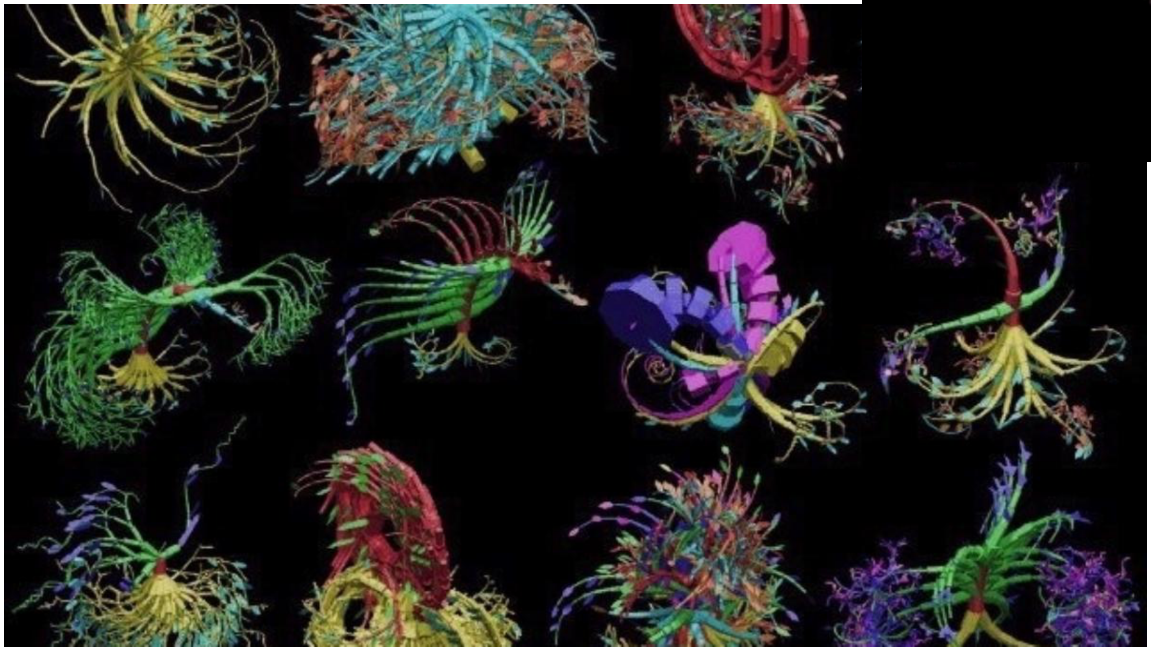
Evolutionary art

- Todd and Latham
- Rutherford
- Karl Sims
- http://www.karlsims.com



Particle systems

- Dyanamic systems of particles
- Model water, plants, fire, smoke
- https://www.youtube.com/watch?v=heW3vn1hP2E
- https://www.youtube.com/watch?v=HtF2qWKM_go