New Perspectives on Social Choice

Eric Pacuit, University of Maryland

Lecture 6: Evaluative Voting

PHIL 808K

Approval Voting: Each voter selects a subset of candidates. The candidate with the most "approvals" wins the election.

S. Brams and P. Fishburn. *Approval Voting*. Birkhauser, 1983.

J.-F. Laslier and M. R. Sanver (eds.). *Handbook of Approval Voting*. Studies in Social Choice and Welfare, 2010.

Under Approval Voting (AV), voters are asked to select the candidates that the voter *approves*.

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The two pieces of information are related, but not derivable from each other

Approving of a candidate is not (necessarily) the same as simply ranking the candidate first.

Why Approval Voting?

www.electology.org/approval-voting

S. Brams and P. Fishburn. *Going from Theory to Practice: The Mixed Success of Approval Voting*. Handbook of Approval Voting, pgs. 19-37, 2010.

Example

Voters	a	b	c	d
1	1	0	1	1
2	0	1	1	0
3	0	1	0	0
4	0	0	0	0
5	1	1	1	1

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3	0	1	0	0
4	0	0	0	0
5	1	1	1	1

1	2	3	4	5
a	b	d	d	a
b	c	b	c	b
c	a	c	b	d
d	d	a	a	c

Example

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1	2	3	4	5
a	b	d	d	a
b	c	b	c	b
c	a	c	b	d
d	d	a	a	c

An AV ballot is **sincere** if, given the lowest-ranked candidate that a voter approves of, he or she also approves of all candidates ranked higher.

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5	1	1	1	1

1	2	3	4	5
a	b	d	d	a
b	c	b	c	b
c	a	c	b	d
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4	0	0	0	0
5	1	1	1	1

1	2	3	4	5
a	*b*	*d*	*d*	*a*
b	*c*	*b*	*c*	*b*
c	*a*	*c*	*b*	*d*
d	*d*	*a*	*a*	*c*

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Approval Voting is more flexible

# voters	2	2	1
	a	b	c
	d	d	a
	b	a	b
	c	c	d

The Condorcet winner is a.

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There is no fixed rule that always elects a unique Condorcet winner.

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The Condorcet winner is a.
Vote-for-1 elects $\{a, b\}$, vote-for-2 elects $\{d\}$, vote-for-3 elects $\{a, b\}$.

Approval Voting is more flexible

AV may elect the Condorcet winner

# voters	2	2	1
	a	b	c
	d	d	a
	b	a	b
	c	c	d

The Condorcet winner is a.
$(\{a\}, \{b\}, \{c, a\})$ elects a under AV.

Possible Failure of Unanimity

# voters	1	1	1
	a	c	d
	b	a	a
	c	b	b
	d	d	c

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	a	c	d
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Approval Winners: a, b

Indeterminate or Responsive?

# voters	6	5	4
	a	b	c
	c	c	b
	b	a	a

Plurality winner: a, Borda and Condorcet winner: c.

Indeterminate or Responsive?

# voters	6	5	4
	a	b	c
	c	c	b
	b	a	a

Plurality winner: a, Borda and Condorcet winner: c.
Any combination of a, b and c can be an AV winner (or AV winners).

Approval Voting vs. Borda

D. Saari and J. Van Newenhizen. *The problem of indeterminacy in approval, multiple, and truncated voting systems*. Public Choice 59, pp. 101- 120, 1998.

S. Brams, P. Fishburn and S. Merrill III. *The responsiveness of approval voting: comments on Saari and Van Newenhizen*. Public Choice 59, pp. 121 - 131, 1998.

D. Saari and J. Van Newenhizen. *Is approval voting an 'unmitigated evil'?: a response to Brams, Fishburn, and Merrill*. Public Choice 59, pp. 133 - 147, 1998.

S. Brams, P. Fishburn and S. Merrill III. *Rejoinder to Saari and Van Newenhizen*. Public Choice 59, pp. 149, 1998.

Characterizing Approval Voting

Suppose X is a set of candidates and $\mathcal{K}$ is the set of non-empty subsets of X.

An **anonymous profile** $\boldsymbol{P}$ is a function $\boldsymbol{P} : \mathcal{K} \rightarrow \mathbb{N}$ assigning the number of voters submitting each profile. For $A \in \mathcal{K}$, $\boldsymbol{P}(A)$ is the number of voters that submit the ballot A.

Let $Prof(X)$ be the set of all profiles over $\mathcal{X}$.

For $x \in X$, $\boldsymbol{P} \in Prof(X)$, let $n(x, \boldsymbol{P}) = \sum_{A \in \mathcal{K}, x \in A} \boldsymbol{P}(A)$.

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For $x \in X$, $\boldsymbol{P} \in Prof(X)$, let $n(x, \boldsymbol{P}) = \sum_{A \in \mathcal{K}, x \in A} \boldsymbol{P}(A)$.

A ballot aggregation function is a mapy $f : Prof(X) \to \mathcal{K}$

Approval Voting is the function:

$$av(\boldsymbol{P}) = \{x \mid x \in X, n(x, \boldsymbol{P}) \geq n(y, \boldsymbol{P}) \text{ for all } y \in X\}$$

Characterizing Approval Voting

Faithfulness: For all $A \in \mathcal{K}$ and profiles $\boldsymbol{P}$, if $\boldsymbol{P}(A) = 1$ and $\boldsymbol{P}(B) = 0$ for all $B \in (\mathcal{K} \setminus \{A\})$, then $f(\boldsymbol{P}) = A$.

Consistency For all profiles $\boldsymbol{P}$, $\boldsymbol{P}'$, if $f(\boldsymbol{P}) \cap f(\boldsymbol{P}') \neq \varnothing$, then $f(\boldsymbol{P}) \cap f(\boldsymbol{P}') = f(\boldsymbol{P} + \boldsymbol{P}')$

Cancellation: For all profiles $\boldsymbol{P}$, if $[n(x, \boldsymbol{P}) = n(y, \boldsymbol{P})$ for all $x, y \in X]$, then $f(\boldsymbol{P}) = X$.

Theorem (Fishburn 1978, Alós-Ferrer 2006). A ballot aggregation function f is approval voting if and only if f satisfies Faithfulness, Consistency and Cancellation.

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Ask the voters to provide both a linear ranking of the candidates and the candidates that they approve.

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Make the ballots more expressive: Dis&Approval voting, RangeVoting, Majority Judgement

R. Sanver. *Approval as an intrinsic part of preference*. Handbook of Approval Voting, 2010.

Let $\mathcal{U}(X)$ be the set of real-valued **utility functions** defined over X, an *aggregation function* is

$$F : \mathcal{U}(X)^n \to \wp(X) - \{\emptyset\}$$

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At the other extreme, rule out any kind of cardinal information and interpersonal comparability, partitions $U(X)^n$ into cells which are *ordinally equivalent*:

$$F : \mathcal{O}(X)^n \to \wp(X) - \{\emptyset\}$$

where $\mathcal{O}(X)$ is the set of *weak orderings* on X.

Preference Approval Voting

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where 0 separates the “good” and “bad” elements.

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"approval is not a strategic action but has an intrinsic meaning: It refers to the alternatives which are qualified as good."

Assumptions

Assume each voter has a (linear) preference over the candidates.

Each voter is asked to rank the candidates from most preferred to least preferred (ties are not allowed).

Voters are then asked to specify which candidates are acceptable.

Consistency Assumption Given two candidates a and b, if a is approved and b is disapproved then a is ranked higher than b.

For example, we denote this approval ranking for a set $\{a, b, c, d\}$ of candidates as follows

$$a \ d \mid c \ b$$

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   2.2 If there is not one majority-preferred candidate because of a cycle among the majority-approved candidates, then the AV winner among them is the PAV winner.

PAV vs. Condorcet

Rule 1

I.	1 voter:	$a\ b \mid c$
II.	1 voter:	$b \mid a\ c$
III.	1 voter:	$c \mid a\ b$

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b is the AV winner.

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b is also the PAV winner.

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b is also the PAV winner.
a is the Condorcet winner.

PAV vs. Condorcet

Rule 2(a)

I.	1 voter:	$a\ b\ c \mid d$
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a is the Condorcet winner.

PAV vs. Condorcet

Rule 2(b)

I.	1 voter:	$d \ a \ b \ c \mid e$
II.	1 voter:	$d \ b \ c \ a \mid e$
III.	1 voter:	$e \mid d \ c \ a \ b$
IV.	1 voter:	$a \ b \ c \mid d \ e$
V.	1 voter:	$c \mid b \ a \ d \ e$

PAV vs. Condorcet

Rule 2(b)

I.	1 voter:	$d\ a\ b\ c \mid e$
II.	1 voter:	$d\ b\ c\ a \mid e$
III.	1 voter:	$e \mid d\ c\ a\ b$
IV.	1 voter:	$a\ b\ c \mid d\ e$
V.	1 voter:	$c \mid b\ a\ d\ e$

a (3 votes), b (3 votes), and c (4 votes) are all majority approved.

PAV vs. Condorcet

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a (3 votes), b (3 votes), and c (4 votes) are all majority approved.
c is the PAV winner.
d is the Condorcet winner.

Example

I.	3 voters:	$a \; b \; c \mid d$
II.	3 voters:	$d \; a \; c \mid b$
III.	2 voters:	$b \; d \; c \mid a$

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c is approved by all 8 voters.

Example

I.	3 voters:	$a\ b\ c \mid d$
II.	3 voters:	$d\ a\ c \mid b$
III.	2 voters:	$b\ d\ c \mid a$

c is approved by all 8 voters.
There is a top cycle $a > b > d > a$ which are all preferred by majorities to c (the AV winner).
a is the PAV winner

Example

I.	3 voters:	$a\ b\ c \mid d$
II.	3 voters:	$d\ a\ c \mid b$
III.	2 voters:	$b\ d\ c \mid a$

a is the PAV winner.
c is the AV winner.
d is the IRV winner.

Example

I.	2 voters:	$a \; c \; b \mid d$
II.	2 voters:	$a \; c \; d \mid b$
III.	3 voters:	$b \; c \; d \mid a$

Example

I.	2 voters:	$a\ c\ b \mid d$
II.	2 voters:	$a\ c\ d \mid b$
III.	3 voters:	$b\ c\ d \mid a$

c is approve by all 7 voters.
a is the least approved candidate.
a is the PAV winner.
$BC(a) = 12$
$BC(c) = 14$

Majoritarian Approval

$$q_i(x; \boldsymbol{P}) = G \text{ iff } x \; \boldsymbol{P}_i \; 0$$

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$$\gamma(\boldsymbol{P}) = \{x \in A \mid n^G(x; \boldsymbol{P}) \geq n/2\}$$

Majoritarian Approval

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$$\gamma(\boldsymbol{P}) = \{x \in A \mid n^G(x; \boldsymbol{P}) \geq n/2\}$$

$f : \mathcal{O}(X \cup \{0\})^n \to \wp(X) - \emptyset$ satisfies **majoritarian approval** iff we have $f(\boldsymbol{P}) \subseteq \gamma(\boldsymbol{P})$ for every $\boldsymbol{P} \in \mathcal{O}(X \cup \{0\})^n$ where $\gamma(\boldsymbol{P}) \neq \emptyset$

Approval Independence

$f : \mathcal{O}(X \cup \{0\})^n \to \wp(X) - \emptyset$ satisfies **approval independence** iff we have $f(\boldsymbol{P}) = f(\boldsymbol{P}')$ for every $\boldsymbol{P}, \boldsymbol{P}' \in \mathcal{O}(X \cup \{0\})^n$ where $\boldsymbol{P}_{|\{x,y\}} = \boldsymbol{P}'_{|\{x,y\}}$ for all $x, y \in X$.

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Fact. Majoritarian approval and approval independence are logically incompatible.

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Fact. Majoritarian approval and approval independence are logically incompatible.

1 voter: $a \mid b\ c$
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a is the winner according to majoritarian approval

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1 voter: $c\ b \mid a$

a is the winner according to approval independence, contrary to majoritarian approval.

Grading

In many group decision situations, people use measures or grades from a **common language of evaluation** to evaluate candidates or alternatives:

- in figure skating, diving and gymnastics competitions;
- in piano, flute and orchestra competitions;
- in classifying wines at wine competitions;
- in ranking university students;
- in classifying hotels and restaurants, e.g., the Michelin $*$

M. Balinski and R. Laraki. *Majority Judgement: Measuring, Ranking and Electing*. The MIT Press, 2010.

- voters have more on their minds than merely comparing candidates, or approving of some and disapproving of others, and wish to express it,

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- voters care about all the results of an election (the distributions of the votes, who is in second, third, down to last place, the spreads between candidates, and so on) and not merely who is the winner, and

- voters have more on their minds than merely comparing candidates, or approving of some and disapproving of others, and wish to express it,

- voters care about all the results of an election (the distributions of the votes, who is in second, third, down to last place, the spreads between candidates, and so on) and not merely who is the winner, and

- voters are dissatisfied with election results that do not reflect their true opinions.

Electing vs. Ranking: Condorcet's Ranking

Condorcet-ranking (also known as Kemeny's rule) associates a score to each possible rank ordering:

- A voter contributes k Condorcet-points to a rank-ordering if his input agrees in k pair-by-pair comparisons.
- The Condorcet-score of a rank-ordering is the sum of its Condorcet-points over all voters.
- The Condorcet-ranking is the ranking that maximizes the Condorcet-score.

Electing vs. Ranking

Is Borda's method good for designating a winner, or a ranking, or both?

Is Condorcet's method good for designating a winner, or a ranking, or both?

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Is Borda's method good for designating a winner, or a ranking, or both?

Is Condorcet's method good for designating a winner, or a ranking, or both?

- Given a method of ranking, the first-placed candidate is the winner.
- Given a method of designating a winner (or loser), the winner is the first-ranked (or last-ranked); the second-ranked is the winner among the remaining candidates, and so on.

Are ranking and designating winners two sides of one coin?

333	333	333	1
A	B	C	A
B	C	A	C
C	A	B	B

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According to Condorcet: $A \succ B \succ C$ and $C \succ A \succ B$ are tied for first.

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According to Borda: A is the winner, B is the loser. Thus, society's order is $A \succ C \succ B$

According to Condorcet: $A \succ B \succ C$ and $C \succ A \succ B$ are tied for first.

- No reasonable **ranking function** must choose $A \succ C \succ B$
- Any reasonable **choice function** must choose $A \succ C \succ B$

There is a fundamental incompatibility between electing and ranking.

Conclusion: The traditional model's inputs are inadequate messages and must be reformulated.

“Arrow’s Paradox” (aka violation of local-α)

OBO Rule used to judge skating competitions

1. Rank the competitors by their number of wins (thereby giving precedence to the Llull and Condorcet idea).
2. Break any ties by using Borda's rule.

Table 1. Scores of competitors given by nine judges (performance plus technical marks).

Name	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	Avg.
T. Eldredge	11.3	11.6	11.3	11.4	11.4	11.7	11.4	11.2	11.5	11.42
C. Li	10.8	11.2^+	11.0	10.9	10.6	11.0	10.8	10.9	11.2	10.93
M. Savoie	11.1	10.8^+	11.1	10.8^+	10.5	10.8	10.6	10.5	11.1	10.81
T. Honda	10.3	11.2	10.9	11.0	10.8	10.9^+	10.4	10.3	10.7	10.72
M. Weiss	10.6	11.1	10.6	10.8	10.4	10.9	10.9	10.4	10.9	10.73
Y. Tamura	09.8	10.8	10.1	10.4	11.0	11.6	10.7	10.6	10.8	10.64

Note. x^+ is ranked above x.

Table 2. Judges' inputs (indicating rank orders of the six competitors).

Name	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9
T. Eldredge	1	1	1	1	1	1	1	1	1
C. Li	3	2	3	3	4	3	3	2	2
M. Savoie	2	5	2	4	5	6	5	4	3
T. Honda	5	3	4	2	3	4	6	6	6
M. Weiss	4	4	5	5	6	5	2	5	4
Y. Tamura	6	6	6	6	2	2	4	3	5

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T. Eldredge	1	1	1	1	1	1	1	1	1
C. Li	3	2	3	3	4	3	3	2	2
M. Savoie	2	5	2	4	5	6	5	4	3
T. Honda	5	3	4	2	3	4	6	6	6
M. Weiss	4	4	5	5	6	5	2	5	4
Y. Tamura	6	6	6	6	2	2	4	3	5

Table 3. Judges' majority votes in all head-to-head comparisons.

	T. Eldredge	C. Li	M. Savoie	T. Honda	M. Weiss	Y. Tamura	Number of wins	Borda score
T. Eldredge	—	9	9	9	9	9	5	45
C. Li	0	—	7	7	8	7	4	29
M. Savoie	0	2	—	5	6	5	3	18
T. Honda	0	2	4	—	5	4	1	15
M. Weiss	0	1	3	4	—	6	1	14
Y. Tamura	0	2	4	5	3	—	1	14

$$\text{Eldredge} \succ_S \text{Li} \succ_S \text{Savoie} \succ_S \text{Honda} \succ_S \text{Weiss} \approx_S \text{Tamura}.$$

Suppose that the order of the performances had been first Honda, then Weiss, Tamura, Savoie, Li, and Eldredge. After each performance, the results are announced.

After Honda, Weiss and Tamura perform we have:

Table 4. Judges' inputs, three competitors.

Name	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9
T. Honda	2	1	1	1	2	2	3	3	3
M. Weiss	1	2	2	2	3	3	1	2	1
Y. Tamura	3	3	3	3	1	1	2	1	2

Table 5. Majority votes in head-to-head comparisons, three competitors.

	T. Honda	M. Weiss	Y. Tamura	Number of wins	Borda score
T. Honda	—	5	4	1	9
M. Weiss	4	—	6	1	10
Y. Tamura	5	3	—	1	8

$$\text{Weiss} \succ_S \text{Honda} \succ_S \text{Tamura}$$

After Honda, Weiss, Tamura, and Savoie perform we have:

Table 6. Judges' inputs, four competitors.

Name	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9
M. Savoie	1	3	1	2	3	4	3	2	1
T. Honda	3	1	2	1	2	2	4	4	4
M. Weiss	2	2	3	3	4	3	1	3	2
Y. Tamura	4	4	4	4	1	1	2	1	3

Table 7. Majority votes in head-to-head comparisons, four competitors.

	M. Savoie	T. Honda	M. Weiss	Y. Tamura	Number of wins	Borda score
M. Savoie	—	5	6	5	3	16
T. Honda	4	—	5	4	1	13
M. Weiss	3	4	—	6	1	13
Y. Tamura	4	5	3	—	1	12

$$\text{Savoie} \succ_S \text{Weiss} \approx_S \text{Honda} \succ_S \text{Tamura.}$$

After Honda, Weiss and Tamura perform we have:

$$\text{Weiss} \succ_S \text{Honda} \succ_S \text{Tamura}$$

After Honda, Weiss, Tamura, and Savoie perform we have:

$$\text{Savoie} \succ_S \text{Weiss} \approx_S \text{Honda} \succ_S \text{Tamura}.$$

After everyone performs we have:

$$\text{Eldredge} \succ_S \text{Li} \succ_S \text{Savoie} \succ_S \text{Honda} \succ_S \text{Weiss} \approx_S \text{Tamura}.$$

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- Should there be a "no opinion" option?

Evaluative Voting: Examples

Approval Voting: voters can assign a single grade "approve" to the candidates. The candidates with the most approvals are the winner.

Dis&Approval Voting: voters can approve or disapprove of the candidates. The candidates with the greatest scores are the winners.

Score Voting: voters can assign any grade from a fixed set of grades to the candidates. The candidate with the greatest sum of the scores is the winner.

Majority Judgement: voters can assign any grade from a fixed set of grades to the candidates. The candidate with the greatest median score is the winner.

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Fixe a common grading language consisting of, for example, the integers $\{0, 1, 2, \ldots, 10\}$

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Suppose A's grades are $\{7, 7, 8, 8, 9, 9, 9, 10\}$. The average grade is 8.375

Suppose B's grades are $\{9, 9, 9, 9, 9, 10, 10, 10\}$. The average grade is 9.375

So, Score Vote (Range Vote) ranks B above candidate A.

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www.electology.org/score-voting and rangevoting.org

Majority Judgement

Fix a common grading language. For example, $\{0, 1, 2, \ldots, 10\}$

The candidate with the largest median grade is declared the winner.

The *median* grade is the grade that is in the middle of the list when the grades are ordered (If there is an even number of judges, then the median grade is the lowest grade in the middle interval.)

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Suppose B's grades are $\{6, 6, 6, 6, 9, 9, 9, 10\}$: The median grade is 6.

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