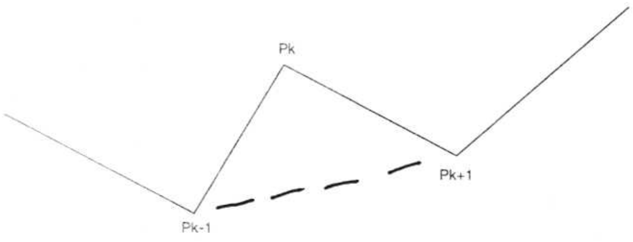
CMSC427 Piecewise parametric curve exercises

1. If you use a Hermite curve to interpolate through the points of a polyline, to insure C1 continuity at points you have to be careful not to not use uneven tangents – eg, at point Pk compute the left tangent to Pk-1, and the right tangent to Pk+1. How can you compute input tangents to get a smooth curve?



Use the vector $V = P_{k+1} - P_{k-1}$

as the tangent @ P_k

This makes the tangent the same for the left and right curves @ P_k.

2. Here's the Hermite curve we developed in class. If you carry through the matrix math, what are the coefficients a and b for the cubic and quadratic terms?

$$P(t) = [t^3 \quad t^2 \quad t \quad 1]\begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \\ \boldsymbol{c} \\ \boldsymbol{d} \end{bmatrix} = [t^3 \quad t^2 \quad t \quad 1]\begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \boldsymbol{P0} \\ \boldsymbol{P1} \\ \boldsymbol{T0} \\ \boldsymbol{T1} \end{bmatrix}$$

$$a = [2 \ -2 \ 1 \ 1]\begin{bmatrix} P_0 \\ P_1 \\ T0 \\ T1 \end{bmatrix}$$

$$= 2P_0 - 2P_1 + T0 + T1$$

$$b = [-3 \ 3 \ -2 \ -1]\begin{bmatrix} P_0 \\ P_1 \\ T_0 \\ T_1 \end{bmatrix}$$

$$= -3P_0 + 3P_1 - 2T_0 - T1$$

3. Using the derivation of the Hermite cubic curve as done in the lecture materials, apply that derivation to create a similar parametric form of a Hermite quadratic curve. Given three points P0, P1, P2, calculate a quadratic form of the Hermite curve. Have the curve interpolate through P1 and P2, and use P0-P1 as the tangent for the first endpoint.

What are the blending functions from this solution?

The curve is $P(t) = at^2 + bt + c$

$P'(t) = 2at + b$

To solve for a, b, c with tangent @ $t=0$ equal to $x_1 - x_0$

$P(0) = \underline{x_0 = c}$

$P(1) = \underline{x_1 = a + b + c}$

$P'(0) = \Delta x = \underline{b = x_1 - x_0}$

Solving for a: $x_1 = a + b + c$

$= a + (x_1 - x_0) + x_0$

so $a = 0$

A straight line from P_0 to P_1 is the solution

The equation is $P(t) = (x_1 - x_0)t + x_0$

$= t x_1 + (1-t) x_0$

Blending functions are

t and $(1-t)$

4. Given the Hermite curve we computed in class, what would be the first derivative of the curve? Represent it as below, as the product of a vector of powers of t and a basis matrix.

$$P(t) = [t^3 \quad t^2 \quad t \quad 1]\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = [t^3 \quad t^2 \quad t \quad 1]\begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \boldsymbol{P0} \\ \boldsymbol{P1} \\ \boldsymbol{T0} \\ \boldsymbol{T1} \end{bmatrix}$$

$$P'(t) = \underline{[3t^2 \ 2t \ 1 \ 0]}\begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} P0 \\ P1 \\ T0 \\ T1 \end{bmatrix}$$