6 Fallacies of the Traditional Model in Voting

During the Middle Ages there were all kinds of crazy ideas, such as that a piece of rhinoceros horn would increase potency. Then a method was discovered for separating the ideas—which was to try one to see if it worked, and if it didn't work, to eliminate it. This method became organized, of course, into science.
—Richard P. Feynman

Several centuries of work on the theory of social choice have produced very substantial contributions, notably, in identifying a host of important properties or criteria that should (or should not) be satisfied by a mechanism that amalgamates the beliefs, desires, or wills of individuals into a decision of society.

Arrow's paradox must be avoided: a method should satisfy independence of irrelevant alternatives, that is, the presence or absence of some candidate should not cause a change of winner between two others. Condorcet's paradox must be avoided: a method should yield a transitive order of finish among the competitors. A method should be monotonic: a winning candidate who receives more votes or rises in the rank-orders of candidates must remain the winning candidate. A unanimous decision among individual voters must be the decision of society. Mechanisms should make voters' optimal strategies be those messages that honestly express their beliefs; or, if no such mechanism can be found, then one that best resists strategic manipulation and best incites the electorate to express themselves honestly must be found.

Regrettably, the theory shows that even when voters eschew strategic voting and honestly express their convictions, there exists no method that satisfies the essential criteria, unless it is assumed that voters have very restricted types of unrealistic views. The impossibility and incompatibility theorems prove that the traditional model harbors internal inconsistencies. The reason for this conundrum is the basic paradigm of social choice: voting depends on comparisons between pairs of candidates—one is better than another—so voters have

lists of preferences in their minds. Instead of inputs that evaluate the *absolute merits* of candidates, the inputs compare the *relative standings* of candidates. But even the idea of comparing is questionable: if the decision or output is to be a rank-order of the candidates, should not the voters be asked to compare the relative merits of the various possible rank-orders rather than only the relative merits of candidates?

6.1 Unrealistic Inputs

Every bit as damning as the logical inconsistencies of the theory is the fact that the traditional paradigm of voting theory—that voters, when confronted by a set of candidates, compare them or rank-order them—is simply wrong. Voters do not go to the polls with rank-orders of the candidates in their minds. The French presidential elections of 2002 and 2007 had, respectively, sixteen and twelve candidates. Instead of effecting a rank-ordering a voter ignored most candidates as unacceptable and looked upon a few with varying intensities of approval or disapproval. The model that underlies the theory simply does not correspond to reality. Experimental evidence proves this conclusively. Information drawn from three electoral experiments refute the traditional view as well as several other preconceived ideas.

The Orsay experiment (see chapters 1 and 15) tested the majority judgment, so voters' inputs were expressed in a common language of grades—*Excellent, Very Good, Good, Acceptable, Poor*, and *To Reject* [1]—evaluating the merits of candidates. Of the 2,360 who voted, 1,752 officially participated; 1,733 ballots were valid. Contrary to the predictions of some, the voters had no difficulty in filling out the ballots, usually doing so in about one minute. In fact, every member of the team conducting the experiment had the impression that the participants were very glad to have the means of expressing their opinions concerning *all* the candidates, and were delighted with the idea that candidates would be assigned final grades.[2] An effective argument to persuade reluctant voters to participate was that the majority judgment allows a much fuller expression of a voter's opinions. The actual system offered voters only thirteen possible *messages*: to vote for one of the twelve candidates or to vote for none. Several participants actually stated that the experiment had induced them to vote for the first time: finally, a method that permitted them to express

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1. *Très Bien, Bien, Assez Bien, Passable, Insuffisant*, and *A Rejeter*.

2. A collection of television interviews of participants prepared by Raphaël Hitier, a journalist of *I-Télé*, confirms these impressions. Also, a questionnaire used in the ILC experiment (see chapter 17) shows that voters prefer using three number grades rather than two.

themselves. The majority judgment offered voters more than 2 billion possible messages with which to express themselves (with twelve candidates and six grades, there are $6^{12} = 2,176,782,336$ possible messages). The voters' relative ease of expression in the face of so vast a choice shows that assigning grades is cognitively simple, certainly much simpler than ranking candidates (as any teacher or professor faced with ranking students will attest). Of the 1,733 valid majority judgment ballots, 1,705 were different. It is surprising that they were not all different. Had all those who voted in France in 2007 (some 36 million) cast different majority judgment ballots, fewer than 1.7% of the possible messages would have been used. Those that were the same among the 1,733 valid messages of the experiment contained only *To Rejects* or accorded *Excellent* for one or several candidates and *To Reject* for all the others.

Voters were particularly happy with the grade *To Reject* and used it the most. There was an average of 4.1 of *To Reject* per ballot and an average of 0.5 of no grade (which, in conformity with the stated rules, was counted as a *To Reject*). Voters were parsimonious with high grades and generous with low ones (see table 6.1). Only 52% of voters used a grade of *Excellent*; 37% used *Very Good* but no *Excellent*; 9% used *Good* but no *Excellent* and no *Very Good*; 2% gave none of the three highest grades. The opinions of voters are richer, more varied and complex by many orders of magnitude than they are allowed to express with any current system.

The highest grades were often multiple (see table 6.2). In all, *more than 33% of the ballots gave the highest grade to at least two candidates.* Thus one of every three voters did not designate a single best candidate. This shows that many voters either saw nothing (or very little) to prefer among several candidates, or at the least, they were very hesitant to make a choice among two, three, or more candidates. Moreover, many voters did not distinguish between the leading candidates: 17.9% gave the same grade to Bayrou and Sarkozy (10.6% their highest grade to both), 23.3% the same grade to Bayrou and Royal (11.7% their highest grade to both), and 14.3% the same grade to Sarkozy and Royal (4.1% their highest grade to both). Indeed, 4.8% gave the same grade to all three (4.1% their highest grade to all three: all who gave their highest grade to Sarkozy and Royal also gave it to Bayrou). These are significant percentages: many elections are decided by smaller margins. These are valid, significant inputs of opinion that are completely ignored by the traditional model.

This finding is reinforced by a poll conducted on election day (by TNS Sofres–Unilog, Groupe LogicaCMG, April 22, 2007) that asked at what moment voters had decided to vote for a particular candidate. Their hesitancy in making a choice is reflected in the answers: 33% decided in the last week, one-third of

Table 6.1
Average Number of Grades per Majority Judgment Ballot, Three Precincts of Orsay, April 22, 2007

	Average No. of Grades per Ballot
Excellent	0.69
Very Good	1.25
Good	1.50
Acceptable	1.74
Poor	2.27
To Reject	4.55
Total	12.00

Note: Of the 4.55 *To Reject*, 0.5 corresponded to no grade.

Table 6.2
Multiple Highest Grades, Three Precincts of Orsay, April 22, 2007

Two or more *Excellent*	11%
Two or more *Very Good*, none higher	16%
Two or more *Good*, none higher	6%

whom (11%) decided on election day itself. For Bayrou voters 43% decided in the last week and 12% on election day; for Sarkozy voters the numbers were 20% and 6%; for Royal voters, 28% and 9%; for Le Pen voters, 43% and 18%. In contrast, the system *forced* them to make a choice of one (or to vote for no one).

Moreover, inputs that are rank-orders—or that simply show preferences between pairs of candidates—ignore how voters *evaluate* the respective candidates (just as the 2002 runoff ignored the respective evaluations of Chirac and Le Pen) except, of course, that one is evaluated higher than the other. Over one-half of the highest grades are less than *Excellent*. Two-thirds of the second highest grades are merely *Good* or worse (see table 6.3). To be first, second, or third in a ranking of at least three candidates carries very different meanings to different voters that are completely ignored by the inputs to the traditional model. This is still another reason that aggregating rank-orders (as do the methods of Condorcet and of Borda, and their combinations) is not meaningful.

The Faches-Thumesnil experiment tested two versions of the alternative vote, so voters' inputs were rank-orders of the candidates (Farvaque, Jayet, and Ragot 2007). The experiment was conducted in two of the eleven voting precincts of Faches-Thumesnil, a small town in France's northernmost department, Nord. Voters were not obliged to rank-order all candidates (as in Australia): a candidate

Table 6.3
Distributions: Highest Grade, Second Highest Grade, Third Highest Grade, Three Precincts of Orsay, April 22, 2007

	Excellent	Very Good	Good	Acceptable	Poor	To Reject
Highest	52%	37%	9%	2%	0%	1%
Second highest	–	35%	41%	16%	5%	3%
Third highest	–	–	26%	40%	22%	13%

Table 6.4
Number of Candidates Rank-Ordered, Faches-Thumesnil Experiment, April 22, 2007

	Number of Candidates Rank-Ordered			
	1–3	4–6	7–11	12
No. of ballots	260	210	53	370
Percent of ballots	29.1%	23.5%	5.9%	41.4%

not on the list of a voter's ballot was considered off the list and thus could never be placed first on the voter's list after elimination of other candidates. Of those who voted officially, 960 (or 60%) participated in the experiment, 67 ballots were invalid, and 893 were valid. Almost 60% of the ballots did not rank-order all candidates and over 50% rank-ordered six or fewer of the twelve candidates, showing that voters are reluctant to rank-order many candidates (see table 6.4).

Admittedly, it is a difficult and time-consuming task to rank-order alternatives, and in any case, whether a voter rank-orders many or few candidates, she is unable to express any sense of how much or how little any of the candidates are appreciated. Suppose there are n candidates. To rank-order them a voter first places some one candidate on a list; then places the second in the slot above or below; then the third in one of the three slots (above, between, below); and so on. This takes $n(n + 1)/2$ time units. And if ties are not allowed, ranking becomes even more difficult. In contrast, it is a much easier task to assign each candidate a grade than to rank-order candidates. In practice, with a natural well-understood language of grades, a voter quickly situates an approximate grade for each candidate (e.g., Sarkozy is *Good* or *Very Good*) and thus takes about $2n$ time units (and in any case, a maximum of mn when there are m grades). Cognitively, assigning grades seems to be a much simpler exercise than ranking candidates. But whatever the reason, ranking a large number of alternatives is clearly very difficult, as is seen by the fact that about 95% of Australian voters rely on predetermined rankings provided by their parties.

The two versions of the alternative vote tested concerned the choice of candidate to eliminate when there is no majority for any candidate among the (current) first places. The Australian system eliminates the candidate listed first the least number of times. The other version eliminates the candidate listed last the greatest number of times. The Australian version makes Sarkozy the winner; the other version makes Bayrou the winner. The Australian version is less favorable to centrists because major candidates of the right and the left are usually either high or low on voters' lists. This may explain why it is used in practice rather than the other version. The other method, sometimes called the Coombs method, guarantees the election of the Condorcet-winner when the preferences are single-peaked and the votes are sincere, which is not true of the first method (see Grofman and Feld 2004; Nagel 2007).

The official first-round results in the two voting precincts of Faches-Thumesnil were very close to the national percentages (table 6.5). The voters' rank-orders make it possible to compute the results of the face-to-face confrontations (table 6.6). They yield the same unambiguous order of finish among the four significant candidates as did the polls on March 28 and April 19 (see table 2.11). Once again the Condorcet-order agrees with the Borda-ranking: Bayrou ≻ Sarkozy ≻ Royal ≻ Le Pen.

Table 6.5
Official First-Round Votes, National and Two Precincts of Faches-Thumesnil, April 22, 2007

	Sarkozy	Royal	Bayrou	Le Pen	Besancenot	de Villiers
National	31.2%	25.9%	18.6%	10.4%	4.1%	2.2%
Faches-Thumesnil	29.7%	25.5%	19.7%	12.0%	3.7%	2.4%

	Buffet	Voynet	Laguiller	Bové	Nihous	Schivardi
National	1.9%	1.6%	1.3%	1.3%	1.1%	0.3%
Faches-Thumesnil	2.4%	1.4%	1.5%	0.9%	0.5%	0.3%

Table 6.6
Projected Second-Round Results, Faches-Thumesnil Experiment, April 22, 2007

	Bayrou	Sarkozy	Royal	Le Pen
Bayrou	–	52%	60%	80%
Sarkozy	48%	–	54%	83%
Royal	40%	46%	–	73%
Le Pen	20%	17%	27%	–

Note: For example, Sarkozy has 48% of the votes against Bayrou.

6.2   Statistical Left-Right Spectra

The one escape from the inner inconsistencies of the traditional model of social choice occurs when voters have single-peaked preferences relative to a common ordering of the candidates. It comes from the idea that in the political realm candidates may be listed on a line from left to right, voters place themselves somewhere along it, prefer the candidate closest to their position, and dislike candidates more, the more distant they are from their position. Were there such a line, and were it true that voters' preferences for candidates are single-peaked, inputs of rank-orders would satisfy the aims of the traditional theory: the winner would be the Condorcet-winner, and the order of finish would be transitive in conformity with the face-to-face votes. The reality, long recognized, is that there is no such left-to-right line for which preferences are single-peaked. New experimental evidence confirms it.

An electoral experiment was conducted in parallel with the French presidential election of 2002 in five of Orsay's twelve voting precincts (under the same general conditions as the 2007 Orsay experiment). Its aim was to test approval voting (see chapter 18 for a detailed description of the experiment). The experimental ballot contained a list of the sixteen candidates together with instructions saying:

Rules of approval voting: The elector votes by placing crosses [in boxes corresponding to candidates]. He may place crosses for as many candidates as he wishes, but not more than one per candidate. The winner is the candidate with the most crosses.

On average there were 3.15 crosses per ballot. The total number of different possible messages was $2^{16} = 65,536$. Of the 2,587 valid ballots, 813 were different. Voters had no incentive to vote other than sincerely, namely, if a cross was given to some candidate C, then a cross was given to every candidate preferred to C as well. But if there existed a left-to-right line relative to which the voters' preferences are single-peaked, then the total number of different possible sincere votes would have been 137. The crosses would have to have been consecutive with regard to the alignment along the spectrum: there are 16 sincere messages with one cross, 15 with two consecutive crosses, 14 with three consecutive crosses, ... , 1 with sixteen consecutive crosses, and 1 with no crosses, so in all 137 sincere votes. The large discrepancy between 137 and 813 proves that the single-peaked condition was far from satisfied.

To assume single-peaked preferences is certainly not valid in elections. On the other hand, there is no denying that candidates and their political parties seeking election are commonly described in terms of a left-right spectrum and that this makes sense to political scientists, journalists, and the general public in

France, the U.K., the U.S.A., and throughout the world. The Orsay experiments of 2002 and 2007 both give solid scientific evidence that this is a valid concept.

Ballots from the 2002 experiment with several crosses yield statistical information about how voters favorable to one candidate might transfer their votes to others. For example, an estimate for, say, Bayrou may be computed as follows: among the ballots containing a cross for Bayrou and $k \geq 1$ other crosses, attribute $1/k$ to each of the other candidates with a cross, and find the sum given each candidate. The estimate of the transfer to a candidate is the percentage that the candidate's sum represents of the total sum (see table 6.7). *Statistically, the voters' transfers are almost single-peaked among the important candidates.* For instance, among those who gave a cross to Chirac, Bayrou was the most likely transfer, and the further distant from Chirac on the left-right line, the less likely the transfer. This does not hold for the unimportant candidates. The deviations are strikingly small among the important candidates and are easily explained. Chirac, the incumbent president, often exerted an appeal to voters in excess of the left-right spectrum (e.g., 16% of Chevènement voters go to Chirac, only 14% to Bayrou); crosses were sometimes given to the far right and the far left as expressions of opposition (e.g., more Gluckstein voters go to Le Pen than to Bayrou).

Table 6.7
Estimated Transfers of Votes to Important Candidates, Based on 2002 Orsay Experiment

	Left ⟵ Mamère	Jospin	Chevènement	Bayrou	Chirac	⟶ Right Le Pen	Ten Others
Gluckstein	15%*	9%	5%	2%	4%	5%	60%
Laguiller	14%	20%*	9%	4%	7%	5%	45%
Hue	13%	33%*	10%	3%	2%	2%	37%
Besancenot	20%	21%*	9%	5%	3%	3%	39%
Mamère	–	38%*	8%	7%	4%	1%	42%
Taubira	15%	28%*	10%	8%	4%	0%	35%
Jospin	26%*	–	15%	8%	5%	1%	45%
Chevènement	8%	20%*	–	14%	16%	6%	36%
Bayrou	6%	10%	13%	–	27%*	4%	40%
Chirac	3%	5%	13%	24%*	–	10%	45%
Madelin	3%	4%	9%	22%	32%*	6%	24%
Lepage	7%	12%	12%	17%*	16%	2%	34%
Boutin	4%	4%	6%	23%*	17%	5%	41%
Le Pen	3%	3%	13%	10%	26%*	–	45%
Saint-Josse	3%	6%	10%	10%	23%*	9%	39%
Mégret	1%	1%	5%	11%	22%	36%*	24%
Average transfer	9.4%	14.3%	9.9%	11.2%	13.7%	6.4%	

Note: Boldface indicates important candidates.
    Percentages with asterisks are the largest in their rows.

A left-right line—among all the candidates shown in column 1 of table 6.7, it would go from top to bottom—is constructed as follows. The three candidates receiving the highest average transfers are singled out: Jospin, Bayrou, and Chirac. One is the principal candidate of the left, one of the center, and one of the right. The statistics show that Jospin voters favor Bayrou (8%) over Chirac (5%) and Chirac voters favor Bayrou (24%) over Jospin (5%), so Bayrou must be placed in the center if one seeks order along a line consistent with single-peaked transfers. The choice of Jospin on the left and Chirac on the right is arbitrary (but in keeping with the political meaning of the directions). With sixteen candidates, the average transfer is $100/15 \approx 6.7\%$; the important candidates are those with a larger average, except for Le Pen (important because he survived the first round).

A candidate is classified to the left (to the right) if her voters transfer to the principal candidate of the left more than (less than) to the principal candidate of the right. So, for example, Laguiller is to the left (20% to Jospin, 7% to Chirac), and Madelin is to the right (4% to Jospin, 32% to Chirac). The result—Gluckstein through Chevènement to the left, Chirac through Mégret to the right—is consistent with the media's and the generally accepted classification of the candidates. The precise order of all the candidates on the left-right line can be found in various ways and so relies to a certain extent on an arbitrary rule. In this case, Chevènement is closer to Bayrou than Jospin and thus is classified as center left. To the left of Jospin the candidates are listed according to increasing total transfers to the two center candidates: Gluckstein gives them 7%, Laguiller 13%, . . . , Taubira 38%. To the right of Chirac the candidates are listed according to decreasing total transfers to the two center candidates: Madelin 31%, Lepage 29%, . . . , Mégret 16%. The single peak in the rows is accompanied by a single peak in the columns among the important candidates: in Bayrou's row the percentages among the important candidates decrease the further they are from Bayrou, and the same is true for Bayrou's column. In the rows of the unimportant candidates a single-peaked property holds: Madelin's peak is Chirac, and the further away, the smaller the percentage. When it is possible to achieve single peaks in rows and columns, the order is clearly unique: it defines the *statistical left-right spectrum*.

It is amusing to note that if the left-right line were as shown in table 6.7 and the median-voter mechanism proposed in Moulin (1980) were applied to the national vote (see table 2.9), Bayrou would have missed being the winner in 2002 by a hair; Bayrou's vote plus that of the candidates to his left was 49.7%, so Chirac would have been the winner. Were Lepage (with her 1.9% of the vote) classified between Bayrou and Chirac—not unreasonable in view of the transfers of her voters to candidates of the left—the median-voter mechanism would

have elected her. In 2002 the election of Bayrou with 7% of the first-round votes or of Lepage with 2% is unacceptable: neither was a major candidate.[3]

The estimates of transfers in 2007 are given in table 6.8 and are computed for, say, Bayrou as follows. Among the ballots whose highest grade goes to Bayrou, either $k \geq 1$ other candidates are given the same grade or Bayrou is the only candidate with that grade and there are $k \geq 1$ candidates who are given the next highest grade. Attribute $1/k$ to each of the other candidates in either case, and find the sum accorded to each candidate. The estimate of the transfer to a candidate is the percentage his sum represents of the total sum. Exactly the same rules are used to determine candidates of the left and the right and the order among them. In this case, there is only one center candidate, Bayrou. The left (from Buffet to Royal) and the right (from Sarkozy to Le Pen) correspond to the usual media designations. Important candidates are Le Pen and those whose average transfer is above $100/11 = 9.1\%$. Once again, *statistically, the voters' transfers are almost single-peaked among the important candidates.* The single peak in the rows (one small exception for the important candidate Le Pen) is accompanied by a single peak in the columns (two small exceptions, Besancenot and Le Pen). In two practical political situations *single-peaked transfers are real when seen in terms of probabilities.*

More strikingly than in 2002, the bulk of the transfers go to the important candidates: to Besancenot (far left), to Royal (moderate left), to Bayrou (center), and to Sarkozy (right). In fact, if the grades are used as determinants of preference among the three major candidates, 4.1% expressed the preference Royal $\succeq$ Sarkozy $\succ$ Bayrou and 5.8% Sarkozy $\succeq$ Royal $\succ$ Bayrou. So 90.1% of the ballots agree with the single-peaked preferences hypothesis on the left-right line among the three, going from Royal to Bayrou to Sarkozy (though among more candidates this is not true).

Not surprisingly, Bayrou is the choice of the median-voter nationally with respect to the left-right line of table 6.8: Bayrou's vote plus that of the candidates to his left was 57.7%; his vote plus that of the candidates to his right was 60.9%. These numbers are close to the estimates that are available of face-to-face confrontations with Royal (thus against the left) and with Sarkozy (thus against the right). A poll taken two days before the election shows the same

3. The 2002 Orsay experiment allows estimates to be made of the face-to-face races. To compute the estimate between two candidates, a vote is given to one whenever he is given a cross and the other is not. Jospin (19.5%) and Bayrou (9.9%) did better in the Orsay official vote than nationally, and Le Pen (10.0%) did worse. The estimates show Jospin winning against Chirac (with 53%), Bayrou (with 56%) and Le Pen (with 75%); Chirac winning against Bayrou (with 54%) and Le Pen (with 80%); Bayrou winning against Le Pen (with 74%). Jospin is at once the Condorcet-winner and the Borda-winner, and the Condorcet- and Borda-rankings are the same as well: Jospin $\succ_S$ Chirac $\succ_S$ Bayrou $\succ_S$ Le Pen.

Table 6.8
Estimated Transfers of Votes to Important Candidates, Based on 2007 Orsay Experiment

	Left ⟵ Besancenot	Royal	Bayrou	Sarkozy	⟶ Right Le Pen	Seven Others
Buffet	28%*	24%	5%	2%	2%	39%
Laguiller	32%*	17%	14%	11%	3%	23%
Bové	17%*	13%	15%	9%	2%	44%
Schivardi	29%*	11%	17%	5%	8%	30%
Besancenot	–	26%*	18%	3%	2%	51%
Voynet	13%	34%*	24%	9%	1%	19%
Royal	11%	–	44%*	10%	1%	35%
Bayrou	6%	34%	–	36%*	2%	22%
Sarkozy	2%	15%	43%*	–	12%	28%
Nihous	14%	11%	18%	19%*	7%	31%
de Villiers	2%	4%	9%	60%*	11%	14%
Le Pen	4%	8%	6%	38%*	–	44%
Average transfer	14.5%	18.1%	19.5%	18.4%	4.5%	

Note: Boldface indicates important candidates.
  Percentages with asterisks are the largest in their rows.

Table 6.9
Transfers of Votes to Important Candidates, Polling Results, April 20, 2007

	Left ⟵ Besancenot	Royal	Bayrou	Sarkozy	⟶ Right Le Pen	Eight Others	Not Counted
Royal	12%	–	34%*	5%	4%	27%	18%
Bayrou	7%	28%*	–	25%	3%	15%	22%
Sarkozy	3%	10%	37%*	–	7%	19%	24%
Le Pen	1%	12%	8%	31%*	–	23%	25%

Source: Polling Results by BVA.
Note: Percentages with asterisks are the largest in their rows. To compare them with the percentages in table 6.8, the percentages in this table must be normalized.

qualitative results (though they are national estimates, not Orsay estimates; see table 6.9).

6.3   Borda's and Condorcet's Bias for the Center

It is striking that in the 2007 election (for which there is so much polling and experimental evidence), the Condorcet-winner and the Borda-winner—those centuries-old opposing concepts—are consistently one and the same candidate (Bayrou). Why? The evidence of tables 6.7 and 6.8 suggests that when there is a statistical left-right spectrum, a voter's second choice is most likely to be a major candidate (protest voters are an exception). So if there are two major candidates,

each beats a minor candidate by a large margin, the Condorcet-winner is also the Borda-winner and must be one of the two major candidates. When there are three major candidates, the Condorcet- and Borda-winners are again limited to those three candidates, and the analysis may be restricted to them.

Suppose three candidates on the left-right spectrum, A the major candidate of the left, B the major candidate of the center, and C the major candidate of the right, are respectively the favorites of $x_A\%$, $x_B\%$, and $x_C\%$ of the voters. Let γ be the probability that a voter who prefers C votes for B when B opposes A; α be the probability that a voter who prefers A votes for B when B opposes C; and β be the probability that a voter who prefers B votes for A when A opposes C. The expected pair-by-pair votes are given in table 6.10.

There is a *statistical left-right spectrum* if the matrix of transfers is single-peaked in rows and in columns. With three candidates this occurs when $\alpha > \frac{1}{2}$, $\gamma > \frac{1}{2}$, and $\max\{\beta, 1 - \beta\} < \min\{\alpha, \gamma\}$, which holds when β is close to $\frac{1}{2}$. When, in addition, α and γ are sufficiently large, implying $v_{BA} > v_{CA}$ (B against A wins more votes than C against A), and symmetrically, $v_{BC} > v_{AC}$, there is a *strong statistical left-right spectrum*, meaning that the matrix of pairwise votes is single-peaked in rows and in columns. In this case, it is easy to prove that there always exists a Condorcet-winner. The experimental evidence from both the 2002 and 2007 Orsay experiments shows that a strong statistical left-right spectrum existed among the three principal candidates (table 6.11).

When the centrist candidate B is the Condorcet-winner, then B is necessarily the Borda-winner. For $v_{BC} > v_{AC}$, and $v_{BA} > 50\%$ implies $v_{AB} < 50\%$, so $v_{BA} + v_{BC} > v_{AB} + v_{AC}$, and the symmetric argument gives the same

Table 6.10
Pair-by-Pair Votes among Three Candidates

vs.	Left ⟵ A	B	⟶ Right C
A	–	$v_{AB} = (x_A + (1 - \gamma)x_C)\%$	$v_{AC} = (x_A + \beta x_B)\%$
B	$v_{BA} = (x_B + \gamma x_C)\%$	–	$v_{BC} = (x_B + \alpha x_A)\%$
C	$v_{CA} = (x_C + (1 - \beta)x_B)\%$	$v_{CB} = (x_C + (1 - \alpha)x_A)\%$	–

Table 6.11
Strong Statistical Left-Right Spectrum, Pairwise Votes, 2002 and 2007 Orsay Experiments

2002	Jospin	Chirac	Le Pen	2007	Royal	Bayrou	Sarkozy
Jospin	–	56%	75%	Royal	–	44%	52%
Chirac	47%	–	80%	Bayrou	56%	–	60%
Le Pen	25%	20%	–	Sarkozy	48%	40%	–

Table 6.12a
Number of Wins among Royal, Bayrou, and Sarkozy Only, 2007 Orsay Experiment

	Left ⟵ Royal	Bayrou	⟶ Right Sarkozy	Tie	Cycle
First-past-the-post winner	4,274	1,772	3,574	380	–
Two-past-the-post winner	3,410	4,671	1,225	694	–
Majority judgment-winner	1,462	7,573	956	9	–
Condorcet-winner	772	8,894	65	246	23
Borda-winner	369	9,526	67	38	–

Note: Ten thousand samples of 101 ballots, which were drawn from 1,733 ballots.
   "Cycle" indicates a Condorcet paradox.

Table 6.12b
Number of Wins among All Candidates, Winner Always Royal, Bayrou, or Sarkozy, 2007 Orsay Experiment

	Left ⟵ Royal	Bayrou	⟶ Right Sarkozy	Tie	Cycle
First-past-the-post winner	2,324	2,260	5,379	37	–
Two-past-the-post winner	3,175	5,830	801	194	–
Majority judgment-winner	1,290	7,756	943	11	–
Condorcet-winner	623	9,152	5	184	36
Borda-winner	348	9,639	0	13	–

Note: Ten thousand samples of 101 ballots, which were drawn from 1,733 ballots.
   "Cycle" indicates a Condorcet paradox.

conclusion when comparing B with C. On the other hand, if A is the Condorcet-winner, then the Borda-winner is either A or B. For $v_{BC} > v_{AC} > 50\% > v_{CB}$ and $v_{BA} > v_{CA}$ imply that C cannot be the Borda-winner; and symmetrically, if C is the Condorcet-winner, then the Borda-winner is either C or B. So the Borda-winner favors the centrist candidate more than does the Condorcet-winner. However, with only a statistical left-right spectrum it is entirely possible for the Condorcet paradox to occur in theory (by varying the data in table 6.10) and in practice, as the experimental evidence shows (see tables 6.12a, 6.12b, 6.14a, 6.14b).

Evidence from the 2007 Orsay experiment supports these arguments and observations. Two sets of independent random drawings were made. In one, 10,000 samples from 101 ballots were drawn from the 1,733 valid ballots in order to compare the behavior of the principal methods applied only to the three major candidates, Bayrou, Royal, and Sarkozy (table 6.12a). In the other, conducted separately, 10,000 random samples from 101 ballots were drawn to compare the behavior of the principal methods applied to all the candidates

(table 6.12b). In every case one of the three major candidates was the winner. To compute the winners by one or another of the methods a candidate was accorded the vote of a ballot if she had the highest grade; when there was a tie among k candidates for the highest grade on a ballot, each was attributed $\frac{1}{k}$.

The results in tables 6.12a and 6.12b clearly show that as one passes from one method to another down the list—from first-past-the-post to Borda—the centrist candidate is more and more favored. Borda's method favors the centrist candidate Bayrou slightly more than Condorcet's, and Condorcet's much more than the majority judgment. At the opposite end of the spectrum, the first- and two-past-the-post methods disfavor the centrist candidate in comparison with the majority judgment. The nine and eleven ties in the majority judgment mean ties in the majority-gauge (not the majority-value): a 0.001 probability of a tie with only 101 voters is sufficiently small. The twenty-three and thirty-six occurrences of the Condorcet paradox show that the preferences among the candidates is not single-peaked and that though there is a statistical left-right spectrum, it is not strong. One of the twenty-three Condorcet paradoxes of table 6.12a showed Bayrou with 59% against Sarkozy, Sarkozy with 52.5% against Royal, and Royal with 52% against Bayrou. The striking contrast between tables 6.12a and 6.12b is the large increase in Sarkozy first-past-the-post wins when there are twelve candidates rather than three: it reflects a large number of occurrences of Arrow's paradox coming from the dispersion of votes among candidates of the left. And, of course, the more candidates there are, the more Borda favors the centrist. The majority judgment is unaffected by the number of candidates: the small differences are due to the independently drawn samples.

The official first-round votes in the three precincts of the 2007 Orsay experiment were quite different from the official first-round votes nationally (table 6.13). In particular, Royal's 29.9% in Orsay was above her 25.5% nationally, Bayrou's 25.5% in Orsay was much above his 18.6% nationally, Le Pen's 5.9% in Orsay much below his 10.4% nationally. So it is no surprise to see Bayrou—the choice of the median-voter in the official first-round vote in the

Table 6.13
Official First-Round Votes, National and Three Precincts of Orsay, April 22, 2007

	Sarkozy	Royal	Bayrou	Le Pen	Besancenot	de Villiers
National	31.2%	25.9%	18.6%	10.4%	4.1%	2.2%
Orsay	29.0%	29.9%	25.5%	5.9%	2.5%	1.9%
	Buffet	Voynet	Laguiller	Bové	Nihous	Schivardi
National	1.9%	1.6%	1.3%	1.3%	1.1%	0.3%
Orsay	1.4%	1.7%	0.8%	0.9%	0.3%	0.2%

Table 6.14a
Number of Wins among Royal, Bayrou, and Sarkozy Only, 2007 Orsay Experiment

	Left ← Royal	Bayrou	→ Right Sarkozy	Tie	Cycle
First-past-the-post winner	1,678	42	8,089	191	–
Two-past-the-post winner	2,145	820	6,470	565	–
Majority judgment-winner	1,288	4,001	4,701	10	–
Condorcet-winner	671	6,462	1,993	669	205
Borda-winner	484	7,109	2,225	182	–

Note: Ten thousand samples of 101 ballots, which were drawn from a sample of 501 ballots representative of the national vote. The same approach was used as in estimating first-round results on the basis of majority judgment ballots; the percentage of votes of each candidate in the sample of 501 ballots came close to that of the candidate's national vote. In this sample, Sarkozy had 30.7%, Royal 25.5%, and Bayrou had 18.7%. (Le Pen 9.3%.)
  "Cycle" indicates a Condorcet paradox.

Table 6.14b
Number of Wins among All Candidates, Winner Always Royal, Bayrou, or Sarkozy, 2007 Orsay Experiment

	Left ← Royal	Bayrou	→ Right Sarkozy	Tie	Cycle
First-past-the-post winner	2,061	50	7,874	15	–
Two-past-the-post winner	2,174	716	6,731	379	–
Majority judgment-winner	1,309	4,034	4,649	8	–
Condorcet-winner	616	6,538	2,002	630	214
Borda-winner	354	9,608	26	12	–

Note: Ten thousand samples of 101 ballots, which were drawn from a sample of 501 ballots representative of the national vote.
  "Cycle" indicates a Condorcet paradox.

voting precincts of the Orsay experiment and in the nation—so often the winner (see tables 6.12a and 6.12b). Accordingly, parallel sets of independent random drawings were made from a subset of 501 ballots (of the 1,733 valid ballots) whose estimated first-round votes were representative of the national vote. In one, 10,000 samples from 101 ballots were drawn from the 501 to compare the methods applied to the three major candidates (table 6.14a); in the other, conducted separately, 10,000 samples from 101 ballots were drawn from the 501 to compare the methods applied to all the candidates (table 6.14b).

The results show more dramatically how Borda's method and to a lesser extent Condorcet's method favor the centrist candidate and how the first- and two-past-the-post methods penalize him, while in contrast the majority judgment appears to be more evenhanded. Note, in particular, the chaotic

behavior in the centrist's Borda-wins when there are twelve rather than only three candidates.

This has practical significance: most thoughtful commentators reject election mechanisms that *systematically* elect the centrist candidate. As the well-known popularizer of science William Poundstone wrote, "We want a system that doesn't *automatically* exclude [moderate] candidates from winning. We also want a system that doesn't make it easy for any goof who calls himself a moderate to win" (2008, 211). On the other hand, the fact that Bayrou had merely forty-two and fifty wins with first-past-the-post when by all reasonable estimates Bayrou was the Condorcet- and Borda-winner seems derisory. *A good election mechanism should eliminate extremes and give all major poles—left, center, and right—a fighting chance to win.*

To this day, the Condorcet-winner and the Borda-ranking dominate the thinking in the theory of social choice: they continue to be proposed and reproposed, alone and in combinations. Agreement between them would therefore seem to be a happy concurrence giving a particularly valid result. But both of these mechanisms are heavily biased in favor of moderate candidates. Major candidates of the right and the left, such as Sarkozy and Royal, often elicit strong support and strong opposition, so they are given high or low evaluations. A moderate candidate, on the other hand, is often placed second or third. Face-to-face confrontations and rank-orders ignore how voters evaluate the respective candidates (just as the 2002 French presidential runoff merely compared Chirac and Le Pen but did not evaluate them).

The ballots of the 2007 Orsay experiment show that these evaluations are significant: two-thirds of the second highest grades are merely *Good* or worse, three-quarters of the third highest grades are *Acceptable* or worse (see table 6.15). Both Condorcet and Borda ignore evaluations; they rely only on comparisons. When there are twelve candidates, a voter's list gives 11 points to the first candidate, 10 to the second, 9 to the third, and so on. The difference between being first, second, or third on the list is marginal, especially in the presence of many candidates. Perhaps this exaggerated bias in favor of moderate candidates explains why these mechanisms are hardly ever used in practice.

Table 6.15
Distributions: Highest Grade, Second Highest Grade, Third Highest Grade, Three Precincts of Orsay, April 22, 2007

	Excellent	*Very Good*	*Good*	*Acceptable*	*Poor*	*To Reject*
Highest	52%	37%	9%	2%	0%	1%
Second highest	–	35%	41%	16%	5%	3%
Third highest	–	–	26%	40%	22%	13%

Chapter 19 compares the qualitative properties of methods in more detail. The majority judgment results are given here only as a point of comparison. The first- and two-past-the-post systems both systematically eliminate centrist candidates (when they exist). France and the U.K. are excellent examples. The centrist candidate Bayrou did not survive the first round in 2002 or 2007, either in the precincts of the Orsay experiments or in the entire nation. In the British general elections of 2005 the Liberal Democrats won only 9.6% of the seats for 22% of the votes, in those of 2010 they won but 8.8% of the seats for 23% of the votes.

6.4   Conclusion

More experimentation is called for. Nevertheless, the experimental evidence already shows the following:

· The inputs imputed to voters in the traditional theory of social choice—relative comparisons of candidates or rank-orders of candidates—are completely unrealistic. Voters do not think in those terms and do not wish to express themselves in those terms.

· Most voters—three-fifths of them—refuse to rank-order all candidates when they are asked to do so.

· Many voters—one-third of them—refuse to single out one preferred candidate when they have the opportunity to give the same evaluations to more than one candidate.

· Many voters—one-half of them—refuse to declare their preferred candidate *Excellent*. To be first in a rank-order has very different meanings, so aggregating rank-orders is meaningless.

· The fact that there is a statistical left-right spectrum according to which the preferences of many voters are single-peaked shows why the Borda- and Condorcet-winners are often the same and why both of these mechanisms (more so Borda's) strongly favor centrist candidates.

· "The free communication of thoughts and opinions is one of the most precious rights of man." (*Déclaration des droits* 1789, article 11). Not one of the electoral systems used in practice—whether it be the Australian rank-order or the one vote allowed in England, France, and the United States—gives voters anywhere near the freedom of expression they wish to have.

The traditional model of social choice has been tried in theory and in practice and does not work. By Richard Feynman's definition of science, it must be eliminated.