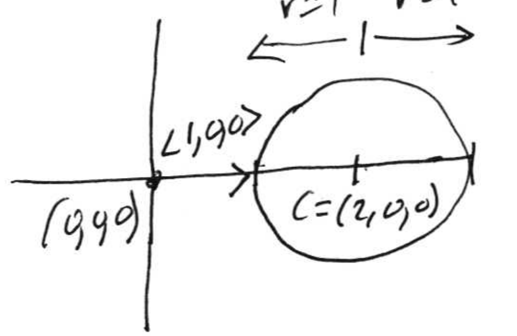
CMSC427 Fall 2020
Exercises for Tuesday, Oct. 27th
Global illumination: intersections in ray tracing

Exercises:

1. Try out these cases for ray-sphere intersection.

Ray $p(t) = (0,0,0) + t < 1,0,0>$	with sphere $c = (2,0,0)$ and radius = 1
Ray $p(t) = (0,0,0) + t < 1,0,0>$	with sphere $c = (-2,0,0)$ and radius = 1
Ray $p(t) = (0,0,0) + t < 1,0,0>$	with sphere $c = (0,0,0)$ and radius = 4
Ray $p(t) = (0,0,0) + t < 1,1,1>$	with sphere $c = (1,1,0.8)$ and radius = 1
Ray $p(t) = (0,0,0) + t < 1,1,1>$	with sphere $c = (4,4,0)$ and radius = 2

The first case:



By inspection, The intersection is at $t_0 = 1$, $p = (1,0,0)$

also at $t_1 = 3$, $p = (3,0,0)$

By equations (scratchapixel),

$L = C - \text{origin} = (2,0,0) - (0,0,0) = \langle 2,0,0 \rangle$

$D = \langle 1,0,0 \rangle \qquad t_{ca} = L \cdot D = 2, \geq 0$

$d = \sqrt{L \cdot L - t_{ca}^2} = \sqrt{\langle 2,0,0 \rangle \cdot \langle 2,0,0 \rangle - 2^2} = 0 \geq 0$

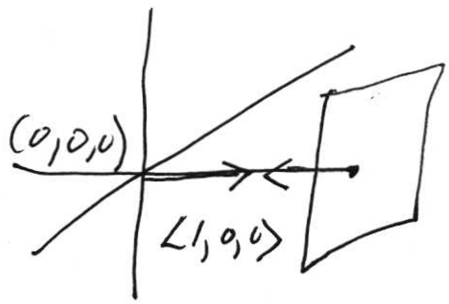
$t_{hc} = \sqrt{\text{radius}^2 - d^2} = \sqrt{1 - 0} = 1$

$t_0 = t_{ca} - t_{hc} = 2 - 1 = \underline{1}$

$t_1 = t_{ca} + t_{hc} = 2 + 1 = \underline{3}$

2. Try out these cases for ray-plane intersection.

Ray $p(t) = (0,0,0) + t < 1,0,0>$ with point (2,0,0) and normal (-1,0,0)
Ray $p(t) = (0,0,0) + t < 1,0,0>$ with point (2,0,0) and normal (0,1,0)
Ray $p(t) = (0,0,0) + t < 1,1,1>$ with point (3,3,3) and normal (1,1,0)



Case 1: By inspection
The intersection is
at $t=2$, $p=(2,0,0)$

$\bar{O} = (0,0,0)$ $R = \langle 1,0,0 \rangle$

By equation

origin

$N = \langle -1,0,0 \rangle$

$V_0 = (2,0,0)$

Distance to origin

$D = \cancel{N\cdot O}\ N \cdot V_0 = -2$

$$t_{hit} = \frac{N \cdot \bar{O} + D}{-N \cdot R}$$

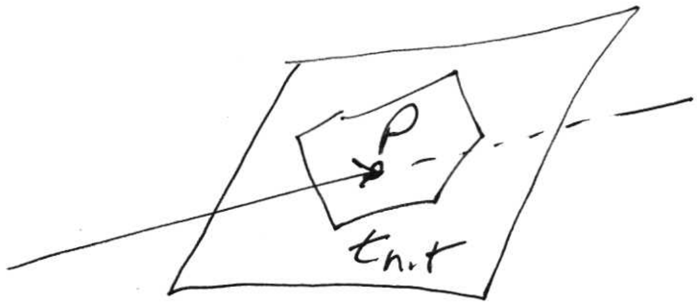
$$= \frac{0 + -2}{-\langle -1,0,0\rangle \cdot \langle 1,0,0\rangle}$$

$$= \frac{-2}{-1} = 2$$

$$P_{hit} = \bar{O} + t_{hit} R = \bar{O} + 2\langle 1,0,0 \rangle$$
$$= (2,0,0)$$

3. Extend the ray triangle intersection algorithm to a general polygon.

The idea is that you first



compute the hit point for the ray and plane.

We then compute the inside-outside test for each edge in the polygon, as with the triangle. This only works if the polygon is convex, though

4. Work out the equations for ray cylinder intersection for a vertical cylinder case~~s~~ defined as follows:

Centered at (0,0,0) $a^2 + b^2 = r^2$ should be $x^2+y^2=r^2$

Centered at (a,b,0) $(x-a)^2 + (y-b)^2 = r^2$

The ray is defined a $P(t)=P_0+tV$

<u>(0,0,0) case</u>

$$(P_0^x+tv_x)^2+(P_0^y+tv_y)^2=r^2$$

gives $P_0^{x\,2}+2tv_x+t^2v_x^2+P_0^y+2tv_y+t^2v_y^2=r^2$

or $t^2(v_x^2+v_y^2)+t(2v_x+2v_y)+P_0^{x\,2}+P_0^{y\,2}-r^2=0$

by quadratic formula $a=v_x^2+v_y^2$

$b=2v_x+2v_y$

$c=P_0^x-P_0^y-r^2$

and $$t=\frac{-(2v_x+2v_y)\pm\sqrt{\cancel{4}(2v_x+2v_y)^2-4(v_x^2+v_y^2)c}}{2(v_x^2+v_y^2)}$$

If $t>0$, and discriminate $\neq 0$, solutions give hit points. Note that if $V=\langle 0,0,0\rangle$ the ray is ill-defined and so is the ~~[illegible]~~ quadratic equation.