CMSC427 Midterm
Fall 2020

University honor pledge. I pledge on my honor that I have not given nor received any unauthorized assistance on this assignment/examination.

Signature: SOLUTIONS

Instructions:

• Partial credit will be given, which is easiest when you write clearly, explain your answers, and show the equations you use.
• You shouldn't need a calculator for calculations – setting up equations will be enough – but may use one if you wish. If you use Octave, Wolfram Alpha, or another site to compute note that – but the exam should not require it.
• The exam is open book and open notes. You may use the internet, but the boundary is that you may not copy answers from websites or others.

Grade

Section	Worth	Received
I. Vectors and operations	25	
II. Curves and surfaces	20	
III. Transformations	30	
IV. Polygons and meshes	15	
V. Shading	10	
Total	100	

Section I: Vectors and operations (25 pts)

1. (5 pts) Without computing the angle explicitly, is the angle between this pair of vectors in the previous problem greater than, less than, or equal to 90 degrees? v1=<1,1,0> and v2=<-2,0,1>.

$v1 \cdot v2 = \langle 1,1,0\rangle \cdot \langle -2,0,1\rangle = -2$

Since $V1 \cdot V2 < 0$ The angle is greater then $90°$ - obtuse.

2. (6 pts) Using vector operations, how would you determine if four points P1, P2, P3 and P4 are coplanar?

Use 3 points to compute a plane

$n = (P2-P1) \times (P3-P1)$ normal

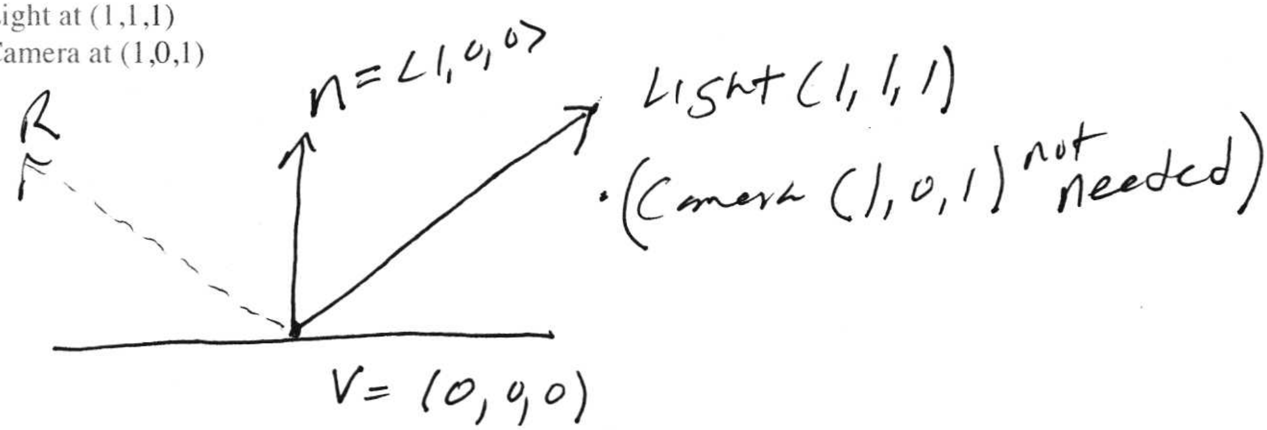
$p = P1$ point on plane

So the point-normal form.

Now take $(P4-P1) \cdot n$ and if that's zero P4 is on the plane.

3. (4 pts) Reflection vector. Given this data for a vertex in a mesh, compute the reflection vector R.

Vertex at (0,0,0)
Normal <1,0,0>
Light at (1,1,1)
Camera at (1,0,1)



$\bar{n} = \langle 1,0,0 \rangle$

$L = \langle 1,1,1 \rangle$

$$R = 2(L \cdot \vec{n})\hat{n} - L$$
$$= 2(\langle 1,1,1 \rangle \cdot \langle 1,0,0 \rangle)\, n - \langle 1,1,1 \rangle$$
$$= 2(1)\langle 1,0,0 \rangle - \langle 1,1,1 \rangle$$
$$= \langle 2,0,0 \rangle - \langle 1,1,1 \rangle = \langle 1,-1,-1 \rangle$$

4. (10 pts) Given a 3D parametric surface P(u,v) = < R*cos(v), h*v + d, u^2 >, compute the parametric normal as a function of (u,v).

$$\frac{\partial P(u,v)}{\partial u} = \langle 0, 0, 2u \rangle$$

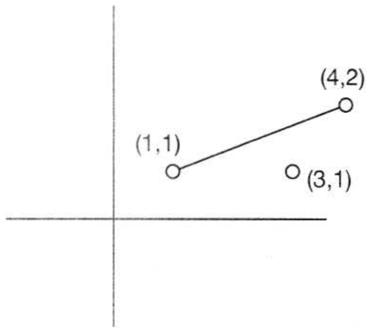
$$\frac{\partial P(u,v)}{\partial v} = \langle -R\sin v, h, 0 \rangle$$

$$\frac{\partial P}{\partial u} \times \frac{\partial P}{\partial v} = \begin{vmatrix} i & j & k \\ 0 & 0 & 2u \\ -R\sin v & h & 0 \end{vmatrix}$$

$$= \langle 2uh, -2uR\sin v, 0 \rangle$$

Section II: Curves and surfaces (20 pts)

1. (8 pts) Given the two points (1,1) and (4,2), in the diagram below, give the following:



a) A parametric equation for the line.

$p_0 = (1,1)\quad p_1 = \langle 4,2)$

$v = p_1 - p_0 = \langle 3,1\rangle$

$P(t) = tv + p_0$

$= t\langle 3,1\rangle + (1,1)$

b) The point-normal form of the same line.

$\text{normal} = v^{\perp} = \langle -1, 3\rangle$

$p = p_0$ point on line

For any point q, q is on line ~~#~~ if

$(q-p)\cdot n = 0$

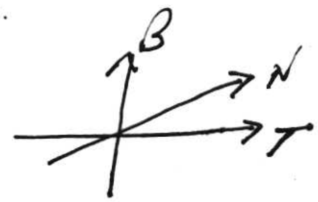
c) What is the distance from (3,1) to the line?

$\bar{n} = \dfrac{v^{\perp}}{|v^{\perp}|} = \dfrac{\langle -1,3\rangle}{\sqrt{10}}$

$d = |(3,1) - (1,1)\cdot \bar{n}|$

$= \left|\langle 2,0\rangle \cdot \dfrac{\langle -1,3\rangle}{\sqrt{10}}\right| = \dfrac{2}{\sqrt{10}} = \dfrac{\sqrt{2}}{\sqrt{5}}$

2. (8 pts) Compute the three vectors of the Frenet frame for the parametric curve
p(t) = < 3t, t^2, sin t > Don't normalize for this exercise.



$$p'(t) = \langle 3, 2t, \cos t \rangle$$

$$p''(t) = \langle 0, 2, -\sin t \rangle$$

$$p' \times p'' = \begin{vmatrix} i & j & k \\ 3 & 2t & \cos t \\ 0 & 2 & -\sin t \end{vmatrix} = \langle -2t\sin t - 2\cos t,\ 3\sin t,\ 6 \rangle$$

Tangent is p', Normal p'', and Binormal $p' \times p''$

3. (4 pts) Assume the parametric curve in the previous problem gives the location, and through the Frenet frame, the direction and orientation of the camera. Explain how you would construct the camera matrix from the curve and frame?

We have $at = p(t)$
Assuming the vectors are normalized, Then

$$M_c = \begin{vmatrix} & T & & -T\cdot at \\ & N & & -N\cdot at \\ & B & & -B\cdot at \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Section III: Transformations (25 pts)

1. (7 pts) Show that 2D homogenous scale and rotation matrices do not commute by giving an example.

Let $M_S = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ and $M_R = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ $\theta = 90°$

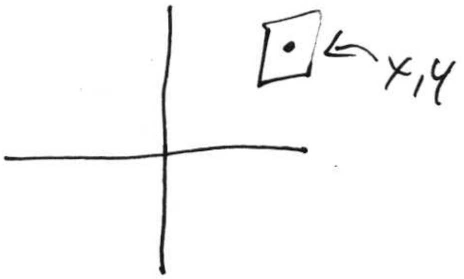
Then $M_S * M_R = \begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix}$

$M_R * M_S = \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}$

2. (8 pts) Give matrices to rotate and translate a shape around its center of mass. Assume the center of mass is at (x,y), and the rotation is theta. Since it's in 2D the rotation isn't around a specific axis – we're in the xy plane. Give the matrices, and then give the composition of matrices you'd need to perform the operations in the right order. Do not calculate out the final matrix.

With $M_{R(\theta)}$, $M_{T(x,y)}$

$M_{T(x,y)} = \begin{bmatrix} 1 & 0 & x \\ 0 & 1 & y \\ 0 & 0 & 1 \end{bmatrix}$



$M_{R(\theta)} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$M = M_{T(x,y)} * M_{R(\theta)} * M_{-T(x,y)}$

3. (10 pts) Given the input data below, compute the camera coordinate system xc, yc and zc, and the camera matrix.

at = (1,0,1) lookAt = (1,0,0) up = < 0, 1, 0>

$$Z_c = at - lookAt = (1,0,1) - (1,0,0) = \langle 0,0,1\rangle$$

$$X_c = UP \times Z_c = \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \langle 1,0,0\rangle$$

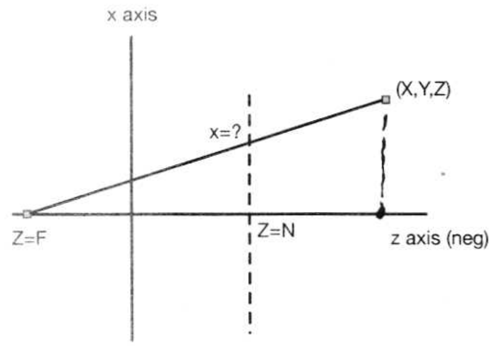
$$Y_c = Z_c \times X_c = \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = \langle 0,1,0\rangle$$

$$d = \langle x_c \cdot at,\ y_c \cdot at,\ z_c \cdot at\rangle$$

$$= \langle 1,\ 0,\ 1\rangle$$

$$M_c = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. (5 pts) Derive a perspective equation for x from the following diagram, and give the perspective matrix that results. This is the most general case, where the focal point at Z=F, and the image plane at Z=N, can vary relative to the origin.



$$\frac{x}{F-N} = \frac{X}{F+Z}$$

$$x = \frac{X(F-N)}{F+Z}$$

$$M_p = \begin{bmatrix} F-N & 0 & 0 & 0 \\ 0 & F-N & 0 & 0 \\ 0 & 0 & F-N & 0 \\ 0 & 0 & 1 & F \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} X(F-N) \\ Y(F-N) \\ Z(F-N) \\ F+Z \end{bmatrix}$$

After homogeneous normalization we have

$$\begin{bmatrix} X(F-N)/(F+Z) \\ Y(F-N)/(F+Z) \\ Z(F-N)/(F+Z) \\ 1 \end{bmatrix}$$

Section IV: Polygons and meshes (15 pts)

1. (5 pts) Given a polygon of n points, what's the brute force algorithm to determine if the points represent a simple polygon, and approximately what the big O efficiency of the algorithm.

If there are n points there are n edges. The brute force algorithm tests each pair of edges for intersection, so $O(n^2)$.

More precisely, ~~a~~ $\frac{n(n+1)}{2}$

2. (5 pts) What's the approximate space efficiency for N vertices of using a triangle strip over using triangles, given that the latter requires you to send each vertices multiple times, once for each triangle?

The reduction is to about 1/3 of the original list since you don't need to send a vertex multiple times for each triangle it belongs to.

3. (5 pts) An indexed polygon mesh data structure can maintain three lists, vertices, normals and faces. What information does a face have?

A face must list the vertices and associated normals.

Section V: Shading (10 pts)

1. (4 pts) In the basic shading model that we've covered, what's the role of the ambient term? How does it cover up for flaws in how the model approximates lighting in a scene?

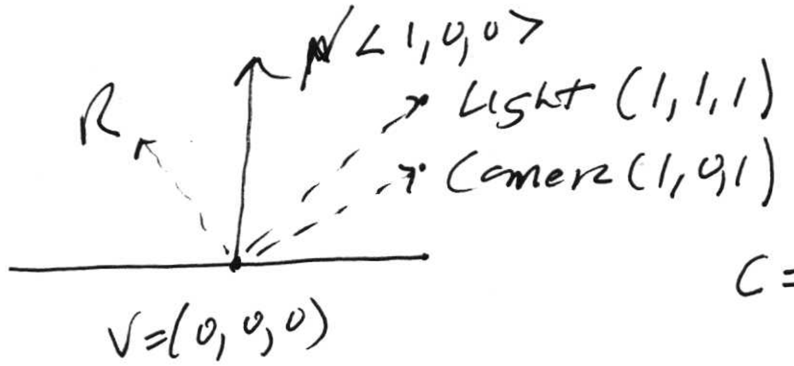
The shading model we have used has only direct illumination - light direct from source to surface. The ambient term accounts for light bouncing around the scene - global illumination, indirect lighting

2. (6 pts) Given the following data for a scene:

Vertex at (0,0,0) with kd = 0.80, ks = 0.70, cl = 100, p = 2
Normal <1,0,0>
Light at (1,1,1)
Camera at (1,0,1)

Set up the computation for the specular component with the Phong model. You may assume you have the reflection vector R from Section I, problem 3.

(Your R vector need not be correct).



C = Camera - Vertex
$= \langle 1,0,1 \rangle$

$$\text{Specular term} = ks * cl * (C \cdot R)^p$$
$$= (0.70)(100)(\langle 1,0,1\rangle \cdot \langle 1,-1,-1\rangle)^2$$
$$= (0.7)(100)(0)^2 = 0$$

adequate for answer