CMSC427 Fall 2020
Rotations and quaternions

Exercises:
1. Compute the average of the two angle-axis rotations (angle=0, vector=<1,0,0>) and (angle=π/4, vector=<0,1,1>) and renormalize.

Average the angle and the two vectors, and renormalize the resultant vector.

Angle averaged = π/8	Vectors averaged	= <0.5,0.5,0.5>
	Renormalized	= <0.5774, 0.5774, 0.5774>

2. Compute the product of the two quaternions p=< 2, 1, 1, 0> and q=<1,0,1,1>.

p * q = (2 + 1*i + 1*j + 0*k) * (1 + 0*i + 1*j + 1*k)

= (2 + 2j + 2k) + (i + ij + ik) + (j + jj + jk) = (2 + 2j + 2k) + (i + k – j) + (j -1 + i)
= 1 + 2i + 2j + 3k

3. Compute the conjugate and length of p. What is the inverse of p?

p* = (2 - 1*i - 1*j - 0*k)

|p| = 2*2 + 1*1 + 1*1 + 0*0 = 6

4. Work out for yourself that pp* = $a^2 + b^2 + c^2 + d^2$

p x q = (2 + 1*i + 1*j + 0*k)* (2 - 1*i - 1*j - 0*k)

= (2*2 – 2i – 2j – 0k) + (2i – i*i – i*j – 0*i*k) + (2j – j*i – j*j – 0*j*k) + (0*q) // from k
= (4 – 2i – 2j) + (2i + 1 – k) + (2j + k + 1)
= 4 + (-2i + 2i) + (-2j + 2j) + (-k + k) + 1 + 1
= 4 + 1 + 1 + 0 = 6

5. Give two unit vectors in the same family as the pure unit quaternion p=< 0,1,1,0>

p=< 0,1,1,0>. I mean here two vectors in the same direction, so the vector parts are parallel.

p1 = <0.577,0.577,0.577,0> p2 = <0.333, 0.6667, 0.6667, 0>

6. Work out the rotation of a vector by angle-axis (angle= π/4, vector=<1,0,0>) using Rodrigues' formula. Let θ=π/4, and u = <1,0,0>, then we have

R(v) = (cos π/4)v+(1−cos π/4) <1,0,0>, (<1,0,0>, ·v)+(sin π/4)(<1,0,0>, ×v).

7. Work out for yourself the multiplication of two quaternions p, q and show it fits the standard formula with dot and cross products.

This a long, brute force development not worth doing on the final -see the end of the Mount lecture notes.

8. What is the quaternion for a rotation by angle theta around the x-axis? The z-axis?

The general formula for a rotation quaternion is $q = (\cos(\theta/2), \sin(\theta/2)*u)$

So for an x-axis rotation is $q = (\cos(\theta/2), \sin(\theta/2)*\langle 1,0,0\rangle) = \cos(\theta/2) + \sin(\theta/2)i$

For a z-axis rotation $q = q = (\cos(\theta/2), \sin(\theta/2)*\langle 0,0,1\rangle) = \cos(\theta/2) + \sin(\theta/2)k$

9. Compute the combined rotation from a 45 degree rotation around the x-axis followed by a 90 degree rotation around the z-axis (use #8 to find the quaternions involved.)

The general formula for a rotation quaternion is $q = (\cos(\theta/2), \sin(\theta/2)*u)$

$q1 = (\cos(45/2), \sin(45/2)*\langle 1,0,0\rangle) = \cos(22.5)+ \sin(22.5)i = a + bi$

$q2 = (\cos(90/2), \sin(90/2)*\langle 0,0,1\rangle) = \cos(45) + \sin(45)k = c + dk$

Combined we have $q = q2 * q1 = (a + bi) * (c + dk) = ac + adk + bci + bdik = ac + bci -bdj + adk$