New Perspectives on Social Choice

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Lecture 8: Perpetual Voting

PHIL 808K

51	49
a	b
b	a

51% of the voters have a *slight* preference for a over b and 49% of the voters have a *strong* preference for b over a.

- Utilitarian considerations suggests that b should win.
- Majoritarian considerations suggests that a should win.

80	20
a	b
b	a

80% of the voters *strictly prefer* a over b and 20% of the voters have an "*extremely strong*" preference for b over a.

- Utilitarian considerations suggests that b should win??
- Majoritarian considerations suggests that a should win.

75	25
a	b
b	a

75% of the voters *strictly prefer a* over b and 25% of the voters *strictly prefer b* over a. If a wins, then this will cause harm to the 25% of voters that prefer b to a; and if b wins, this will cause some annoyance to the 75% of the voters that prefer a to b.

How do we weigh the preference of the majority while avoiding harm to the minority?

75	25
a	b
b	a

75% of the voters *strictly prefer* a over b and 25% of the voters *strictly prefer* b over a. If a wins, then this will cause harm to the 25% of voters that prefer b to a; and if b wins, this will cause some annoyance to the 75% of the voters that prefer a to b.

How do we weigh the preference of the majority while avoiding harm to the minority?

- Not all questions should be decided by a vote.
- Education, deliberation, etc. to change the rankings of the enough of the 75% of the voters to ensure that b is the majority opinion.

- Aggregating utilities vs. aggregating grades
- Comparing different types of *scoring* rules (evaluative voting and positional scoring rules)

Grades vs. Utilities

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In the real world the deep preferences or utilities of a judge or a voter are a very complicated function that depends on a host of factors....We contend that the deep preference of judges or voters *cannot* be the inputs of a practical model of voting. *A judge's input is simply a message, no more no less*. But her input chosen strategically, depends, on her deep preferences or utilities.

(Balinksi and Laraki, p. 184)

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Grades vs. Utilities

The language of grades has nothing to do with utilities (viewed as measures of individual satisfaction). Grades are absolute measures of merit. In the context of voting and judging, utilities are relative measures of satisfaction. Formally, a distinction must be made between two different types of scales of measurement. An *absolute* scale measures each entity individually (height, area, merit). A *relative* scale measure each entity with respect to a collective of like entities (velocity, satisfaction). Were voters to be asked their satisfaction as inputs, adjoining or eliminating candidates would alter their answers, provoking the possibility of Arrow's paradox. A common language must be an absolute scale of measurement. (Balinksi and Laraki, p. 185)

A. Dhillon and J.-F. Mertens. *Relative Utilitarianism*. Econometrica, Vol. 67, No. 3, pp. 471 - 498, 1999.

Axiomatize **relative utilitarianism**: normalize the nonconstant individual von Neumann-Morgenstern utility functions to have infimum zero and supremum one, and taking the sum as social utility.

Utility Functions

A **utility function** on a set X is a function $u : X \to \mathbb{R}$

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A preference ordering is **represented** by a utility function iff x is (weakly) preferred to y provided $u(x) \geq u(y)$

L. Narens and B. Skyrms . *The Pursuit of Happiness Philosophical and Psychological Foundations of Utility*. Oxford University Press, 2020.

Let X and V be nonempty sets with $|X| \geq 3$ and V finite.

Let $\mathcal{U}(X)$ be the set of all functions $u : X \to \mathbb{R}$

A **profile** is a function $\boldsymbol{U} : V \to \mathcal{U}(X)$, write $\boldsymbol{U}_i$ for voter i's utility function on X in profile $\boldsymbol{U}$.

A **Social Welfare Functional (SWFL)** is a function f mapping profiles of utilities to asymmetric relations on X. So for each profile $\boldsymbol{U}$, $f(\boldsymbol{U})$ is the social preference order on X.

U	x	y	z
a	3	1	8
b	3	2	1
c	1	4	1

P	a	b	c
	z	x	y
	x	y	x z
	y	z	

U	x	y	z
a	3	1	8
b	300	2	1
c	1	4	1

P	a	b	c
	z	x	y
	x	y	x z
	y	z	

U	x	y	z
a	3	1	8
b	300	2	1
c	1	400	1

P	a	b	c
	z	x	y
	x	y	x z
	y	z	

Von Neumann-Morgenstern Axioms

Let $\mathcal{L}$ be the set of lotteries on a set X, P a strict preference relation on $\mathcal{L}$, and I an indifference relation on $\mathcal{L}$.

Preference	preference over $\mathcal{L}$ are complete and transitive
Compound Lotteries	The decision maker is indifferent between every compound lottery and its *simplification*
Independence	For all $L_1, L_2, L_3 \in \text{Ł}$ and $a \in (0, 1]$, $L_1 \ P \ L_2$ iff $[L_1 : a,\ L_3 : (1-a)] \ P \ [L_2 : a,\ L_3 : (1-a)]$ $L_1 \ I \ L_2$ iff $[L_1 : a,\ L_3 : (1-a)] \ I \ [L_2 : a,\ L_3 : (1-a)]$
Continuity	For all $L_1, L_2, L_3 \in \text{Ł}$, if $L_1 \ P \ L_2 \ P \ L_3$, then there exists $a \in (0, 1)$ such that $[L_1 : a,\ L_3 : (1-a)] \ I \ L_2$

Von Neumann-Morgenstern Theorem

$u : \mathcal{L} \to \mathbb{R}$ is **linear** provided for all $L = [L_1 : p_1, \ldots, L_n : p_n] \in$ Ł,

$$u(L) = \sum_{i=1}^{n} p_i \times u(L_i)$$

Von Neumann-Morgenstern Representation Theorem A relation on the set $\mathcal{L}$ of lotteries satisfies Transitivity, Completeness, Compound Lotteries, Independence and Continuity if, and only if, the relation is represented by a **linear utility function**.

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Moreover, $u' : \mathcal{L} \to \mathbb{R}$ also represents the relation if, and only if, there are real numbers $\beta > 0$ and β such that for all lotteries $L \in \mathcal{L}$, $u'(\cdot) = \alpha + \beta u(\cdot)$. ("$u$ is unique up to linear transformations.")

Ordinal Invariance: Given two profiles $\boldsymbol{U}$ and $\boldsymbol{U}'$, let $\boldsymbol{U} \sim_{OM} \boldsymbol{U}'$ if for all $i \in V$ and $x, y \in X$, $\boldsymbol{U}_i(x) \geq \boldsymbol{U}_i(y)$ if and only if $\boldsymbol{U}'_i(x) \geq \boldsymbol{U}'_i(y)$.

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Cardinal Measurability: Given two profiles $\boldsymbol{U}$ and $\boldsymbol{U}'$, let $\boldsymbol{U} \sim_{CM} \boldsymbol{U}'$ if for all $i \in V$, there are $\alpha_i, \beta_i \in \mathbb{R}$ with $\beta_i > 0$ such that for all $x \in X$, $\boldsymbol{U}'_i(x) = \alpha_i + \beta_i \boldsymbol{U}_i(x)$.

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Cardinal Unit Comparability: Given two profiles $\boldsymbol{U}$ and $\boldsymbol{U}'$, let $\boldsymbol{U} \sim_{CUC} \boldsymbol{U}'$ if there is a $\beta \in \mathbb{R}$ with $\beta > 0$ such that for all $i \in V$, there are $\alpha_i \in \mathbb{R}$ such that for all $x \in X$, $\boldsymbol{U}'_i(x) = \alpha_i + \beta \boldsymbol{U}_i(x)$.

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To prove his impossibility theorem, Arrow assumed *OM* invariance (and Sen generalized it to *CM* invariance). Arrow's Theorem does not hold assuming Cardinal Unit Comparability.

Arrow's Epistemological Objection

> The viewpoint will be taken here that interpersonal comparison of utilities has no meaning and, in fact, that there is no meaning relevant to welfare comparisons in the measurability of individual utility...
>
> (Social Choice and Individual Values, p. 9)

Why not take interpersonally comparable utility functions as primitives?

"...the mythical character of any numbers behind preferences: they are just a construct in the observing mathematician's mind, and without any uniqueness property in addition. But if until Arrow this ordinalist position was almost the consensus, apparently his theorem itself, together with the very influential work of Harsanyi, turned the tide partially, and led to the conclusion that interpersonal comparability was a must to obtain SWF's.

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A. Dhillon and J.-F. Mertens. *Relative Utilitarianism*. Econometrica, Vol. 67, No. 3, pp. 471 - 498, 1999.

To summarize, we consider only preferences as empirically meaningful primitives, and hence axiomatize their aggregation coming thus back to the original formulation of social choice theory (Arrow 1963) as a guide to policy recommendations, not as the ultimate foundations of ethics.

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To summarize, we consider only preferences as empirically meaningful primitives, and hence axiomatize their aggregation coming thus back to the original formulation of social choice theory (Arrow 1963) as a guide to policy recommendations, not as the ultimate foundations of ethics. So a priori the question of interpersonal comparisons is not even meaningful in our framework. Following this approach requires that the set A consist of all feasible and just alternatives: the ethical question lies in the choice of A and it is this that will determine whatever implicit “interpersonal comparisons” occur. The questions of justice being accounted for by the choice of A, the choice within A is to be handled by an appropriate generalization of majority voting: the SWF. (Dhillon and Mertens, p. 477)

A. Dhillon and J.-F. Mertens. *Relative Utilitarianism*. Econometrica, Vol. 67, No. 3, pp. 471 - 498, 1999.

..in a large population satisfying certain statistical regularities, not only is the Condorcet winner almost guaranteed to exist, but it is almost guaranteed to also be the utilitarian social choice. So for such populations, Condorcet and Bentham agree. (p. 85-6)

M. Pivato. *Condorcet meets Bentham*. Journal of Mathematical Economics, 59, pp. 58 - 65, 2015.

- ✓ Aggregating utilities vs. aggregating grades
- Comparing different types of *scoring* rules (evaluative voting and positional scoring rules)

Characterizing *Scoring Rules*

Voting Methods

$$F : \mathcal{D} \to \mathcal{R}$$

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- What is the range? $\mathcal{R} = O(X)$ or $\mathcal{R} = \wp(X) - \emptyset$

- What is the domain? $\mathcal{D} = O(X)^n$, $\mathcal{D} = L(X)^n$ or $\mathcal{D} = \mathcal{B}^n$

- Do we focus on a fixed set of voters?

Variable Population Model

Let $\mathbb{N}$ be the set of "potential" voters.

Let $\mathcal{V} = \{V \mid V \subseteq \mathbb{N}$ and V is finite$\}$ be the set of all voting blocks.

For $V \in \mathcal{V}$, a **profile** for V is a function $\pi : V \to \mathcal{P}$ where $\mathcal{P}$ is $O(X)$, $L(X)$ or some set $\mathcal{B}$ of "ballots".

Let $\Pi^{\mathcal{P}}$ be the set of all profiles based on $\mathcal{P}$.

Variable Population Model

Two profiles $\pi : V \to \mathcal{P}$ and $\pi' : V' \to \mathcal{P}$ are *disjoint* if $V \cap V' = \emptyset$

If $\pi : V \to \mathcal{P}$ and $\pi' : V' \to \mathcal{P}$ are *disjoint*, then $(\pi+\pi') : (V \cup V') \to \mathcal{P}$ is the profile where for all $i \in V \cup V'$,

$$(\pi + \pi')(i) = \begin{cases} \pi(i) & \text{if } i \in V \\ \pi'(i) & \text{if } i \in V' \end{cases}$$

Variable population model: $F : \Pi^{\mathcal{B}} \rightarrow (\wp(X) - \emptyset)$ where $\mathcal{B}$ is some fixed set of "ballots".

Five characterization results:

1. Positional Scoring Functions ($\mathcal{B} = L(X)$)
2. Borda Count ($\mathcal{B} = L(X)$)
3. Approval Voting ($\mathcal{B} = \wp(X) - \emptyset$)
4. Plurality Rule ($\mathcal{B} = L(X)$)
5. Range Voting ($\mathcal{B} = \mathcal{S}$)

Positional Scoring Rules

$F : \Pi^{L(X)} \rightarrow (\wp(X) - \emptyset)$

Suppose there are m candidates. Let $\{s_1, \ldots, s_m\}$ be a set of scores,

$\boldsymbol{N}_\pi(j, A) = \{i \mid i \text{ ranks } A \text{ in the } j\text{th position}\}$ and
$Score_\pi(A) = \sum_{j=1}^{m} s_j \times |\boldsymbol{N}_\pi(j, A)|$

F is a scoring function if $F(\pi) = \{A \mid Score_\pi(A) \geq Score_\pi(B) \text{ for all } B \in X\}$

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F is a scoring function if $F(\pi) = \{A \mid Score_\pi(A) \geq Score_\pi(B)$ for all $B \in X\}$

Consistency: Suppose that π and π' are disjoint profiles. If $F(\pi) \cap F(\pi') \neq \emptyset$, then $F(\pi + \pi') = F(\pi) \cap F(\pi')$.

Positional Scoring Rules

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F is a scoring function if $F(\pi) = \{A \mid Score_\pi(A) \geq Score_\pi(B) \text{ for all } B \in X\}$

Overwhelming Majority: Suppose that a group of voters V elects a candidate A and a disjoint group of voters Y elects a different candidate B. Then there must be some number m such that the population consisting of the subgroup V together with m copies of Y will elect B.

Positional Scoring Rules

Theorem. A social choice correspondence $F : \Pi^{L(X)} \rightarrow (\wp(X) - \emptyset)$ satisfies anonymity, neutrality, consistency and overwhelming majority if and only if F is a scoring rule.

H. P. Young. *Social Choice Scoring Functions*. SIAM Journal of Applied Mathematics, 28, pgs. 824 - 838, 1975.

J. H. Smith. *Aggregation of Preferences with Variable Electorate*. Econometrica, 41, pgs. 1027 - 1041, 1973.

Borda Count

Borda Count is a scoring rule with scores $s_j = m - j$, where m is the number of candidates.

Cancellation: For all profiles π, if $|\boldsymbol{N}_\pi(A\ P\ B)| = |\boldsymbol{N}_\pi(B\ P\ A)|$ for all pairs of candidates $A, B \in X$, then $F(\pi) = X$.

Faithfulness: If there is only one voter $(V = \{i\})$ and $\pi : \{i\} \to L(X)$, then $F(\pi) = \{A\}$ where A is voter i's top-ranked candidate according to the linear order $\pi(i) \in L(X)$.

Borda Count

Theorem. A social choice correspondence $F : \Pi^{L(X)} \to (\wp(X) - \emptyset)$ satisfies anonymity, neutrality, consistency, cancellation and faithfulness if and only if F is Borda Count.

H. P. Young. *An axiomatization of Borda's rule*. Journal of Economic Theory, 9, pgs. 43 - 52, 1974.

S. Nitzan and A. Rubinstein. *A further characterization of Borda ranking method*. Public Choice, 36, pgs. 153 - 158, 1981.

Approval Voting

$F : \Pi^{\mathcal{B}} \rightarrow (\wp(X) - \emptyset)$, where $\mathcal{B} = \{B \subseteq X \mid B \neq \emptyset\}$.

Theorem. A social choice correspondence $F : \Pi^{\mathcal{B}} \rightarrow (\wp(X) - \emptyset)$ satisfies anonymity, neutrality, consistency*, cancellation* and faithfulness* iff F is approval voting.

C. Alós-Ferrer. *A simple characterization of approval voting*. Social Choice and Welfare, 27:3, pgs. 621 - 625, 2006.

P. Fishburn . *Axioms for Approval Voting: Direct Proof*. Journal of Economic Theory, 19, pgs. 180 - 185, 1978.

Plurality Rule

$F : \Pi^{L(X)} \to (\wp(X) - \emptyset)$

For a linear ordering $R \in L(X)$, $T(R)$ is the top ranked candidate.

For a profile π, let $T(\pi) = (T(\pi_i))_{i \in V}$, where V is the domain of π.

For $A \in X$, let $Top_\pi(A) = \{i \mid T(\pi(i)) = A\}$.

F is plurality rule if $F(\pi) = \{A \mid |Top(A)| \geq |Top(B)|$ for all $B \in X\}$

Tops-Only: For all profiles π and π' on the same set V of voters, if $T(\pi) = T(\pi')$ then $F(\pi) = F(\pi')$

Plurality Rule

Theorem. A social choice correspondence $F : \Pi^{L(X)} \rightarrow (\wp(X) - \emptyset)$ satisfies anonymity, neutrality, consistency, faithfulness and tops-only if and only if F is plurality rule.

Y. Sekiguchi. *A Characterization of Plurality Rule*. Economic Letters, 116:3, pgs. 330-332, 2012.

S. Ching. *A Simple Characterization of Plurality Rule*. Journal of Economic Theory, 71, pgs. 298 - 302, 1996.

Range Voting

$F : \Pi^{\mathcal{S}} \to (\wp(X) - \emptyset)$, where $\mathcal{S}$ is a (possibly infinite) set of 'signals'.

F is **at least as expressive as** G if the voters can express any profiles of opinions via F which they could have expressed via F.

F **admits minority overrides** if regardless of the size of the set of voters in a profile and the weight of existing public opinion, a single voter can always cast a vote that changes the outcome.

Range Voting

Theorem. Range voting is the most expressive voting rule which satisfies consistency, neutrality, over-whelming majority and does not admit minority overrides.

M. Pivato. *Formal utilitarianism and range voting*. Mathematical Social Sciences, 2013.

Additional Readings

V. Merlin. *The Axiomatic Characterizations of Majority Voting and Scoring Rules*. Mathematical Social Sciences, 161, pp. 81 - 109, 2003.

M. Pivato. *Variable-population voting rules*. Journal of Mathematical Economics, 49, pp. 210 - 221, 2013.

60	40
a	b
b	a

Majority decision: a

E.g., a group of friends meet every week to go to lunch at either Cava or Seoul Spice. Assuming that the preferences do not change, if more than $1/2$ of them prefer Cava, then they will go to Cava every week.

60	40
a	b
b	a

Majority decision: a

E.g., a group of friends meet every week to go to lunch at either Cava or Seoul Spice. Assuming that the preferences do not change, if more than $1/2$ of them prefer Cava, then they will go to Cava every week.

There are many ways to solve this problem:

1. Probabilistic voting rules: e.g., each week choose with the lottery $[a : 60, b : 40]$
2. Fair division: If they meet 10 times, then 6 times go to Cava and 4 times go to Seoul Spice.

60	40
a	b
b	a

60	40
a	b
b	a

60	40
a	b
b	a

60	40
a	b
b	a

$\cdots$

From Static to Dynamic Choices

Suppose there are two agents, and we can choose an alternative that gives one a reward of 3, and the other a reward of 0; or vice versa; or an alternative that gives each of them 1.

Within a round, the last alternative maximizes **Nash welfare** (the product of the utilities).

J. Nash (1950. *The bargaining problem*. Econometrica, 18(2), pp. 155 - 162.

From Static to Dynamic Choices

If this scenario is repeated every round, then it would be better to alternate between the first two alternatives, so that each agent obtains 1.5 per round on average.

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Of course, initially, say in the first round, we may not realize we will have these options every round, and so we may choose the last alternative; but if we do have these options every round, we should eventually catch on to this pattern and start alternating.

From Static to Dynamic Choices

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Of course, initially, say in the first round, we may not realize we will have these options every round, and so we may choose the last alternative; but if we do have these options every round, we should eventually catch on to this pattern and start alternating.

Ideally, we would maximize the long-term Nash welfare, that is, the product of the long-run utilities (which are the sums of each agent's rewards), rather than, for example, the sum of the products within the rounds.

R. Freeman, S. Maijid Zahedi, and V. Conitzer. *Fair and Efficient Social Choice in Dynamic Settings*. Proceedings of IJCAI 2017.

60	40
a_1	b_1
b_1	a_1

Winner: a_1

65	35
a_2	b_2
b_2	a_2

Winner: a_2

45	55
a_3	b_3
b_3	a_3

Winner: b_3

30	70
a_4	b_4
b_4	a_4

Winner: b_4

$\cdots$

E.g., a group of friends are planning a party and meet and different times to decide when to have the party, what catering to use, how much money to spend on the catering, what drinks to get, etc.

60	40
a_1	b_1
b_1	a_1

Winner: a_1

65	35
a_2	b_2
b_2	a_2

Winner: a_2

45	55
a_3	b_3
b_3	a_3

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Winner: b_4

$\cdots$

E.g., a group of friends are planning a party and meet and different times to decide when to have the party, what catering to use, how much money to spend on the catering, what drinks to get, etc.

- A sequence of different decision to be made by a fixed set of voters for which inclusion and participation is desirable.
- Can we guarantee a fair share of the decision power for each voter?
- Not a good model for large-scale elections and/or when there are extreme or dangerous opinions from the voters.

Fix a set of voters $N = \{1, \ldots, n\}$

A decision instance is a tuple (N, A, C) where

- C is a set of alternatives
- $A = (A(1), \ldots, A(n))$ is an approval profile, where for all $v \in N$, $A(v) \subseteq C$

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k-decision history: $(N, \overline{A}, \overline{C}, \overline{w})$, where $(N, \overline{A}, \overline{C})$ is a k-decision sequence and for each $i \in \{1, \ldots, k\}$, $w_i \in C_i$ is the alternative chosen in round i.

Perpetual Voting Rule

Input:

- k decision instances together with chosen alternatives $w_1, \ldots, w_k$
- A decision instance (N, A_{k+1}, C_{k+1}) for round $k+1$.

Output: a winning alternative w_{k+1}.

Satisfaction

Given an outcome $\overline{w} = (w_1, \ldots, w_k)$, the **satisfaction** of voter v with $\overline{w}$ is the number of decisions that she approved of:

$$sat_k(v, \overline{w}) = |\{i \mid 1 \leq i \leq k \text{ and } w_i \in A_i(v)\}|$$

Weighted Approval Method

A rule is a weighted approval method if

- Voter v has weight $\alpha_k(v)$ in round k
- Initially, weights are $\alpha_1(v) = 1$ for all $v \in N$
- There exists a weight function h such that for all $v \in N$, $\alpha_{k+1}(v) = h(v, \mathcal{H})$
- The rule selects an alternative $w_{k+1} \in C_{k+1}$ with maximal weighted approval score.

Win-Based Weighted Approval Method

A rule is a weighted approval method if

- Voter v has weight $\alpha_k(v)$ in round k
- Initially, weights are $\alpha_1(v) = 1$ for all $v \in N$
- There exists a non-increasing function g such that for all $v \in N$,

$$\alpha_{k+1}(v) = \begin{cases} g(\alpha_k(v)) & \text{if } w \in A_k(v) \\ \alpha_k(v) & \text{if } w \notin A_k(v) \end{cases}$$

- The rule selects an alternative $w_{k+1} \in C_{k+1}$ with maximal weighted approval score.

Perpetual AV:

$$\alpha_{k+1}(v) = \alpha_k(v) = 1$$

Perpetual Unit Cost:

$$\alpha_{k+1}(v) = \begin{cases} \alpha_k(v) + 1 & \text{if } w_k \notin A_k(v) \\ \alpha_k(v) & \text{if } w_k \in A_k(v) \end{cases}$$

Perpetual Reset:

$$\alpha_{k+1}(v) = \begin{cases} \alpha_k(v) + 1 & \text{if } w_k \notin A_k(v) \\ 1 & \text{if } w_k \in A_k(v) \end{cases}$$

Example: Perpetual AV

Suppose that $N = \{1, 2, 3, 4\}$, the same four alternatives $\{a, b, c, d\}$ are used in each round, and alphabetic tie-breaking is used.

	1	2	3	4	winner
weights	1	1	1	1	
A_1	$\{a\}$	$\{a\}$	$\{b\}$	$\{c, d\}$	a
weights	1	1	1	1	
A_2	$\{a\}$	$\{a, b, c\}$	$\{d\}$	$\{c\}$	a
weights	1	1	1	1	
A_3	$\{a\}$	$\{b\}$	$\{a, c\}$	$\{b\}$	a
weights	1	1	1	1	
A_3	$\dots$				

Example: Perpetual Unit Cost

Suppose that $N = \{1, 2, 3, 4\}$, the same four alternatives $\{a, b, c, d\}$ are used in each round, and alphabetic tie-breaking is used.

	1	2	3	4	winner
weights	1	1	1	1	
A_1	$\{a\}$	$\{a\}$	$\{b\}$	$\{c, d\}$	a
weights	1	1	2	2	
A_2	$\{a\}$	$\{a, b, c\}$	$\{d\}$	$\{c\}$	c
weights	2	1	3	2	
A_3	$\{a\}$	$\{b\}$	$\{a, c\}$	$\{b\}$	a
weights	2	2	3	3	
A_3	$\cdots$				

Example: Perpetual Reset

Suppose that $N = \{1, 2, 3, 4\}$, the same four alternatives $\{a, b, c, d\}$ are used in each round, and alphabetic tie-breaking is used.

	1	2	3	4	winner
weights	1	1	1	1	
A_1	$\{a\}$	$\{a\}$	$\{b\}$	$\{c, d\}$	a
weights	1	1	2	2	
A_2	$\{a\}$	$\{a, b, c\}$	$\{d\}$	$\{c\}$	c
weights	2	1	3	1	
A_3	$\{a\}$	$\{b\}$	$\{a, c\}$	$\{b\}$	a
weights	1	2	1	2	
A_3	$\cdots$				

Perpetual PAV:

$$\alpha_{k+1}(v) = \frac{1}{sat_k(v, \overline{w}) + 1} = \begin{cases} \alpha_k(v) & \text{if } w_k \notin A_k(v) \\ \frac{\alpha_k(v)}{\alpha_k(v)+1} & \text{if } w_k \in A_k(v) \end{cases}$$

Example: Perpetual PAV

Suppose that $N = \{1, 2, 3, 4\}$, the same four alternatives $\{a, b, c, d\}$ are used in each round, and alphabetic tie-breaking is used.

	1	2	3	4	winner
weights	1	1	1	1	
A_1	$\{a\}$	$\{a\}$	$\{b\}$	$\{c, d\}$	a
weights	$\frac{1}{2}$	$\frac{1}{2}$	1	1	
A_2	$\{a\}$	$\{a, b, c\}$	$\{d\}$	$\{c\}$	c
weights	$\frac{1}{2}$	$\frac{1}{3}$	1	$\frac{1}{2}$	
A_3	$\{a\}$	$\{b\}$	$\{a, c\}$	$\{b\}$	a
weights	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	
A_3	$\cdots$				

Perpetual Consensus:

- The weights of satisfied voters is reduced by a total of $n = |N|$ split equally among them.
- Some voters may have negative weights. Voters with negative weights are not taken into account when determining the winning alternative.
- After each decision, the weight of all voters is increased by 1.

$$N_k^+(c) = \{v \in N \mid c \in A_k(v) \text{ and } \alpha_k(v) > 0\}$$

$$\alpha_{k+1}(v) = \alpha_k(v) + 1 = \begin{cases} \alpha_k(v) & \text{if } v \notin N_k^+(w_k) \\ \alpha_k(v) + 1 - \frac{n}{|N_k^+(w_k)|} & \text{if } v \in N_k^+(w_k) \end{cases}$$

Example: Perpetual Consensus

Suppose that $N = \{1, 2, 3, 4\}$, the same four alternatives $\{a, b, c, d\}$ are used in each round, and alphabetic tie-breaking is used.

	1	2	3	4	winner
weights	1	1	1	1	
A_1	$\{a\}$	$\{a\}$	$\{b\}$	$\{c, d\}$	a
weights	0	0	2	2	
A_2	$\{a\}$	$\{a, b, c\}$	$\{d\}$	$\{c\}$	c
weights	1	1	3	-1	
A_3	$\{a\}$	$\{b\}$	$\{a, c\}$	$\{b\}$	a
weights	0	2	2	0	
A_3	$\cdots$				

Perpetual Nash: Maximize Nash welfare, i.e., the product of voters' utilities, where the voters' utility is their satisfaction if their satisfaction is larger than 0; voters with a satisfaction of 0 have a utility of some small constant, e.g., $\epsilon = n^{-n}$

$$u_{k+1}(v, c) = \begin{cases} \max(sat_k(v, \overline{w}), \epsilon) & \text{if } c \notin A_{k+1}(v) \\ sat_k(v, \overline{w}) + 1 & \text{if } c \in A_{k+1}(v) \end{cases}$$

The Nash score of c is $nash_{k+1}(c) = \Pi_{v \in N} u_{k+1}(v, c)$. The alternative with the maximum Nash score is chosen.

Support, Quota

The support of a voter $v \in N$ in round j is defined as

$$supp_j(v) = \frac{1}{n} \max_{c \in A_j(v)} |\{u \in V \mid A_j(u) = c\}|$$

The quota of voter $v \in N$ in round j is

$$qu_j(v) = \sum_{i \leq j} supp_i(v)$$

The support of a voter in round j is the proportion of voters that can collectively agree on some alternative that v approves. The quota of voter v in round j is v's cumulative support from round 1 to j.

Simple Proportionality

We say that a k-decision sequence $(N, \overline{A}, \overline{C})$ is simple if $A_1 = \cdots = A_k$, $C_1 = \cdots = C_k$ and $|A_1(v)| = 1$ for all $v \in N$.

For any simple n-decision sequence $(N, \overline{A}, \overline{C})$ with $|N| = n$, we say that $\overline{w} \in \overline{C}$ is proportional if $sat_n(v, \overline{w}) = qu_n(v)$ for every $v \in N$.

A perpetual voting rule satisfies simple proportionality if for any simple $|N|$-decision sequence $(N, \overline{A}, \overline{C})$, the rule is proportional.

Proposition 1. AV, Perpetual Equality, Perpetual Reset, and Perpetual Unit-Cost fail simple proportionality.

Theorem 1. Perpetual PAV, Perpetual Consensus, Perpetual Nash, and Perpetual Quota satisfy simple proportionality.

Independent of Uncontroversial Decisions

An approval profile A is **uncontroversial due to** c if $\bigcap_{v \in N} A(v) = \{c\}$.

Given a k-tuple $L = (l_1, \ldots, l_k)$ and $i \in \{0, \ldots, k\}$, let $L \oplus_i x$ be the $(k+1)$-tuple $(l_1, \ldots, l_i, x, l_{i+1}, \ldots, l_k)$.

A perpetual voting rule $\mathcal{R}$ is *independent of uncontroversial decisions* if for any k-decision sequence $(N, \overline{A}, \overline{C})$, approval profile A for C that is uncontroversial due to c, and $i \in \{0, \ldots, k\}$, it holds that

$$\mathcal{R}(N, \overline{A} \oplus_i A, \overline{C} \oplus_i C) = \mathcal{R}(N, \overline{A}, \overline{C}) \oplus_i c$$

Proposition 2. Perpetual PAV, Perpetual Nash, and Perpetual Reset fail independence of uncontroversial decisions.

Theorem 2. AV, Perpetual Equality, Perpetual Quota, Perpetual Unit-Cost, and Perpetual Consensus satisfy independence of uncontroversial decisions

Dry Spells

Given a k-decision history $(N, \overline{A}, \overline{C}, \overline{w})$, we say that a voter $v \in N$ has a **dry spell of length** ℓ if there exists $t \leq k - \ell$ such that $sat_t(v, \overline{w}) = sat_{t+\ell}(v, \overline{w})$, i.e., voter v is not satisfied by any choice in rounds $t+1, \ldots, t+\ell$.

Let $d : \mathbb{N} \to \mathbb{N}$. A perpetual voting rule has a **dry spell guarantee of** d if, for any decision sequence, no voter has a dry spell of length $d(|N|)$.

A perpetual voting rule $\mathcal{R}$ has **bounded dry spells** if $\mathcal{R}$ has a dry spell guarantee of some d.

Proposition 3. AV, Perpetual PAV, Perpetual Equality, Perpetual Quota, Perpetual Nash, and Perpetual Unit-Cost have unbounded dry spells.

Theorem 4. Perpetual Consensus has a dry spell guarantee of at least $\frac{n^2+3n}{4}$.

Perpetual Lower Quota

A measure to which degree voters receive at least a "fair share" of favorable choices, and thus have a fair say in the decision process.

Let $(N, \overline{A}, \overline{C})$ be a k-decision sequence. A k-choice sequence $\overline{w} \in \overline{C}$ satisfies **perpetual lower quota** if for every voter $v \in N$, it holds that

$$sat_k(v, \overline{w}) \geq \lfloor qu_k(v) \rfloor$$

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$$sat_k(v, \overline{w}) \geq \lfloor qu_k(v) \rfloor$$

Proposition 4. There are decision sequences for which no choice sequence exists that satisfies perpetual lower quota.

	1	2	3	$\cdots$	2^n
A_1	$\{0\}$	$\{0\}$	$\{0\}$	$\cdots$	$\{1\}$
A_2	$\{0\}$	$\{1\}$	$\{0\}$	$\cdots$	$\{1\}$
A_3	$\{0\}$	$\{0\}$	$\{1\}$	$\cdots$	$\{1\}$
$\vdots$					
A_n	$\{0\}$	$\{0\}$	$\{0\}$	$\cdots$	$\{1\}$

- The outcome is a 0-1 sequence of length n.
- Each voter agrees with $\frac{n}{2}$ other voters
- For each possible outcome, there is a voter that has satisfaction 0.

Let $(N, \overline{A}, \overline{C}, \overline{w})$ be a k-decision history. The **perpetual lower quota compliance** of $\overline{w}$, $compl(\overline{w})$ is the average proportion of voters in each round that have their perpetual lower quota satisfied:

$$compl(\overline{w}) = \frac{1}{nk} \sum_{i=1}^{k} |\{v \in N \mid sat_i(v, \overline{w}) \geq \lfloor qu_i(v) \rfloor\}|$$

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- Generate voters and alternatives in a two-dimensional Euclidean space:
  - Voters are split in two groups and are placed on the $2d$ plane by a bivariate normal distribution. For the first group (6 voters) both x- and y-coordinates are independently drawn from $\mathcal{N}(-0.5, 0.2)$; for the second group (14 voters) for the second group (14 voters) x- and y-coordinates are independently drawn from $\mathcal{N}(0.5, 0.2)$.

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  - Voters approve all alternatives that have a distance of at most 1.5 times the distance to the closest alternative. This yields approval sets of size 1.8 on average.
- Results are based on $10,000$ instances. For each instance and each voting rule, we compute the perpetual lower quota compliance.

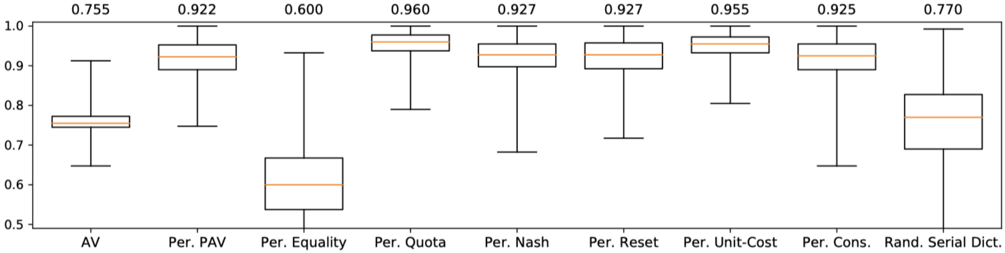
Random Serial Dictatorship (RSD)

In each round, a permutation of voters is selected uniformly at random.

We maintain a set X that starts as the set of all alternatives (in this round). One voter after the other can shrink X further to include only approved alternatives; the set X remains unchanged by voters whose approval set has an empty intersection with X.

As soon as X has cardinality 1, this alternative is chosen. If $|X| > 1$ after all voters are considered, one alternative in X is chosen at random.

Figure 1: Perpetual lower quota compliance (values on top of the diagram are medians)



Gini Coefficient

The **Gini coefficient** is a metric of inequality (often used for income distributions); it is 0 for completely equal distributions and 1 for maximally unequal distributions. We use the Gini coefficient to capture inequality in voters' influence on the decision process.

We define the influence of a voter on a given choice as 1 divided by the number of voters supporting this choice. For example, a voter has an influence of 1 on a choice that everyone but her disagrees with; if a choice is supported by all n agents, then their (individual) influence is $\frac{1}{n}$.

Gini Influence Coefficient

Let $(N, \overline{A}, \overline{C}, \overline{w})$ be a k-decision history. The influence of voter $v \in N$ on the choice sequence $\overline{w}$ is

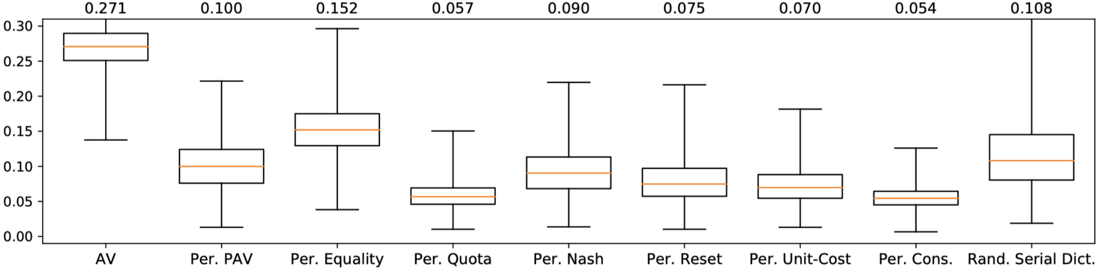
$$inf_k(v, \overline{w}) = \sum_{j \in [k]} \frac{\mathbb{I}_{w_j \in A_j(v)}}{|\{u \in V \mid A_j(u) = w_j\}|}$$

Let a be the average influence $a = \frac{1}{|N|} \sum_{v \in N} infl_k(v)$

The **Gini influence coefficient** of $\overline{w}$ is defined as the Gini coefficient of the sequence $(infl_k(v, \overline{w}))_{v \in N}$.

$$gini_k(\overline{w}) = \frac{1}{2a|N|^2} \sum_{u \in N} \sum_{v \in N} |infl_k(u, \overline{w}) - infl_k(v, \overline{w})|$$

Figure 2: Gini influence coefficient (values on top of the diagram are medians)



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3. In a long-term decision process, compromise becomes a powerful concept. For example, if agents assign different importance to individual decisions, compromise can be found by deciding in favor of agents that consider the issue at hand critical, while assigning a higher priority for future decisions to agents that "yielded".

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4. Agents can strategize by controlling the agenda. The outcome will differ if the order of the decision problems faced by the voters is changed.

Related Work

P. Harrenstein, M.-L. Lackner and M. Lackner. *A Mathematical Analysis of an Election System Proposed by Gottlob Frege*. Erkenntnis, 2020.

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