

CMSC423: Bioinformatic Algorithms, Databases and Tools

Exact string matching:
The Z algorithm

- Recap: Naïve string matching algorithm runs in $O(m n)$
 $m = \text{len}(\text{pattern})$, $n = \text{len}(\text{text})$
- Stop and think: What inefficiencies can you notice in the naïve algorithm?

Intuition

```
T = CCCCCCCCCCCCCC
    ||||X
P = CCCCCG
    ||||X
    CCCCCG
```

Naïve algorithm re-computes information it should already know

Specifically, once the Cs match at the first location, we should know they will match after a 1-letter shift

Can we capture the self-similarity of the string to help matching?

Quantifying self-similarity

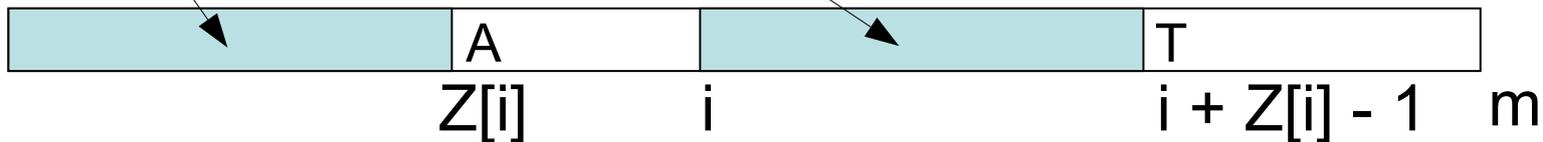
the Z algorithm (Gusfield)

For a string/text T

$Z[i]$ = length of the longest prefix of $T[i..m]$ that matches a prefix of T .
 $Z[i] = 0$ if the prefixes don't match. $Z[0] = 0$ (by definition)

$$T[0 .. Z[i]] == T[i .. i+Z[i] - 1]$$

Shaded areas usually called "Z-boxes"



Quick aside: off-by-1 errors

- Is it i , or $i + 1$?
- Is a range inclusive or exclusive?
- Do coordinates start at 0 or 1?

- These are common issues that arise when implementing string algorithms.
- It is important to carefully trace an example on paper when writing your code. It will save you hours of debugging.



AGGTCCTAGGCGCT

$i = 7$

$Z[i] = 3$

Are the numbers in the diagram above correct, or off by 1?

Example Z values

Please write the Z values below each character in this string.

ACAGGTACAGTTCCCTCGACACCTACTACCTAAG

Example Z values

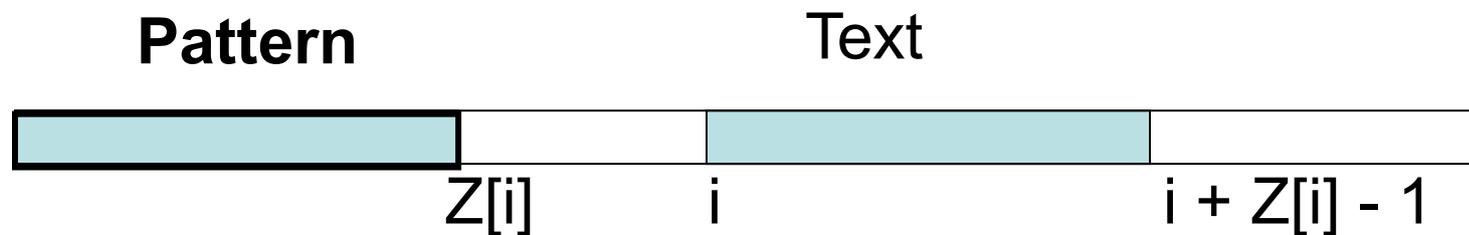
ACAGGTACAGTTCCCTCGACACCTACTACCTAAG
0010004010000000003020002002000110

Stop and Think!

- We only talked about one string
- If you are given a pattern P and a text T , can you use the Z values to find out if the pattern matches the text, and where?

Can the Z values help in matching?

Create string `Pattern$Text` where `$` is not in the alphabet



If there exists i , such that $Z[i] = \text{length}(\text{Pattern})$
Pattern occurs in the Text starting at i

Stop and think: Assuming Z values are computed. What is the runtime?

Example

```
CCTACT$ACAGGTACAGTTCCTCGACACCTACTACCTAAG
01001000100000100002310100106100100410000
```

runtime = $O(m + n)$ (simply scan an array to find the matches)

- Stop and think! What is the largest Z value possible?

NEXT: Can you compute the Z values efficiently?