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# *A model of deliberative and aggregative democracy*

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# A model of deliberative and aggregative democracy\*

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## Abstract

We present a model of collective decision making in which voting and deliberation are treated simultaneously. Political theorists argue that public discussion can lead individuals to change their preferences. This aspect of democracy is typically ignored in models of social choice that focus exclusively on voting. In our model, individuals debate in a public forum and potentially revise their preferences in light of deliberation. Once this process is exhausted, a voting rule is applied to aggregate post-deliberation preferences into a social choice. Restricting attention to three alternatives, we identify conditions under which

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a democracy is truth-revealing in the sense that the deliberation path and the voting rule will always lead to the correct social choice being made, irrespective of the original profile of preferences and size of the electorate (provided the latter is finite). A critical parameter in the model is the persuasion cost. When this is low, a democracy is almost always truth-revealing. When it is high, we have the standard social choice model and truthful revelation is impossible. Moreover, we identify when and only when truthful revelation occurs in an interesting intermediate case.

JEL classification: D71, D72.

Keywords: Deliberative democracy, aggregation, scoring rules.

## 1 Introduction

Political philosophers emphasise that there are two important aspects to democracy: aggregation and deliberation. Aggregation is usually achieved through voting in elections. These elections enable society to make social choices when individual preferences conflict. The theoretical analysis of voting can be traced back to the works of Condorcet and Borda and has been the central paradigm in social choice theory since the seminal work of Black (1958) and Arrow (1963). Importantly, where individual preferences come from is not central to the theory of voting. They are simply the inputs which, when combined with a voting rule, determine the output (the election winner or set of winners).

The well-known paradoxes of social choice theory have led some to conclude that the aggregative aspect (voting) is not as valuable as might first appear. According to this view, elections matter in that they restrain the behaviour of politicians by subjecting them to periodic electoral tests. They are not in general though a way of discovering the “will of the people”. This view is most often associated with the work of Riker (1982).

The importance of the deliberative aspect is associated with philosophers

like Habermas (1996) among others. Habermas argues that public discussion and debate makes people reflect on their preferences. Deliberation is another name for this process of reflection. One possible consequence of deliberation is that people's preferences may change. Some even go so far as to suggest that everyone in society will hold the same post-deliberation preference, thus making the problem of social choice trivial. This view is expressed by Elster (1986). He says (p. 112) that under deliberation "there would not be any need for an aggregation mechanism, since a rational discussion would tend to produce unanimous preferences".

It is fair to say that "deliberationists" are more optimistic than Riker about democracy. For deliberationists, a democracy has certain procedural virtues that go beyond voting. For example, Gutmann and Thompson (2004, p. 7) define a deliberative democracy as a "form of government in which free and equal citizens (and their representatives), justify decisions in a process in which they give one another reasons that are mutually acceptable and generally accessible, with the aim of reaching conclusions that are binding in the present on all citizens but open to challenge in the future". Viewed this way, democracy involves a dynamic process of open and transparent debate, the aim of which is to lead to understandable social choices being made.

Although deliberationists differ on how deliberative democracy should be defined<sup>1</sup>, Dryzek and List observe (2003, p. 1) that they "are uniformly optimistic that deliberation yields rational collective outcomes". We take this to mean the following. A collective outcome (or choice) is rational if and only if no other choice would have been better for the group. By assuming that such a choice exists, many deliberationists are following Cohen's (1986, 1989) "epistemic" theory of democracy. One part of Cohen's theory is his assumption that correct<sup>2</sup> choices exist that are independent of individual preferences and voting. For the purpose of this paper then, we take the deliberationist

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<sup>1</sup>For a sample of these disputes, see Bohman and Regh (1997), Elster (1998), Macedo (1999), Dryzek (2000), Estlund (2002) and Cunningham (2002).

<sup>2</sup>We take correct to mean the same thing as rational.

thesis to be this: by facilitating deliberation, democracy can ensure that the correct social choice is made. This optimistic conclusion contrasts starkly with the pessimism expressed by “aggregationists” like Riker. The objective of this paper is to use formal techniques from social choice theory to clarify the conditions under which the deliberationist thesis is true.

According to deliberationists, public discussion and debate (more generally, communication) can induce preference change.<sup>3</sup> How does public discussion and debate lead to preference change? First, new information is often revealed in public forums. This information can lead us to change our preferences. Second, communication forces us to check the internal consistency of the arguments we use to justify our preferences. On reflection, we may find that consistency is lacking and so revise our preferences. Third, and perhaps most controversially, even if our preferences are consistent, other people might persuade us that they are false. As will become clear later, we interpret preferences as beliefs. These beliefs are either true or false, and in the model people whose beliefs are true can persuade those whose beliefs are false. This modelling assumption should appeal to deliberationists, particularly those of an epistemic persuasion. It seems reasonable to assume that genuine persuasion can occur within a group and, in our model, this is the key to understanding preference change.

For all of these reasons, it seems plausible to imagine that deliberation can cause preferences to change, and that our post-deliberation preferences are superior to our pre-deliberation ones.

But what do we mean by “superior”? One natural interpretation (and the one we adopt in this paper) is closer to the truth (or, equivalently, closer to being correct). The concept of the truth plays no role in the pure theory of social choice, except in work on the Condorcet Jury Theorem.<sup>4</sup> In this paper, we take this concept of an objective truth and implant it in a social choice

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<sup>3</sup>A formal model of this process is suggested by List (2011). List works with judgments which are more general than preferences.

<sup>4</sup>See, for example, List and Goodin (2001).

model that is more in the tradition of Arrow and Black than Condorcet. In our model there is a strict ordering of three alternatives (no ties) that tells us which alternatives are objectively better than which.<sup>5</sup> This ordering is exogenous to the model and is not determined by individual preferences. Recalling our earlier definition, a collective choice is rational if and only if the alternative chosen is a maximal element with respect to this ordering.<sup>6</sup>

This assumption should appeal to many deliberationists, in particular those of an epistemic persuasion like Cohen. But is it true? We interpret this ordering as being a betterness relation for the group.<sup>7</sup> It tells us which alternatives are objectively better for the group than which. Is such a concept meaningful? Given that we commonly make statements of the form “ $A$  is better for group  $\alpha$  than  $B$ ”, it seems reasonable to believe that it is meaningful. For example, it is better for Manchester United to win both the Premiership and the Champions League than just the Premiership. It is better for the citizens of Galway that it is dry today rather than wet. Or, to use an example from Sen (1970b), it would have been better for Rome not to have burned while Nero played his fiddle, even though Nero himself was delighted.<sup>8</sup> We will assume that the betterness relation is transitive,

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<sup>5</sup>A strict ordering  $P$  is a transitive, asymmetric and complete binary relation. See Sen (1970a, p. 9) for definitions.

<sup>6</sup>An alternative  $x$  in the finite set  $\Xi$  is a maximal element with respect to the binary relation  $P$  if and only if  $\neg[\exists y : (y \in \Xi \text{ and } yPx)]$ . Our assumptions about  $P$  ensure that such an alternative exists.

<sup>7</sup>We take this term from Broome (1991, 2004). Welfare economists call it a social preference relation. It should not be interpreted as the “will of the people”. It tells us which alternatives are objectively better for the people, not what the people will. We can think of it as being generated by an underlying Bergson-Samuelson social welfare function, i.e. a function  $E : \Xi \rightarrow \mathbb{R}$  where  $E(y)$  is the social value of alternative  $y$ . Assuming that  $E$  is injective,  $E(x) > E(y) \rightarrow xPy$ . Some might complain that Arrow’s (1951) impossibility theorem (or a variant of the theorem) shows that a non-dictatorial, Bergson-Samuelson social welfare function cannot exist. That would be an error, as pointed out by Fleurbaey and Mongin (2005). This point is conceded by Arrow himself in Arrow (1983), which contrasts starkly with Arrow’s earlier assessment of the Bergson-Samuelson social welfare function expressed in *Social Choice and Individual Values*.

<sup>8</sup>Apparently Nero did not play his lyre (the fiddle had not yet been invented) during the burning of Rome. This myth was created by later Christian tradition as revenge for

asymmetric and complete.

Aggregationists and deliberationists are often claimed to be at loggerheads. The purpose of this paper is to contribute to a reconciliation between them. Our motivation is, therefore, identical to Dryzek and List's. However, Dryzek and List's hypothesis is that deliberation makes it less likely that the assumptions of impossibility theorems hold. Viewed this way, aggregationists (using social choice theory) have identified formal conditions that must be eliminated through the process of deliberation. Whether actual deliberation accomplishes this is an empirical matter, but Dryzek and List's central point is that social choice theory reinforces the importance of deliberation, and does not fundamentally conflict with it.

This paper shares many philosophical commitments in common with Dryzek and List, but approaches the problem from a different direction. Like them, we do not think that deliberation fundamentally conflicts with social choice theory. However, we would argue that an important step in any reconciliation project is to consider whether it is possible to incorporate the process of deliberation into the mathematical structure of social choice theory itself. This is something that Dryzek and List do not consider. Understandably, social choice theory has devoted a considerable amount of time to studying voting rules. It has spent less time developing formal models of the process of deliberation.<sup>9</sup> Is it possible to integrate these two concerns into a single formal framework? The purpose of this paper is to outline such a model: a model of both deliberative and aggregative democracy.

In attempting to do this, we are keeping in the tradition of economic theory. The history of economics is replete with examples of new theoretical models emerging as a consequence of identifying features of the world that

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Nero's persecution of Christians. Sen uses this example to illustrate that it is possible to make partial interpersonal comparisons of well-being. On this see Pivato (2010).

<sup>9</sup>A notable and important exception is List (2011). List models the communication process that could occur in deliberative settings, and proves a new impossibility theorem using social-choice-theoretic techniques. He does not, however, consider voting.



earlier models ignored (or, for simplicity, assumed away). The purpose of new models is to gain an understanding of how these phenomena affect the conclusions of older models. Moreover, one advantage of developing an explicit formal model is that it is intrinsically neutral as to the effect of deliberation. As we will see, sometimes deliberation leads to good social outcomes, but other times it does not.

With this in mind, we now explain the central features of the model. The model itself appears in Section 3, the results in Section 4. The final section contains some concluding remarks.

## 2 Outline of the model

In the standard model of social choice, the inputs into the aggregation procedure are preferences and the output is a social choice or a social ranking. In our model there is a pre-deliberation stage, a post-deliberation stage, and finally an aggregation stage (where a social choice is made). It is important to emphasise that deliberation may occur more than once, we do not necessarily move straight from the pre-deliberation stage to the post-deliberation stage. In fact, the post-deliberation stage is just a stage where no further preference change is possible. At this point, a vote is taken with everyone submitting their final, post-deliberation preferences. The outcome of the voting determines the social choice.

The standard model is represented in Figure 1.

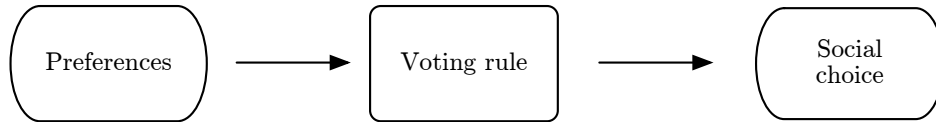


Figure 1: The standard social choice model.

Our model is represented in Figure 2.

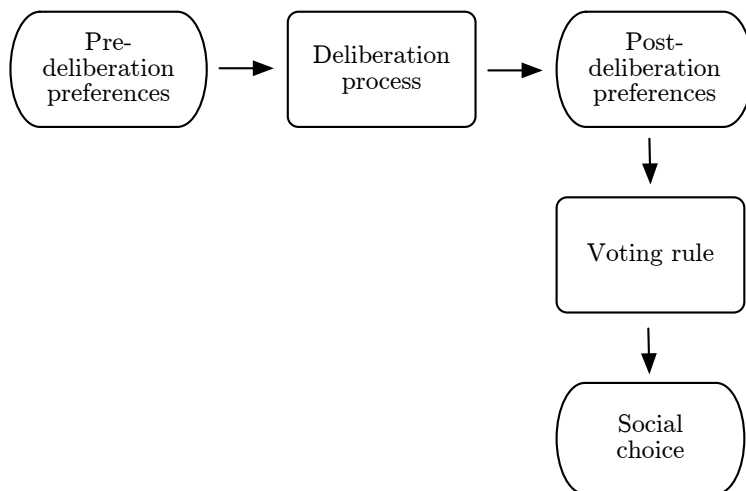


Figure 2: Our model.

Our model makes a radical assumption about preferences, but one that should appeal to deliberationists like Cohen. We assume that an individual’s preferences represent her current beliefs as to which alternatives are better for the group than which. In other words, they represent her best current estimate as to form of the betterness relation. Cohen (1986) calls this a “cognitive account of voting”. He says (p. 34) that this is “the view that voting expresses beliefs about what the correct policies are according to the independent standard, not personal preferences for policies”.

This assumption is radical in that it draws a distinction between what a person prefers and what it better for them. This is unusual in economics. In the model, there is no concept of what is better for a person; a person’s preferences represent her beliefs about what is better for the group. Importantly, these beliefs can change through the process of deliberation. Our formal model, therefore, provides what Cohen refers to (1986, p. 34) as “an account of decision making as a process of the adjustment of beliefs, adjustments that are undertaken in part in light of the evidence about the correct answer that is provided by the beliefs of others”.

A natural and complementary assumption is that throughout the entire

process individuals act sincerely in expressing their preferences, there is no strategic behaviour.<sup>10</sup> So, when an individual expresses her beliefs about what is better for the group (through her preferences), we can take this expression to be a sincere one. One reason for making this assumption is that we want to build a model under assumptions that are particularly favourable to deliberationists. Assuming no strategic behaviour is broadly consistent with a deliberationist world view. As we will see, even with this assumption in place, deliberation does not always guarantee that the correct social choice is made.

Despite this, there is an important sense in which our model is classically strategy-proof. As we will see, if a democracy is “truth-revealing” then at the pre-deliberation stage nobody has any incentive to misrepresent their preferences. The reason for this is that unilateral deviations at any profile do not affect the ultimate social choice. That said, we prefer to place the issue of manipulability to one side. What manipulability means in our model is not at all clear, and there are several places where the issue of strategic behaviour could arise (for instance, individuals could choose not to follow our preference transformation rule which we assume applies universally). For these reasons (and also a desire to keep the model simple), we prefer to interpret behaviour as sincere at all stages in the model. This is in keeping with the spirit of deliberationism, which views preference change as reflecting social learning rather than as an attempt to manipulate outcomes.

Here are the additional details of the model. For simplicity, we assume that there are just three alternatives from which a social choice must be made. We assume that individual preferences are strict. In addition, we assume that at the start of the process (the pre-deliberation stage) at least one individual holds each logically possible preference. There are a finite

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<sup>10</sup>There is an important literature on strategic behaviour in deliberative settings. See, for example, Austen-Smith and Feddersen (2006), Calvert (2006), Hafer and Landa (2007) and Landa and Meirowitz (2009). The pioneering papers are Gibbard (1973) and Satterthwaite (1975).

number of individuals in society.

An important concept in the model is that of a persuasion group. A persuasion group is a set of individuals who engage in a debate with one another. Individuals enter a persuasion group, debate with one another and then leave the group. When an individual leaves a persuasion group, her preferences may be different from those she held when she entered it. This reflects the impact of deliberation on her. Of course, an individual's preferences do not have to change as a consequence of being in a persuasion group. For example, an individual could debate with everyone else and persuade the others that her beliefs are correct. In this case, other people's preferences will change, but not those held by the individual herself.

Persuasion groups can be formed in different ways. The critical parameter determining who can join any particular group is given by what we call the persuasion cost. The persuasion cost is  $\frac{1}{\delta}$  and it takes one of three values,  $\frac{1}{\delta} \in \{\infty, 1, \frac{1}{2}\}$ . The parameter  $\delta$  in this expression is simply the well-known Kemeny measure of distance between preference orderings.<sup>11</sup>

We will briefly explain the concept of Kemeny distance, as it is central to our analysis. Let us call the three social alternatives  $X, Y$  and  $Z$ . To each logically possible strict ordering of the alternatives, we will assign a number. The numbering appears in the following table. Writing  $X \succ Y \succ Z$  means that  $X$  is preferred to  $Y$ ,  $Y$  is preferred to  $Z$ , and  $X$  is preferred to  $Z$ .

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<sup>11</sup>Kemeny and Snell (1962, chapter 2) is an excellent and detailed explanation of Kemeny's metric. They provide an axiomatisation of the metric.

Number	Ranking
1	$X \succ Y \succ Z$
2	$Y \succ X \succ Z$
3	$Y \succ Z \succ X$
4	$Z \succ Y \succ X$
5	$Z \succ X \succ Y$
6	$X \succ Z \succ Y$

Table 1: Numbers assigned to rankings.

We can create a graph with the rankings as vertices. We join two vertices with an edge if the number of pairwise “inversions” needed to convert one ranking into the other is 1. For example, to go from  $X \succ Y \succ Z$  to  $Y \succ X \succ Z$  requires us only to switch the  $\{X, Y\}$  ranking. These two rankings, therefore, are connected by an edge. The Kemeny distance between any two rankings is simply the length of the shortest path between them. The graph is represented in the following figure. For simplicity, we have used each ranking’s number for the vertices rather than the ranking itself.

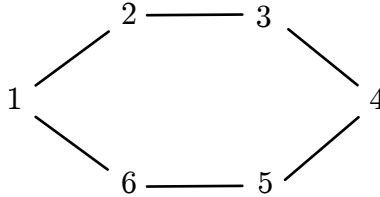


Figure 3: The Kemeny graph.

As we can see, the maximum Kemeny distance between any two rankings is 3.<sup>12</sup>

Each persuasion cost induces a value for  $\delta$ . For example, if  $\frac{1}{\delta} = \infty$  then  $\delta = 0$ . This value of  $\delta$  enables us to construct what we term a “maximal

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<sup>12</sup>We use Kemeny distance in this paper because it is widely known. An argument for its appropriateness as a measure of distance between strict orderings only can be found in Duddy and Piggins (2010). Kemeny and Snell’s axiomatisation covers both weak and strict orderings.

$\delta$ -consistent partition". This is a partition of the set of individuals with the following characteristics.

1. Each part of the partition contains no two individuals who are more than a distance of  $\delta$  away from each other (in terms of their preferences).<sup>13</sup> For example, if  $\delta = 0$  then the only partition that satisfies this requirement is  $\{n(1), n(2), n(3), n(4), n(5), n(6)\}$  where  $n(1)$  is the set of individuals who believe that  $X \succ Y \succ Z$ , and so on. Recall that we assume that each logically possible preference is held by at least one individual at the pre-deliberation stage.
2. The partition is "maximal" in the sense that the number of equivalence classes is as small as possible given the value of  $\delta$ . For example, if  $\delta = 1$  then the partition  $\{n(2) \cup n(3), n(6) \cup n(5), n(1), n(4)\}$  is not maximal since it has 4 parts. The partition  $\{n(1) \cup n(2), n(3) \cup n(4), n(5) \cup n(6)\}$  is maximal as it has only 3 parts. Note that the partition  $\{n(1) \cup n(6), n(5) \cup n(4), n(2) \cup n(3)\}$  is also maximal in this case.
3. Individuals with identical preferences are always in the same part of the partition.

By varying the cost of persuasion ( $\frac{1}{\delta}$ ) we can nest various scenarios into the model.

If the cost of persuasion is infinite, then  $\delta = 0$  and the only maximal  $\delta$ -consistent partition is  $\{n(1), n(2), n(3), n(4), n(5), n(6)\}$ . A persuasion group is simply a part of any maximal  $\delta$ -consistent partition.

In this extreme situation (when the cost of persuasion is infinite) there will be no change in anyone's preferences. We assume that preference change can only occur after a debate has taken place between people who have different beliefs. This necessary condition for preference change is not satisfied here (every persuasion group contains individuals with identical preferences). This

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<sup>13</sup>The parts of a partition are its equivalence classes.

means that no persuasion will take place; everyone’s post-deliberation (final) preferences will be identical to their pre-deliberation (original) preferences. A social choice is then made by applying these preferences to the voting rule.

We would argue that the  $\frac{1}{\delta} = \infty$  case approximates the standard social choice model. In that model, there is an unrestricted domain of preferences. This means that any logically possible profile of preferences<sup>14</sup> is a potential input into the voting rule. Our requirement that at the pre-deliberation stage at least one individual holds each logically possible preference means that our domain is restricted slightly. It is, however, close enough to being unrestricted.

As we will see, more interesting cases arise when  $\frac{1}{\delta} = 1$  and  $\frac{1}{\delta} = \frac{1}{2}$ . In both of these cases, persuasion groups can form containing individuals who are not identical with respect to their preferences. We say that the cost of persuasion is “low” when  $\frac{1}{\delta} = \frac{1}{2}$ . We say that the cost of persuasion is “intermediate” when  $\frac{1}{\delta} = 1$ . We state three formal propositions in this paper, one for each value of  $\frac{1}{\delta}$ . Intuitively speaking, as the cost of persuasion falls, the larger are the persuasion groups.

We shall now explain our preference change rule, which is relevant when the cost of persuasion is low or intermediate. Again, the assumption we make is favourable to deliberationists. Suppose that a persuasion group is formed containing individuals with different preferences, and that these individuals disagree with respect to their  $\{X, Y\}$  preference, for example. Some prefer  $X$  to  $Y$ , others prefer  $Y$  to  $X$ . We assume that the individuals whose beliefs are true (in terms of the underlying betterness relation) can persuade the others that their beliefs are false, and so the latter group will change their  $\{X, Y\}$  preference accordingly. Individuals then leave the persuasion group with their new preferences. In other words, in a persuasion group, any conflict over any pairwise ranking is always resolved in favour of the truth. Of course, if all individuals agree on the ranking of any pair, then they will not change

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<sup>14</sup>A profile is an  $n$ -tuple of orderings, one ordering per person.

their ranking of this pair when they leave the persuasion group.

In a model of genuine deliberation, an assumption is needed to describe how individual preferences change after a debate. We feel that our assumption is a reasonable one to make in this respect. It is certainly more plausible than assuming the converse; that individuals with false beliefs persuade those whose beliefs are true. Another way of thinking about this assumption is that the truth is revealed through the process of debate, and individuals update their beliefs accordingly. Like before, this optimistic assumption is broadly consistent with a deliberationist world view. However, as we will see, even with this assumption in place, deliberation does not always guarantee that the correct social choice is made. The assumption is neutral then in its implication.

There are three concepts left to discuss; deliberation paths, voting rules and truthful revelation.

A deliberation path is a sequence of one or more maximal  $\delta$ -consistent partitions. This sequence starts with the maximal  $\delta$ -consistent partition that forms in the first stage of the deliberation process (immediately after the pre-deliberation preferences), includes all subsequent maximal  $\delta$ -consistent partitions, and ends when no part of the partition contains individuals with different preferences.<sup>15</sup>

To make this clear, consider the following example. Assume that  $\delta = 1$  with the true social ordering being  $X \succ Z \succ Y$ . One possible deliberation path starts with the partition  $\{n(1) \cup n(6), n(5) \cup n(4), n(2) \cup n(3)\}$ . Applying our preference transformation rule, the individuals in  $n(1)$  are persuaded by those in  $n(6)$  that their  $\{Y, Z\}$  preference is false and so change their  $\{Y, Z\}$  preference accordingly. Essentially, the individuals in  $n(1)$  (call them “type-1” individuals) become “type-6” individuals, i.e. their preferences are now identical to those in  $n(6)$ .<sup>16</sup> Similar transformations occur across the other

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<sup>15</sup>We should emphasise that the value of  $\delta$  is constant along any deliberation path.

<sup>16</sup>The numbering here is simply taken from our earlier table.



parts of the partition. The type-3 individuals become type-2 individuals, and the type-4 individuals become type-5 individuals.

Given this, we can construct new sets  $n^*(6) = n(1) \cup n(6)$ ,  $n^*(2) = n(2) \cup n(3)$  and  $n^*(5) = n(4) \cup n(5)$ .

The second step on the deliberation path involves creating a new partition of  $n^*(6) \cup n^*(2) \cup n^*(5)$ . The only maximal  $\delta$ -consistent partition is  $\{n^*(6) \cup n^*(5), n^*(2)\}$ . Again, applying our transformation rule, we now have the sets  $n^{**}(6) = n^*(6) \cup n^*(5)$  and  $n^*(2)$ .

Note that the only maximal  $\delta$ -consistent partition of  $n^{**}(6) \cup n^*(2)$  is  $\{n^{**}(6), n^*(2)\}$ . At this stage, the deliberation path ends. This is because the only persuasion groups that form here contain individuals with identical preferences, and hence no more persuasion is possible.

The reader will be able to verify that another possible deliberation path starts with  $\{n(1) \cup n(2), n(3) \cup n(4), n(5) \cup n(6)\}$ . As we will see later, which path we are on has implications for social choice.

Next, voting rules.

For the purpose of this paper, a voting rule is any member of the family of so-called scoring rules. This family includes the well-known plurality voting rule and the Borda rule among others. The only admissible voting rules we consider in this paper are scoring rules.<sup>17</sup>

Our most important concept is truthful revelation.<sup>18</sup> Our formalism enables us to view a democracy as an ordered pair, the first element of which is the deliberation path, and the second element of which is the voting rule (taken from the family of scoring rules). We say that a voting rule and a deliberation path reveal the truth if the correct social alternative is always chosen irrespective of the original, pre-deliberation profile of preferences and

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<sup>17</sup>Young (1975) provides an axiomatisation of these rules. One could appeal to Young's theorem to argue that the correct voting rule is a scoring rule.

<sup>18</sup>A different, but identically named concept appears in the information economics literature. See Campbell (2006). The name seems particularly appropriate in our context, which is why we have adopted it.

irrespective of the size of the electorate (provided that the electorate is finite). In other words, a truth-revealing democracy will always produce the correct social choice through a combination of deliberation and voting; everyone's original preferences do not matter. The question we ask in this paper is: do truth-revealing democracies exist?

As we will see, when the cost of persuasion is infinite, no truth-revealing democracy exists. Conversely, when the cost of persuasion is low, many truth-revealing democracies exist. In fact, truthful revelation almost always occurs in this setting. Our most interesting result, however, concerns the intermediate case. We identify when and only when truthful revelation occurs in this case. There is one and only one truth-revealing democracy in this setting.

Truthful revelation is a natural property to explore in a model of deliberative and aggregative democracy. The importance of deliberation is reflected in the role of the deliberation path. The importance of aggregation is reflected in the choice of the voting rule. In the case of intermediate persuasion costs, we will learn that a unique truth-revealing democracy exists. If we change the voting rule (without changing the deliberation path) the democracy will no longer reveal the truth. The same happens if the deliberation path is changed without changing the voting rule. This means that deliberation and voting are equally important in the model.

The standard model of social choice ignores deliberation. But, similarly, deliberationists tend to sidestep difficult questions about the choice of voting rule. In the wake of Riker's critique, theorists of democracy have found themselves in one of two camps; aggregationist or deliberationist. The purpose of this paper is to construct a formal bridge between them.

### 3 Model

We use Saari’s geometry of voting to obtain our results.<sup>19</sup> First, we create what Saari calls a representation triangle. In an equilateral triangle identify each vertex with an alternative and define a binary relationship of a point in terms of its proximity to a vertex. Thus, point  $p$  corresponds to the ranking  $X \succ Y$  if and only if  $p$  is closer to vertex  $X$  than to vertex  $Y$ .

This relationship subdivides the equilateral triangle where the open regions (the smallest triangles) correspond to strict rankings without indifference among the alternatives, while the line segments and the baricentric point correspond to rankings with indifference. Given that we only consider strict preference, we can ignore all of these points.

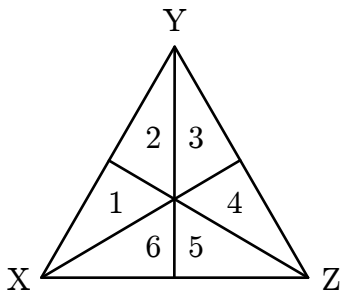


Figure 4: The representation triangle.

Points in each open region correspond to the same ranking. Points in region 1 correspond to the ranking  $X \succ Y \succ Z$ . Points in region 2 correspond to the ranking  $Y \succ X \succ Z$ , and so on. To illustrate which regions correspond to which rankings, the numbering from Table 1 is applied in Figure 4.

To represent a profile, we put numbers in the open regions.

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<sup>19</sup>For an introduction to Saari’s geometry, we recommend Saari (2001) and Saari (1995). Perote-Peña and Piggins (2002) give a simple proof of Arrow’s impossibility theorem using this geometry.

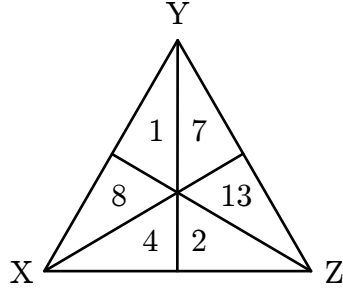


Figure 5: A profile.

In this example, 7 people hold the  $Y \succ Z \succ X$  preference, 4 people hold the  $X \succ Z \succ Y$  preference, and so on. Our model satisfies the normative requirement of anonymity. This says that the names of the individuals do not matter, only the number who hold any particular preference.

The following figure explains how the representation triangle can be used to derive a score for each alternative.

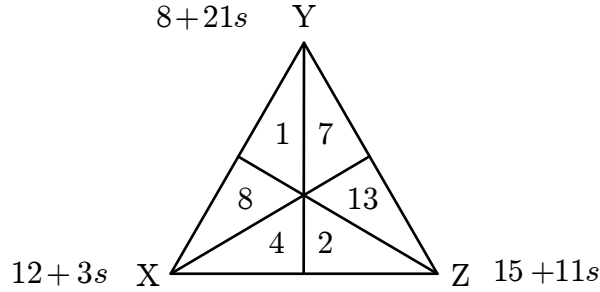


Figure 6: Scoring rules.

The number by each vertex indicates the number of individuals who have that alternative as their top-ranked alternative, plus the number of individuals who have that alternative ranked second. This second term is weighted by  $s \in [0, 1]$ . If  $s = 0$  then the number by each vertex indicates the number of individuals who place that alternative at the top of their ranking. From this information we can identify the plurality winner ( $Z$  in the example). Varying  $s$  from 0 to 1 tells us which alternative will be selected under different

scoring rules. If  $s = 0.5$  then we have the Borda rule,  $s = 1$  corresponds to the antiplurality rule. The antiplurality rule selects the alternative that is ranked bottom by the smallest number of individuals ( $Y$  in the example).

Next, we show how to represent persuasion groups and the objectively correct ranking.

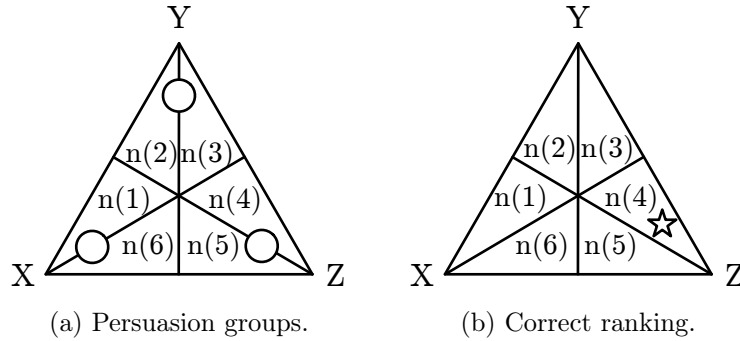


Figure 7

We should emphasise that to keep notation simple we sometimes refer to  $n(1)$  as the number of type-1 individuals (i.e. the number who believe  $X \succ Y \succ Z$ ), and on other occasions we refer to  $n(1)$  as the set of type-1 individuals. However, it is always clear from the context which we mean.

To illustrate this, in Figure 7 the number of individuals who hold the  $X \succ Y \succ Z$  preference is  $n(1)$ , and so on.

However, in Figure 7(a), persuasion groups are indicated by circles connecting distinct ranking regions. So  $n(1) \cup n(6)$  is a persuasion group, as is  $n(2) \cup n(3)$  and  $n(4) \cup n(5)$ . Here we are using  $n(\cdot)$  set-theoretically.

We depict the objectively correct ranking by a star symbol, as in Figure 7(b). As the star is in region 4, the social betterness relation is  $Z \succ Y \succ X$ .

## 4 Results

Let us first consider the  $\frac{1}{\delta} = \infty$  case. Consider the following example.

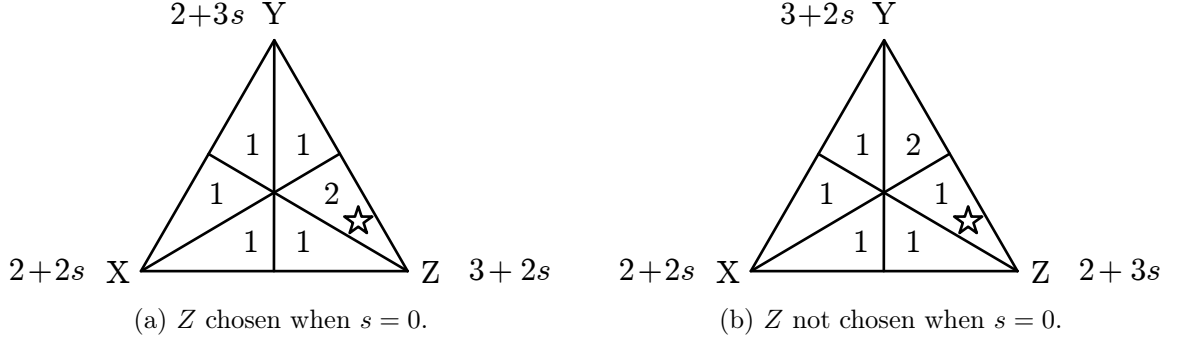


Figure 8:  $\frac{1}{\delta} = \infty$  case.

When  $\frac{1}{\delta} = \infty$  then  $\delta = 0$  and the only maximal  $\delta$ -consistent partition is  $\{n(1), n(2), n(3), n(4), n(5), n(6)\}$ . As we explained earlier, in this case the pre-deliberation profile is identical to the post-deliberation profile. The deliberation path contains just one maximal  $\delta$ -consistent partition.

We apply this logic to the profile in Figure 8(a). Using this profile, we see that  $Z$  is chosen under the plurality rule (when  $s = 0$ ). Moreover,  $Z$  is the correct social choice. However, it is trivial to see that a profile exists where  $Z$  is not chosen under this rule (Figure 8(b)). This argument generalizes. To any profile at which the correct social alternative is chosen (for some particular value of  $s$ ), there exists a profile at which this alternative is not chosen (for the same value of  $s$ ). However, truthful revelation requires that we always make the correct social choice irrespective of profile (and size of electorate).

This yields the following proposition.

**Proposition 1.** *If the cost of persuasion is infinite, then no truth-revealing democracy exists.*

There is a central part of Riker's critique of voting that can be replicated when  $\delta = 0$ . This concerns the point he makes about different voting rules producing different social choices from the same profile of preferences. It is easy to see how this phenomenon can be replicated by simply varying the

value of the  $s$  parameter.<sup>20</sup>

Let us now consider the  $\frac{1}{\delta} = 1$  case and so  $\delta = 1$ . There are two deliberation paths to consider here. Let us consider first the path on which the initial partition is  $\{n(1) \cup n(6), n(5) \cup n(4), n(2) \cup n(3)\}$ . The persuasion groups that form here combine individuals with a common top-ranked alternative into the same part of the partition. We represent this in Figure 9.

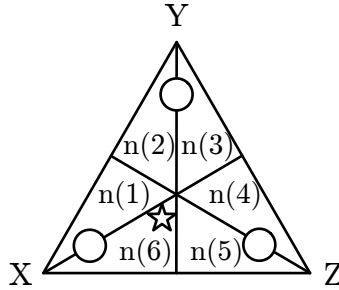


Figure 9: Persuasion groups with common top-ranked alternative.

Without loss of generality, assume that  $X \succ Z \succ W$  is the correct ranking of the alternatives (the social betterness relation).

Our preference change rule implies that, at the next stage, we have the following profile of preferences. To save space we write 1 instead of  $n(1)$ , etc.

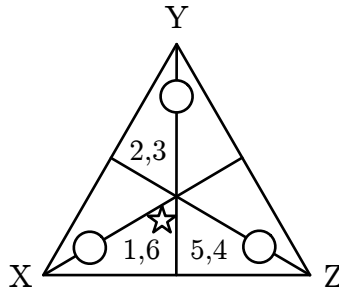


Figure 10: The subsequent profile.

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<sup>20</sup>Saari deals comprehensively and definitively with this issue in the references cited earlier (and in other work).

At this stage, the  $n(2)$  and  $n(3)$  individuals now hold identical preferences. They all believe  $Y \succ X \succ Z$  (they are all type-2 individuals, in other words). In addition, the  $n(1)$  and  $n(6)$  individuals now hold the  $X \succ Z \succ Y$  preference and the  $n(5)$  and  $n(4)$  individuals now hold the  $Z \succ X \succ Y$  preference. The Kemeny distance between the latter two groups is 1.

This means that we can form one more maximal  $\delta$ -consistent partition on our deliberation path. This is depicted in Figure 11. Here we combine individuals with a common bottom-ranked alternative into the same part of the partition.

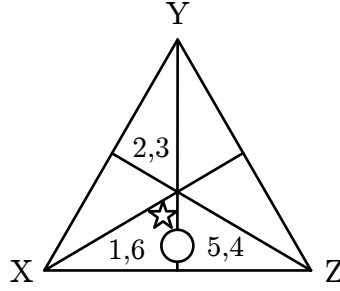


Figure 11: Persuasion group with common bottom-ranked alternative.

Again, our preference change rule implies that, at the next stage, we have the following profile of preferences. This profile is, in fact, the final post-deliberation profile of preferences. This final profile is represented in Figure 12. As we can see, deliberation has not produced unanimous preferences.

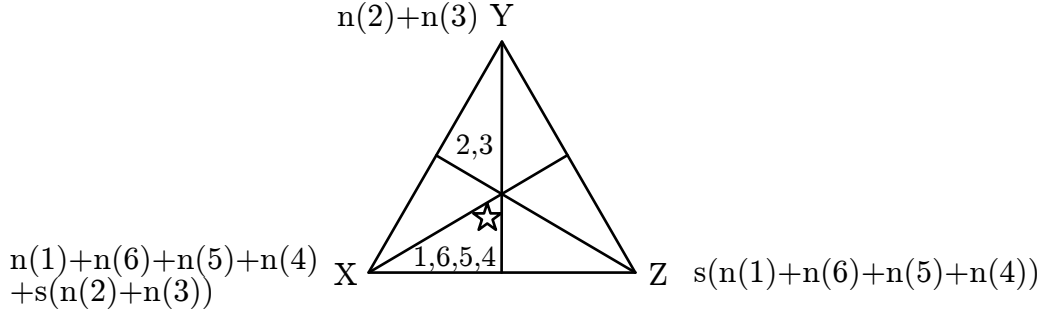


Figure 12: Final, post-deliberation profile.



The score for each alternative is represented by the expression by each vertex. Can a scoring rule guarantee that  $X$  is selected? Clearly, this can only happen if and only if

$$n(1) + n(6) + n(5) + n(4) + s(n(2) + n(3)) > n(2) + n(3).$$

However, this inequality is satisfied if and only if  $s = 1$ . In other words, the antiplurality rule will guarantee that the correct social alternative is chosen when applied to this deliberation path, irrespective of the original profile of pre-deliberation preferences and irrespective of the size of the electorate. A truth-revealing democracy exists.

Is it unique? To answer this question, let us consider the only other possible deliberation path that can arise in this case. The initial partition is  $\{n(1) \cup n(2), n(3) \cup n(4), n(5) \cup n(6)\}$ . This is represented in Figure 13.

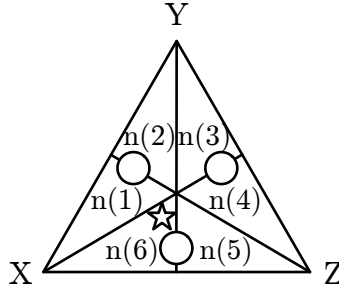


Figure 13: Persuasion groups with common bottom-ranked alternative.

In this case the persuasion groups that form combine individuals with a common bottom-ranked alternative into the same part of the partition.

Our preference change rule implies that, at the next stage, we have the following profile of preferences.

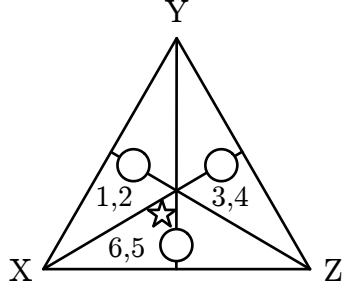


Figure 14: The subsequent profile.

At this stage, the  $n(3)$  and  $n(4)$  individuals now hold identical preferences. They all believe  $Z \succ Y \succ X$  (they are all type-4 individuals, in other words). In addition, the  $n(6)$  and  $n(5)$  individuals now hold the  $X \succ Z \succ Y$  preference and the  $n(1)$  and  $n(2)$  individuals now hold the  $X \succ Y \succ Z$  preference. The Kemeny distance between the latter two groups is 1.

This means that we can form one more maximal  $\delta$ -consistent partition on our deliberation path. This is depicted in Figure 15. Here we combine individuals with a common top-ranked alternative into the same part of the partition.

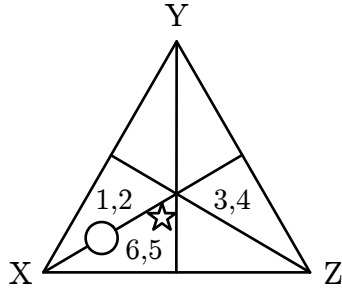


Figure 15: Persuasion group with common top-ranked alternative.

Again, our preference change rule implies that, at the next stage, we have the following profile of preferences. This profile is, in fact, the final post-deliberation profile of preferences. This final profile is represented in

Figure 16. Just like before, deliberation has failed to produce unanimous preferences.

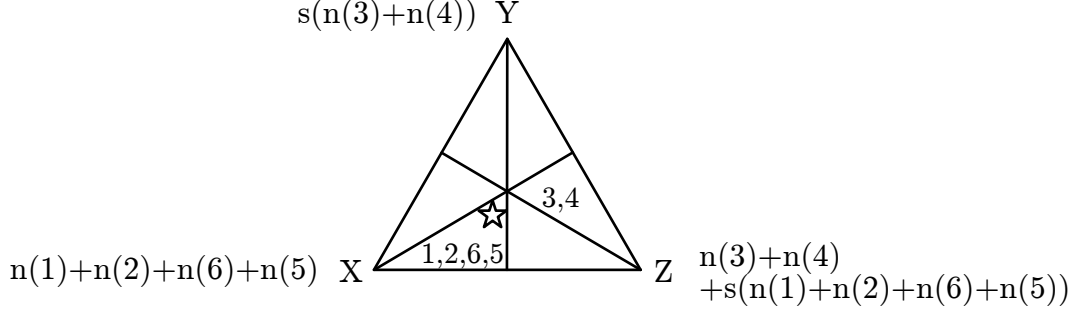


Figure 16: Final, post-deliberation profile.

The score for each alternative is represented by the expression by each vertex. Can a scoring rule guarantee that  $X$  is selected? Clearly, this can happen if and only if

$$n(1) + n(2) + n(6) + n(5) > n(3) + n(4) + s(n(1) + n(2) + n(6) + n(5)).$$

However, there is no value of  $s$  for which this inequality is always satisfied. Truthful revelation does not exist on this deliberation path. Therefore, we have demonstrated the following.

**Proposition 2.** *If the cost of persuasion is intermediate, then a truth-revealing democracy exists. Moreover, this truth-revealing democracy is unique. The deliberation path takes the following form. In the first stage, form persuasion groups consisting of individuals with identical top-ranked alternatives. In the second stage, form persuasion groups consisting of individuals with identical bottom-ranked alternatives. When the deliberation process is completed, apply the antiplurality rule.*

It is important to emphasise that the  $X \succ Z \succ Y$  ranking (the objectively correct one) used in the proof is arbitrary. The same conclusion applies

irrespective of what the true ordering of alternatives happens to be. In other words, the star could be in any region of the representation triangle, and the procedure described above will always “work”. Moreover, nothing else will. To state it another way, it does not matter what the truth actually is, the above procedure will always reveal it.

Only the “right” deliberation path combined with the “right” voting rule (the antiplurality rule) is truth-revealing. This “right” deliberation path combines individuals with a common top-ranked alternative first, and then combines individuals with a common bottom-ranked alternative second. Strikingly, just reversing the order of this deliberation path (combining individuals with a common bottom-ranked alternative first, and then combining individuals with a common top-ranked alternative second), together with the antiplurality rule is not truth-revealing. This asymmetry is surprising. Similarly, using the right deliberation path but changing the voting rule is not truth-revealing either. A small reduction in the value of  $s$  is all that is required to lose the property of truthful revelation.

Another thing worth mentioning is this. Assume that a democracy is truth-revealing. At the pre-deliberation stage nobody has any incentive to misrepresent their preferences. The reason for this is that unilateral deviations at any profile do not affect the ultimate social choice, the correct social choice will be made irrespective. A truth-revealing democracy is classically strategy-proof. Of course, there are other places in our model where the issue of strategic behaviour could conceivably arise. However, we put this issue to one side. Strategic behaviour sits awkwardly with the kinds of processes that deliberationists appeal to, and we have tried to stay faithful to deliberationism in the paper. Deliberationists emphasise social learning, and adopt a concept of agency that, arguably, differs from that of classical game theory.<sup>21</sup>

It is also worth noting that the deliberation paths we consider when  $\delta = 1$  ultimately produce final, post-deliberation profiles that are single-

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<sup>21</sup>Landa and Meirowitz (2009) provide a defence of the game-theoretic approach.

peaked in the sense of Black (1958). The process of deliberation produces single-peakedness, even when the “wrong” deliberation path is used. This possibility has been noted before, by Miller (1991), Knight and Johnson (1994) and Dryzek and List (2003). It is also broadly consistent with empirical evidence (see List, Luskin, Fishkin and McLean (2010)). However, it is worth pointing out an important difference between single-peakedness as it arises here, and the arguments for deliberation-induced single-peakedness made by these other authors. In this paper, single-peakedness does not arise due to agreement about any underlying “dimension” on which the alternatives can be naturally located (the dimension orders the alternatives in a left-right sense). Rather, it emerges as an unintended consequence of the process of persuasion. When debating with one another in a persuasion group, the individuals who are correct persuade those who are incorrect. The ultimate outcome of this process is that the final profile of preferences is single-peaked, but no one intends for this to happen. It does not arise due to agreement about an underlying dimension, the causal mechanism is different. In the model, it is essentially an accident. However, this accident is neutral in its implication: as we have seen, single-peakedness does not guarantee truthful revelation.

Our final case is where  $\frac{1}{\delta} = \frac{1}{2}$ . In this case  $\delta = 2$  and there are several possible deliberation paths. Without loss of generality, assume that the initial partition is  $\{n(6) \cup n(1) \cup n(2), n(3) \cup n(4) \cup n(5)\}$ . Then, Figure 17(a) shows how preferences within these persuasion groups change after the first stage of deliberation. After the second stage, we have a unanimous profile (Figure 17(b)). Of course,  $X$  is chosen if and only if  $s < 1$ .

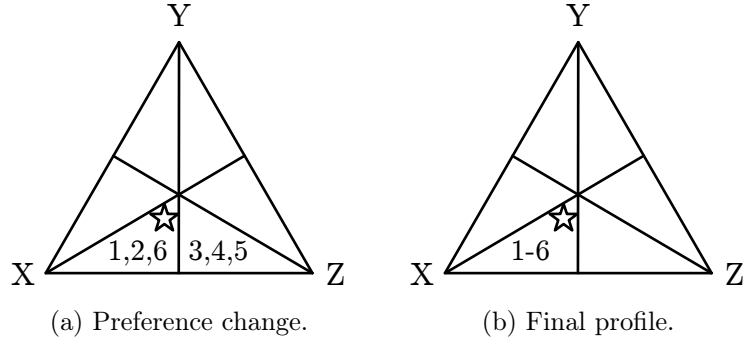


Figure 17

This yields the following.

**Proposition 3.** *If the cost of persuasion is low, then truthful-revelation almost always occurs. Any deliberation path and any voting rule will be truth-revealing, provided that  $s < 1$ .*

As is clear from the analysis above, any deliberation path will produce a unanimous profile, but truthful revelation requires in addition that  $s < 1$ . Interestingly, the only democracies that fail to be truth-revealing are those that use the antiplurality rule (the antiplurality rule produces a tie, not a victory for  $X$ ). Strikingly then, the only voting rule that works in the  $\frac{1}{\delta} = 1$  case is the only one that fails in the  $\frac{1}{\delta} = \frac{1}{2}$  case.

We can think of the  $\frac{1}{\delta} = \frac{1}{2}$  case as corresponding to Elster's view of the impact of deliberation. Deliberation produces unanimity, making the problem of social choice (almost) trivial.

As we have seen, by varying the cost of persuasion it is possible to nest various scenarios into the model.

## 5 Conclusion

We have attempted to construct a model of deliberative and aggregative democracy. The deliberationist tradition assumes that individual preferences

can change after debate, and that the quality of decisions is enhanced through discussion and social learning. The aggregationist tradition (like much of economics) regards individual preferences as immutable and proposes using them for the purpose of making collective decisions where possible.<sup>22</sup>

Some supporters of deliberative democracy, like Elster (1986), have argued that deliberation alone should lead to unanimity and so there is no need to worry about aggregation. Aggregationists have countered that an emphasis on deliberation and the expression of conflicting opinions can make matters worse by leading to the very lack of structure that leads to impossibility theorems.<sup>23</sup> According to this view, it is too optimistic to expect greater consensus from deliberation. Deliberation can, in fact, make preference aggregation more difficult.

In an extensive and less radical discussion, Dryzek and List (2003) have concluded that both approaches to democracy can be reconciled. They argue, among other things, that deliberation can narrow the domain of individual preferences and make them easier to aggregate. Some empirical evidence exists to support this view.

Our model can be seen as a contribution to this debate by making explicit one possible process of preference change through deliberation (a proper deliberation “technology”) that precedes the preference aggregation stage. In our model, a democracy consists of two things: a deliberation path and a voting rule. The extent to which persuasion is possible among dissimilar individuals is limited by the “persuasion cost”, and when no more persuasion is possible, a voting rule chooses an alternative based on the final preference profile.

We are conscious of the fact that our model has limitations, but this is

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<sup>22</sup>For example, economists typically favour policies that make some people better off, without making anyone else worse off. In doing so they take preferences to be a representation of people’s well-being. They are reluctant to favour policies that make some people better off and others worse off.

<sup>23</sup>See van Mill (1996).

true of all models. We restrict attention to three alternatives. Persuasion is a deterministic process, not a stochastic one. Furthermore, no significance is attached to the number of individuals who hold a particular opinion. More realistic models of preference change through deliberation should certainly address these issues. However, our aim is not to produce a realistic model of deliberation, but an analytically tractable one that works in conjunction with aggregation. This sheds some light on the mutual interrelationship between aggregation and deliberation, and suffices to show that in a combined model, the “optimal” voting rule depends critically on the persuasion cost involved, and also on the specific deliberation path chosen. These two things can lead to structure emerging in the preference domain, and this has implications for social choice.

We have attempted to show how the process of deliberation can be incorporated into classical social choice theory. Of course, there may be other ways of achieving this, and different models may turn out to be superior to the one we have described here. However, we feel that the concepts we have introduced could be of value in developing these new models. Ultimately, we advocate the need to accumulate more empirical evidence about the process of deliberation in specific contexts. These insights could then inform the construction of formal models which would help in any reconciliation between aggregation and deliberation.

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